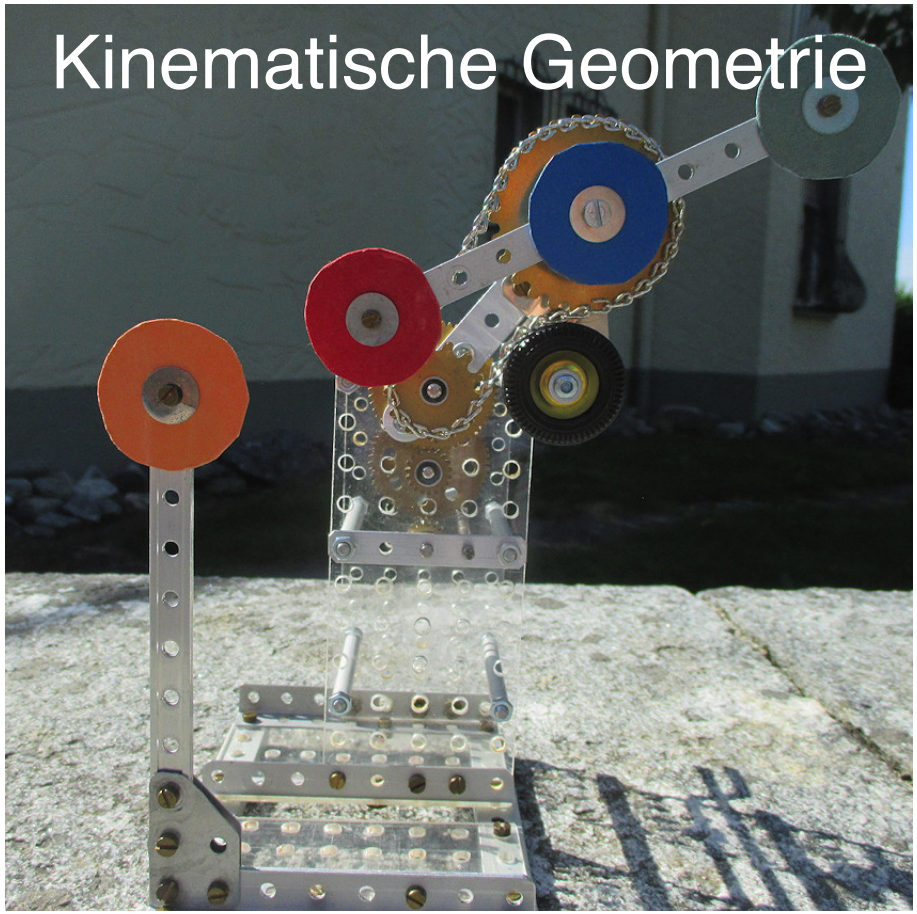
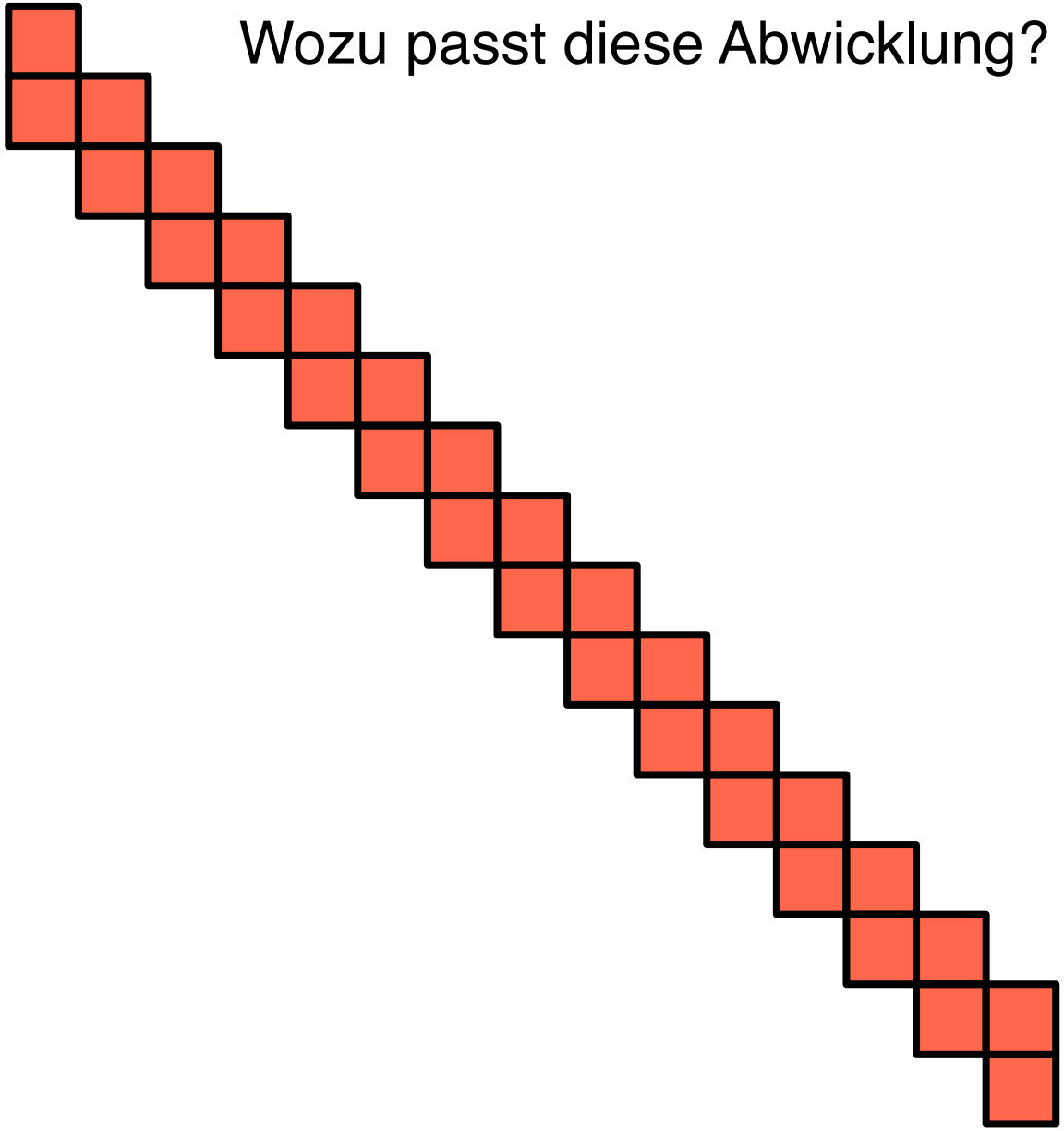
Hans Walser

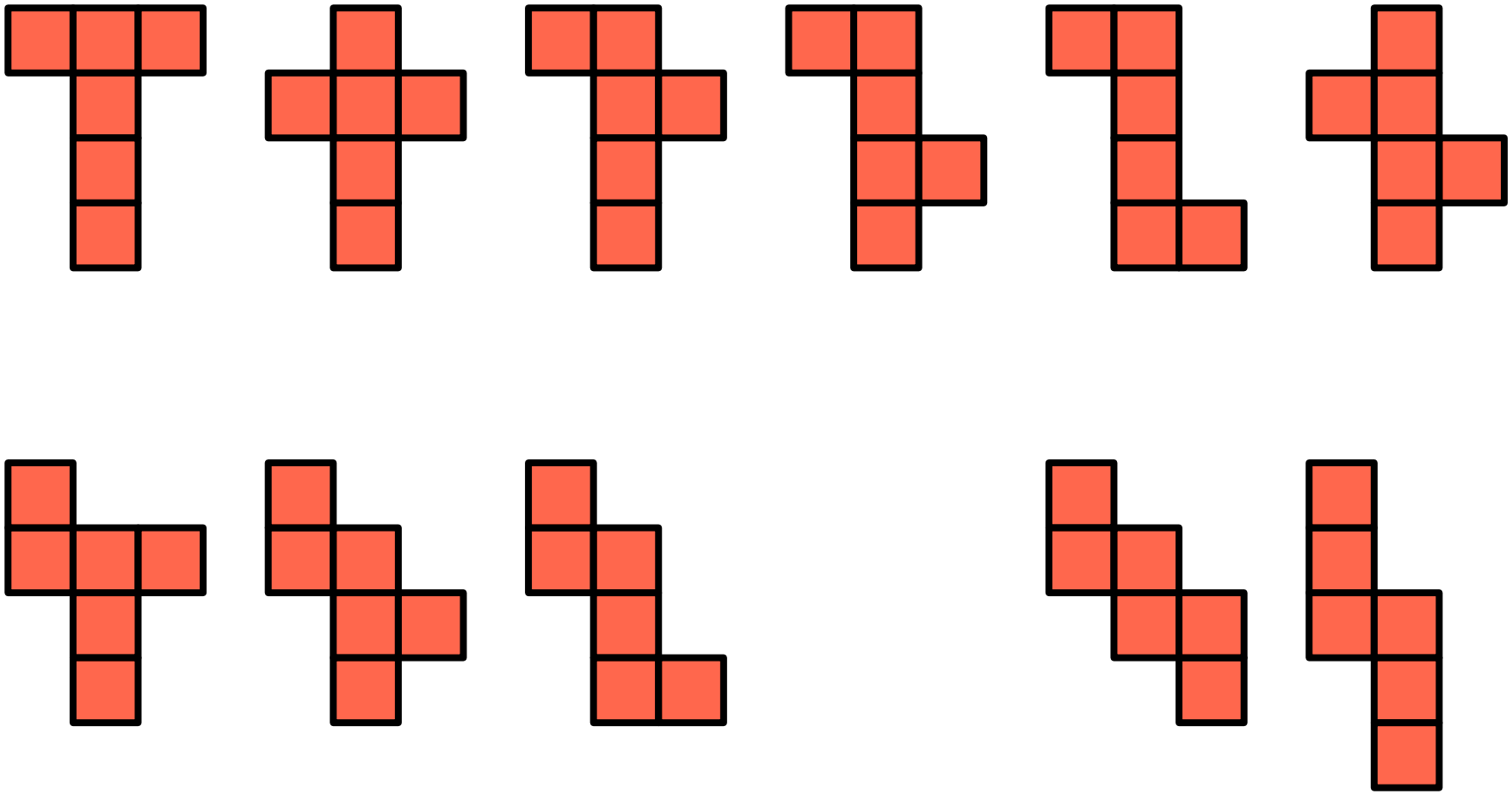


www.walser-h-m.ch/hans/Vortraege/20200117

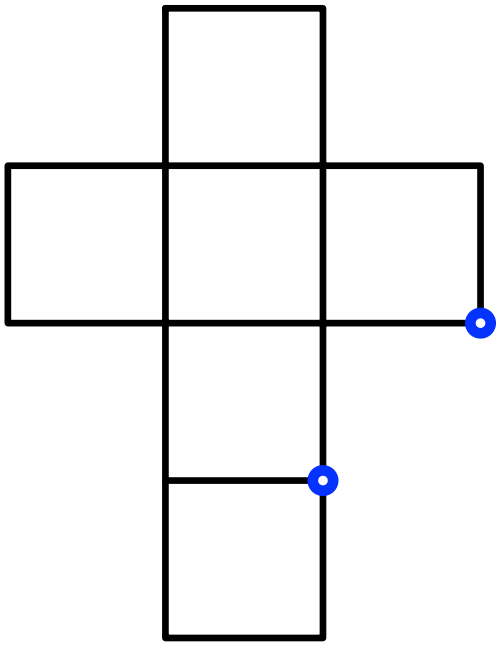
Wozu passt diese Abwicklung? (Unfolding)

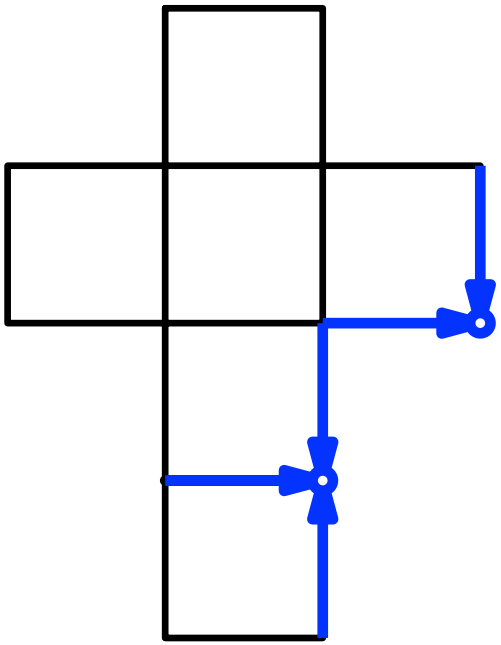


Wozu passen diese Abwicklungen? (Unfoldings)



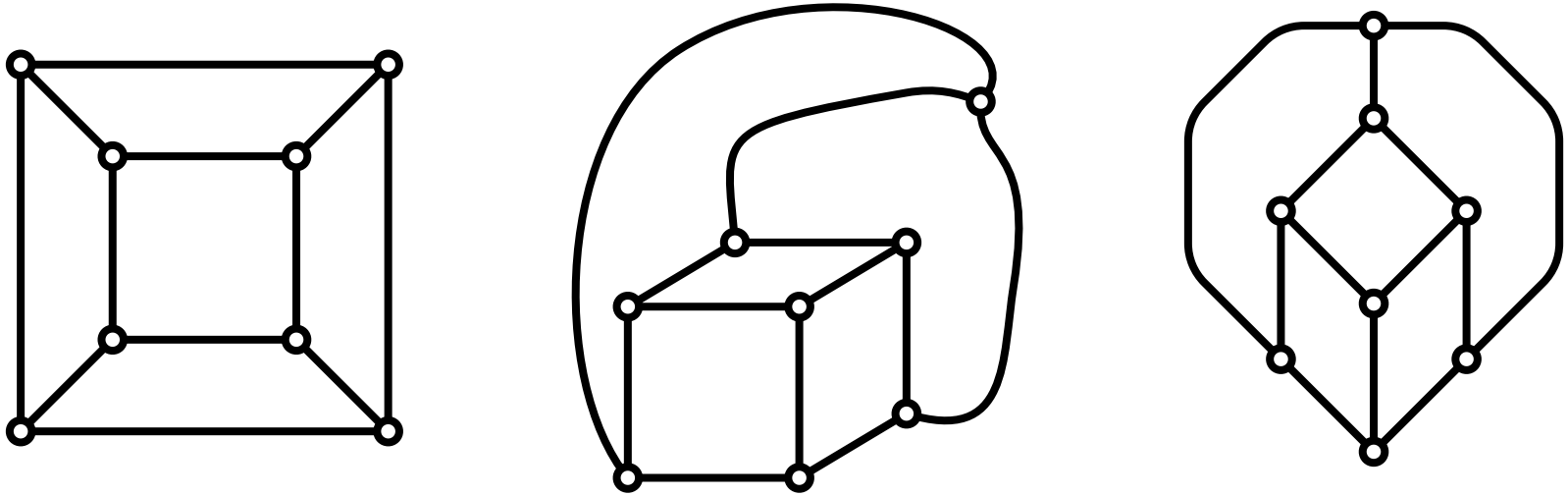
Wozu passen diese Abwicklungen? (Unfoldings)





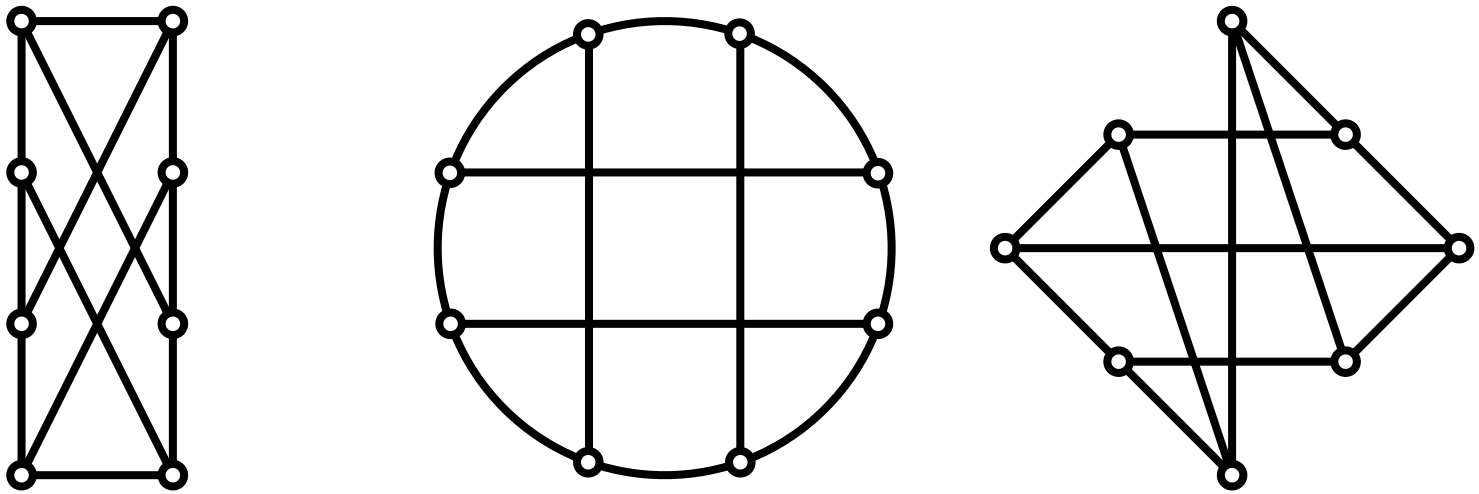
Netz: Knoten und Verbindungen

Würfelnetze



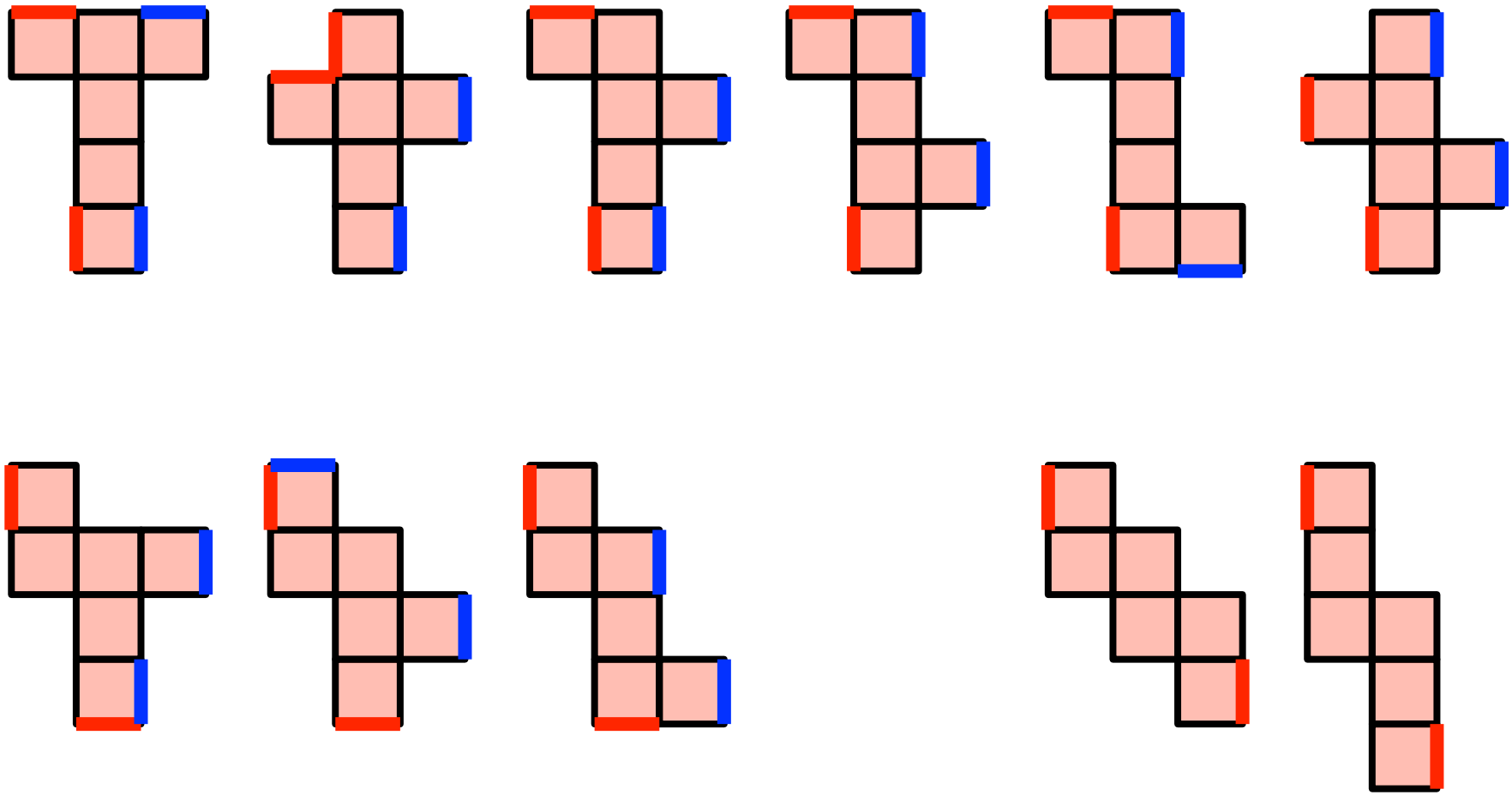
Netz: Knoten und Verbindungen

Würfelnetze mit Überschneidungen



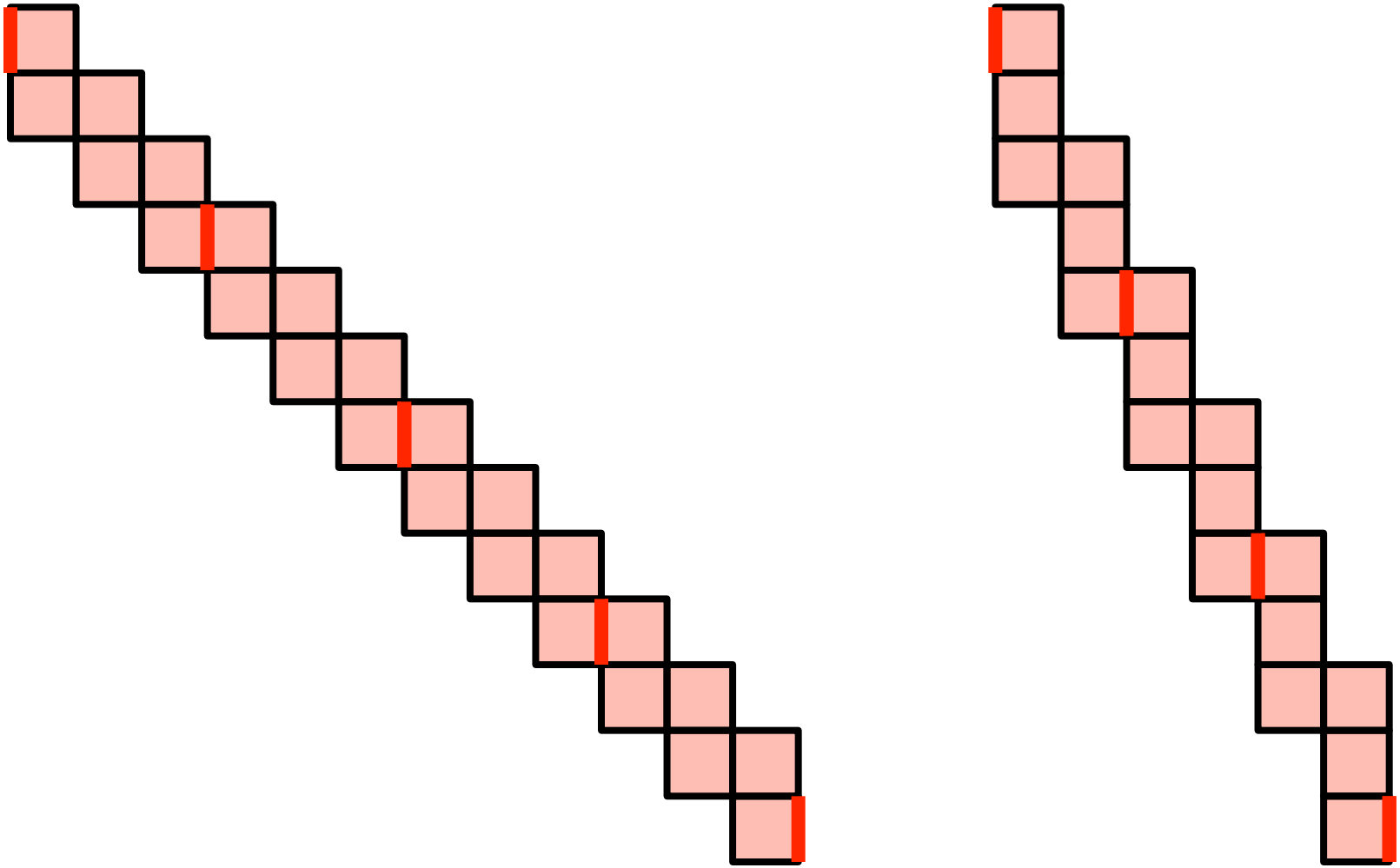
Goldstone, Richard and Suzzi Valli Robert (2019):
Unfoldings oft he Cube. The College Mathematics Journal.
Vol. 50, No. 3, May 2019, 173-184.

Minimal benötigte Klebelaschen

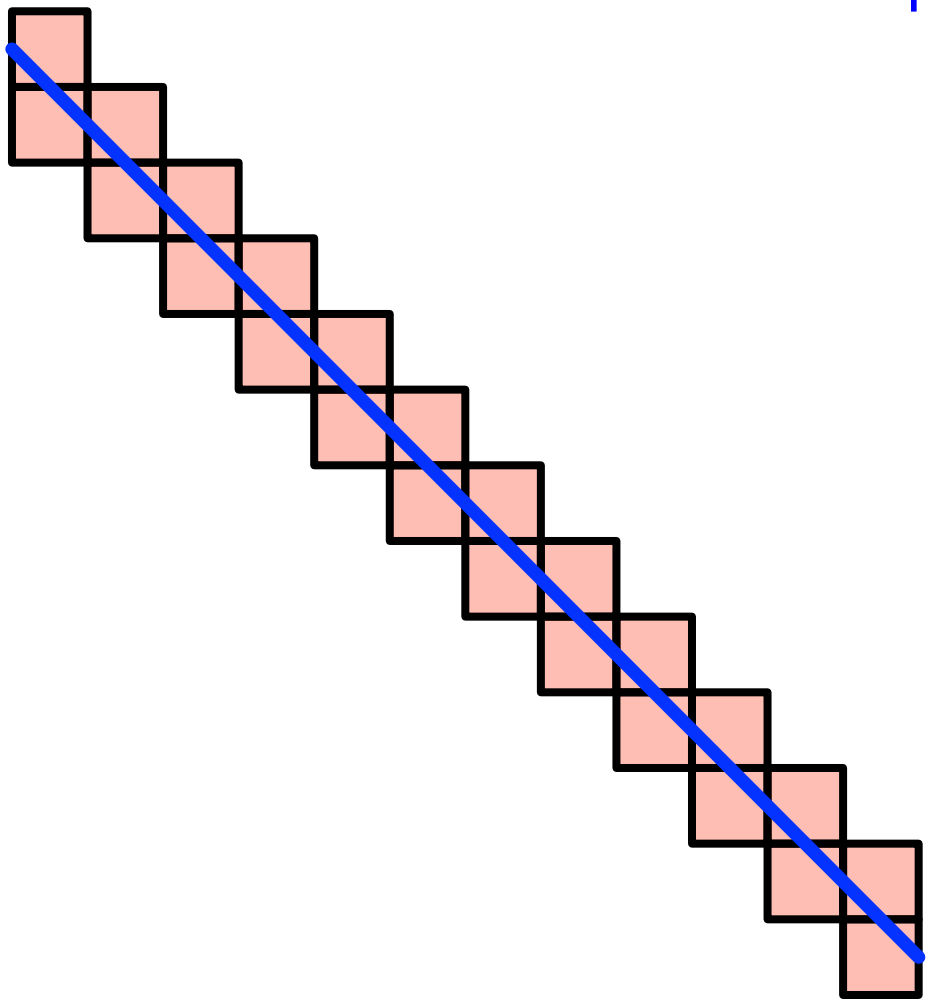


Blechmodell, Fahnenwürfel

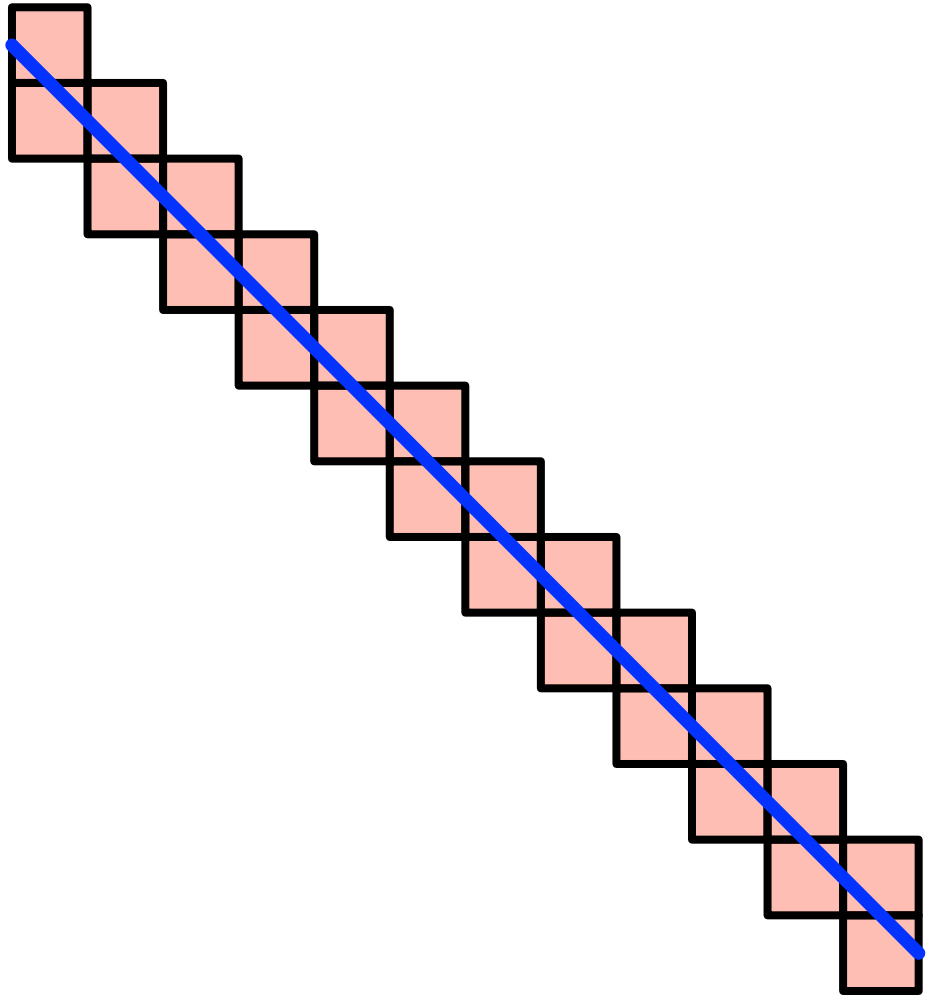
Bandornamente

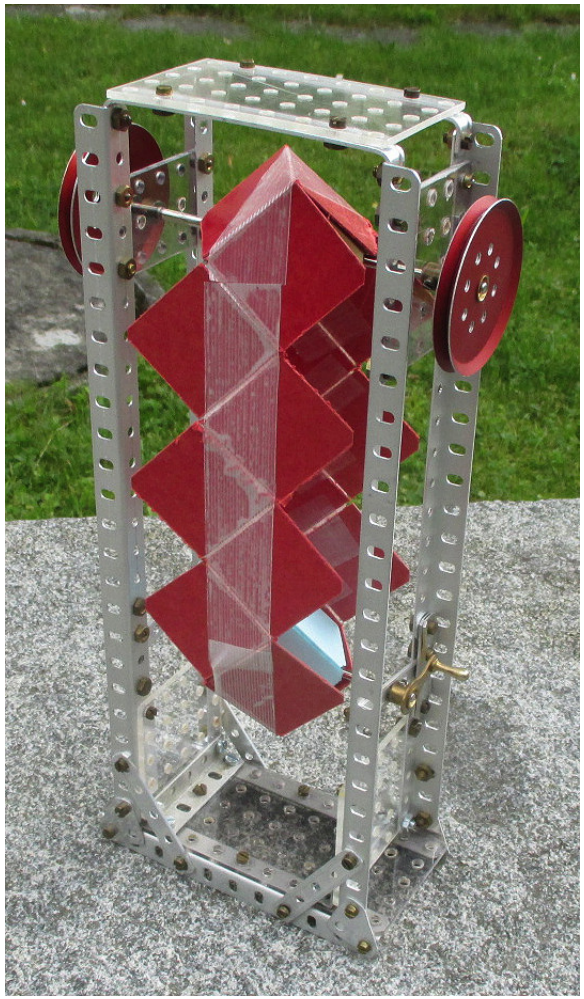


Bandornament
mit Schubspiegelsymmetrie

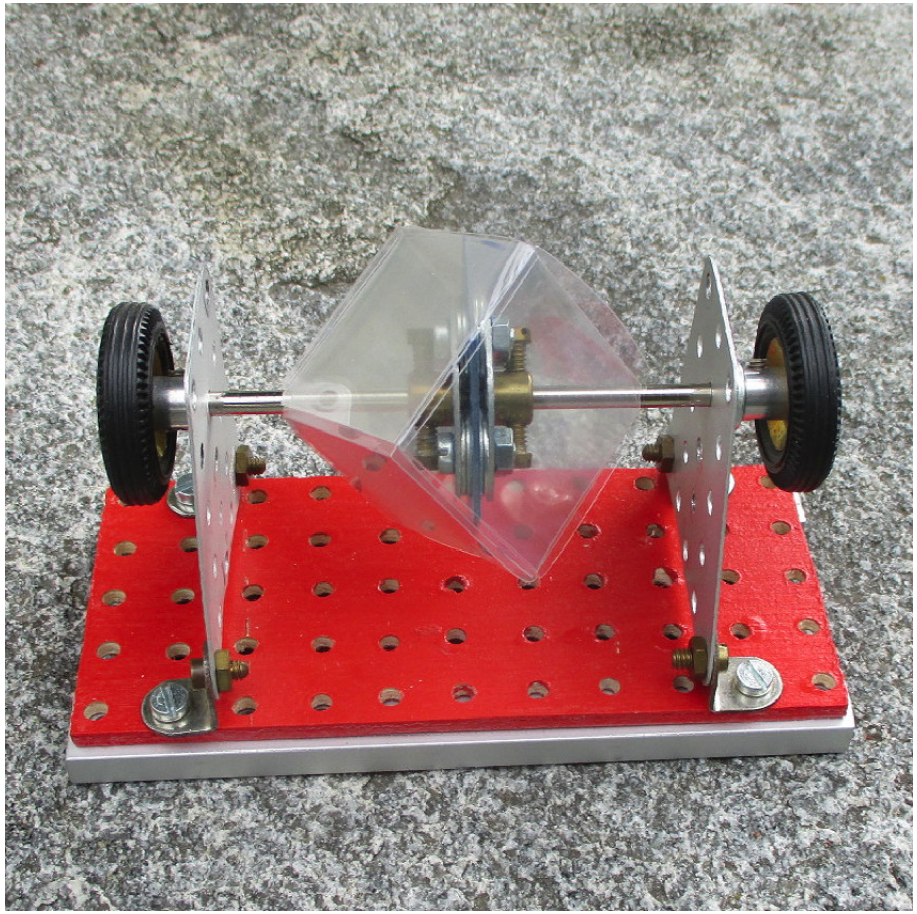


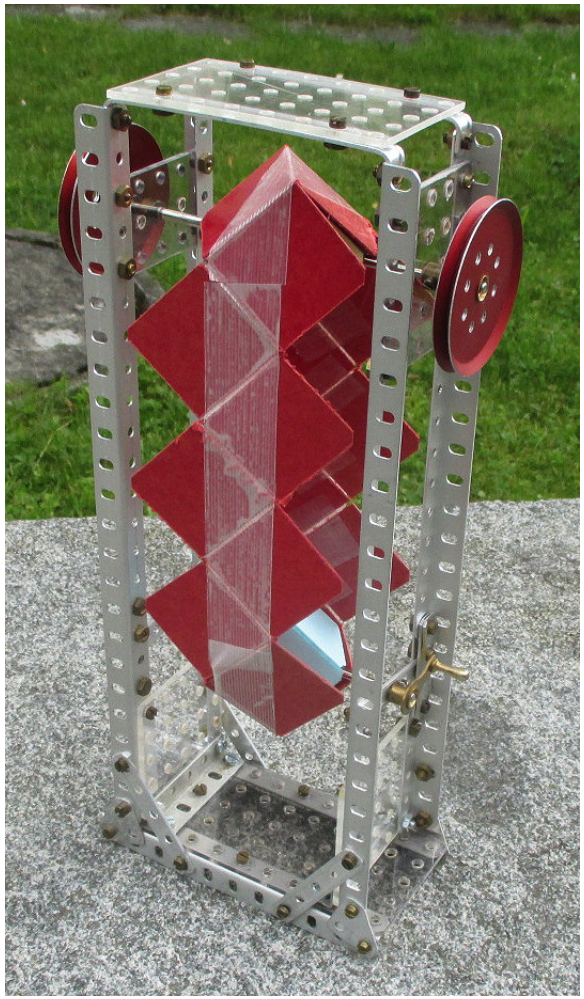
Abwicklung als Transmissionskette





Würfel als Zahnrad





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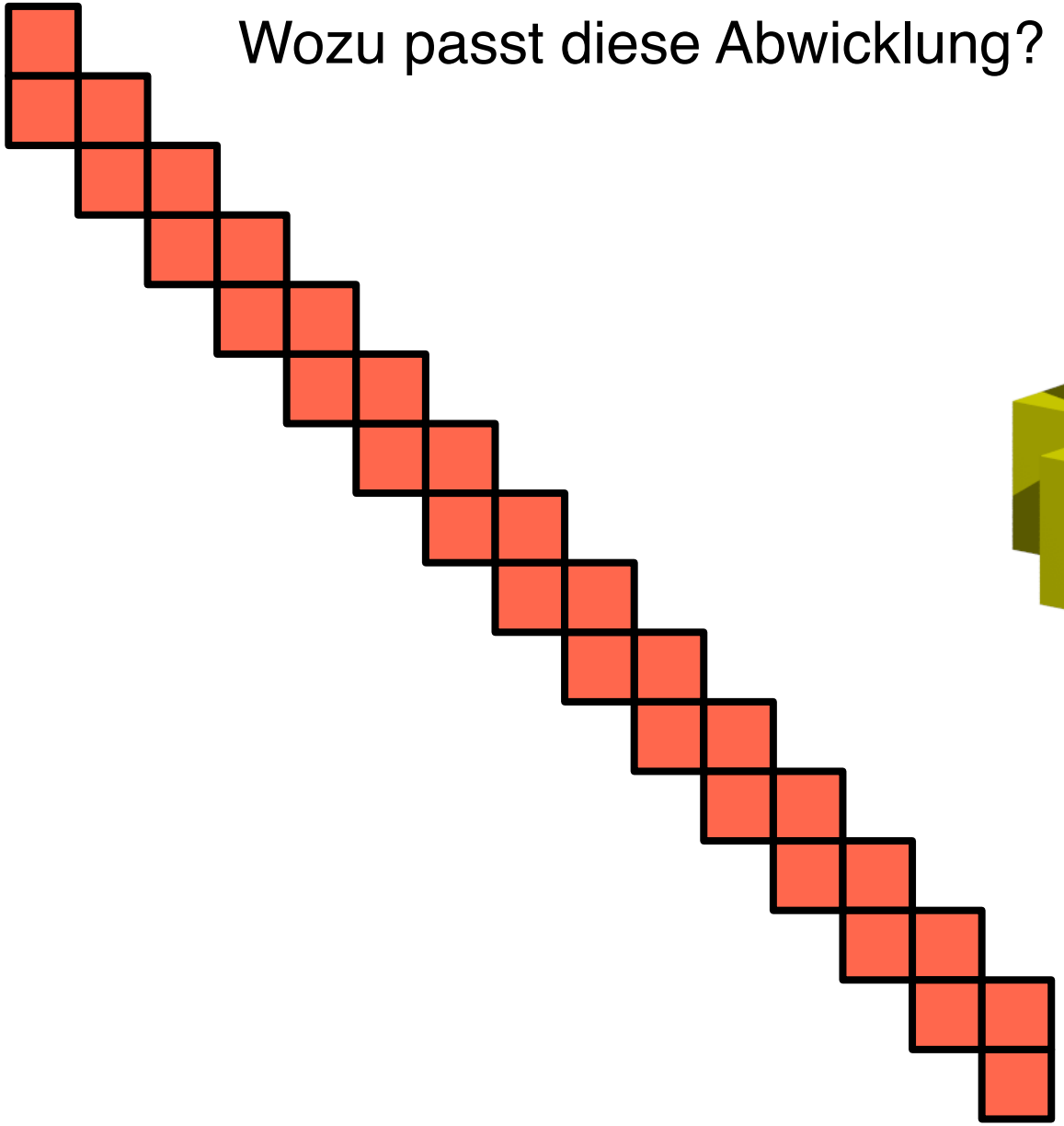
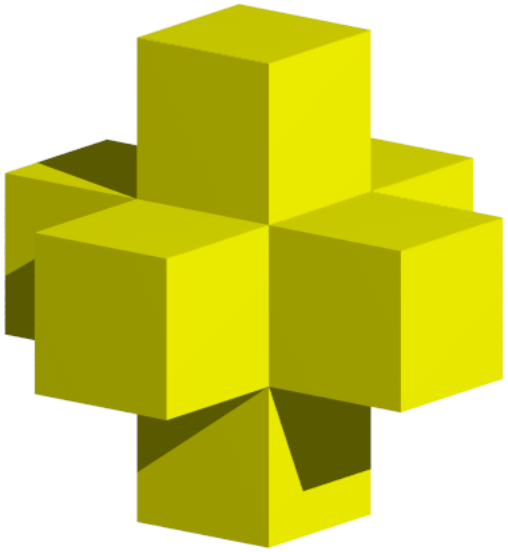
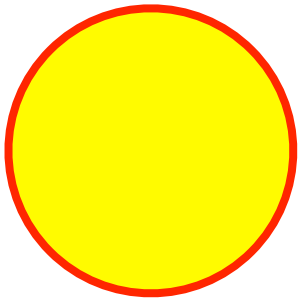




3d-Kreuz

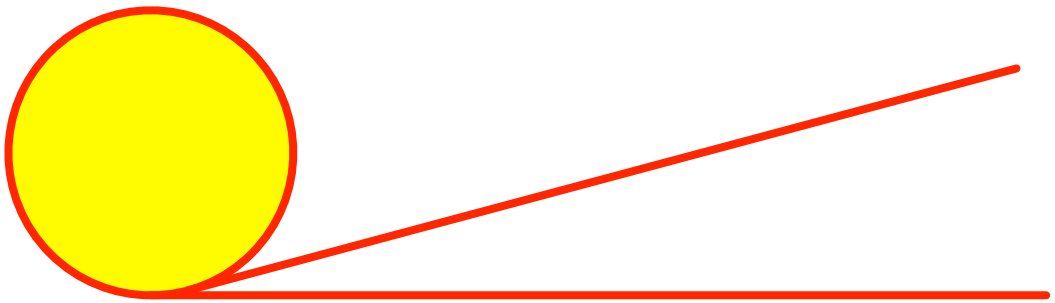
Modell

Aufwickeln zum Kreis



Straff gespannt

Straff gespannt



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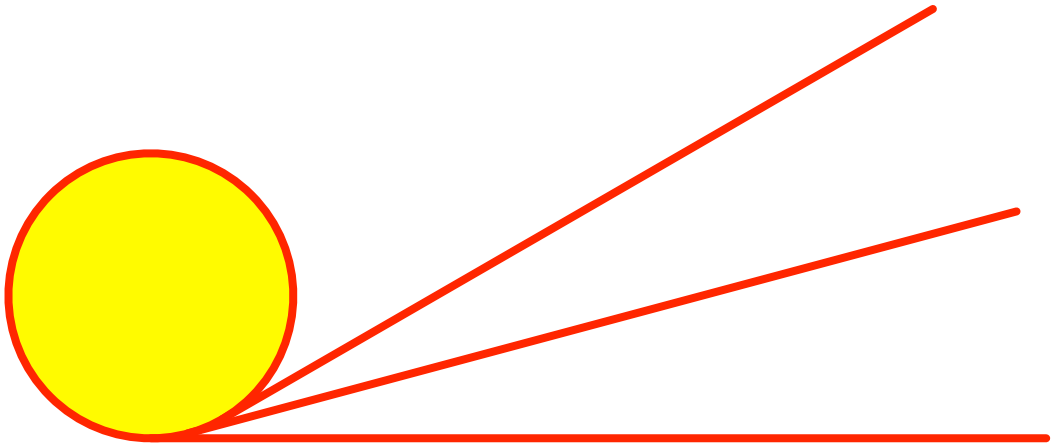

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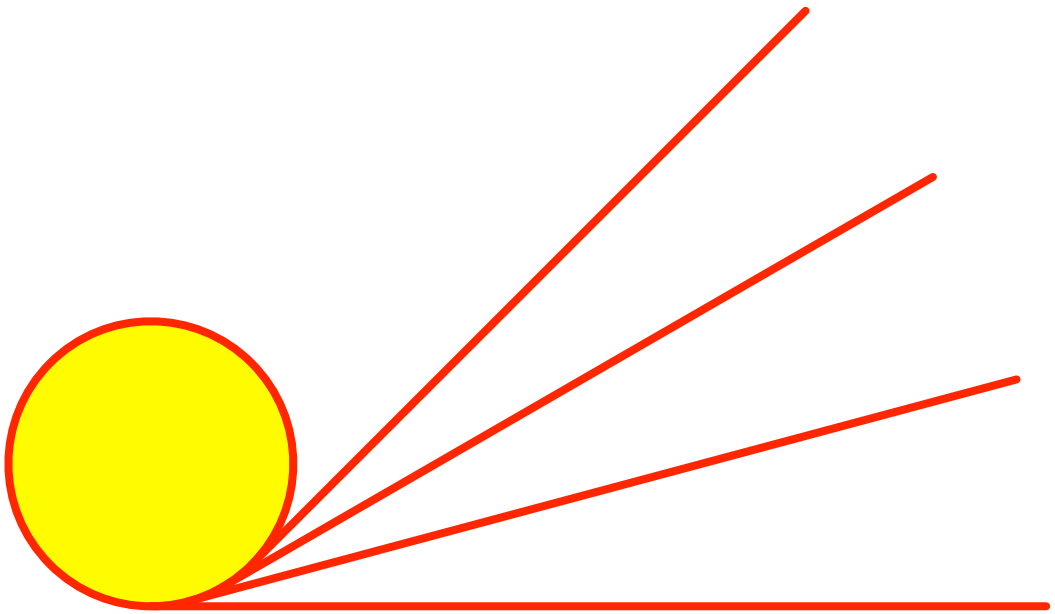

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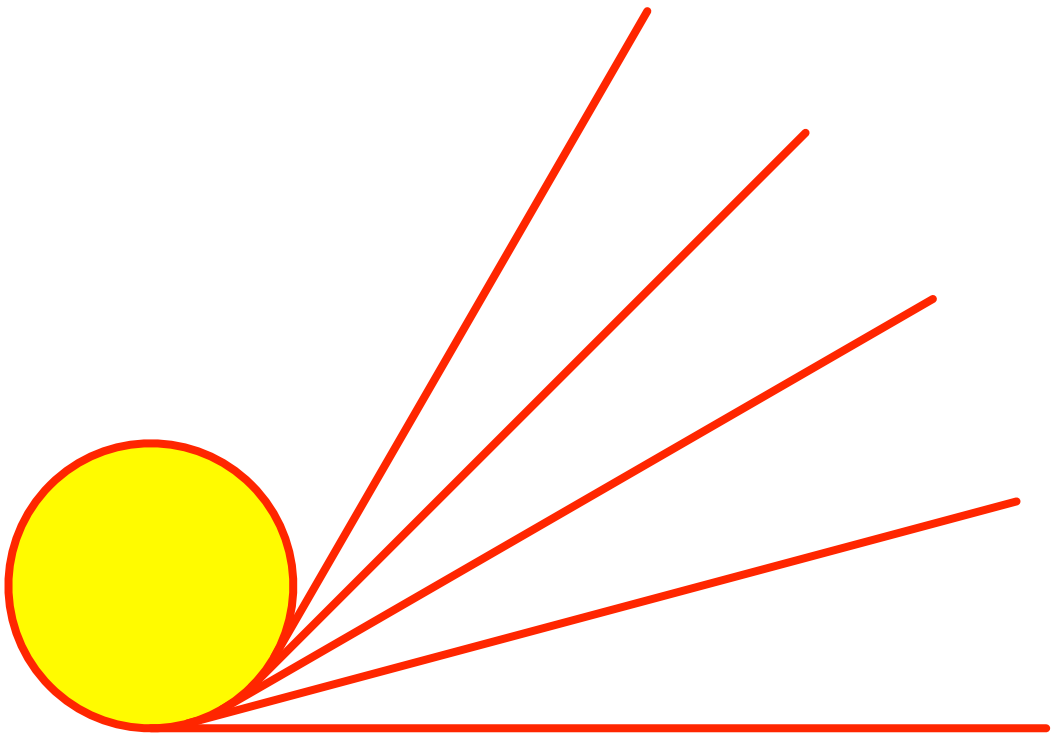

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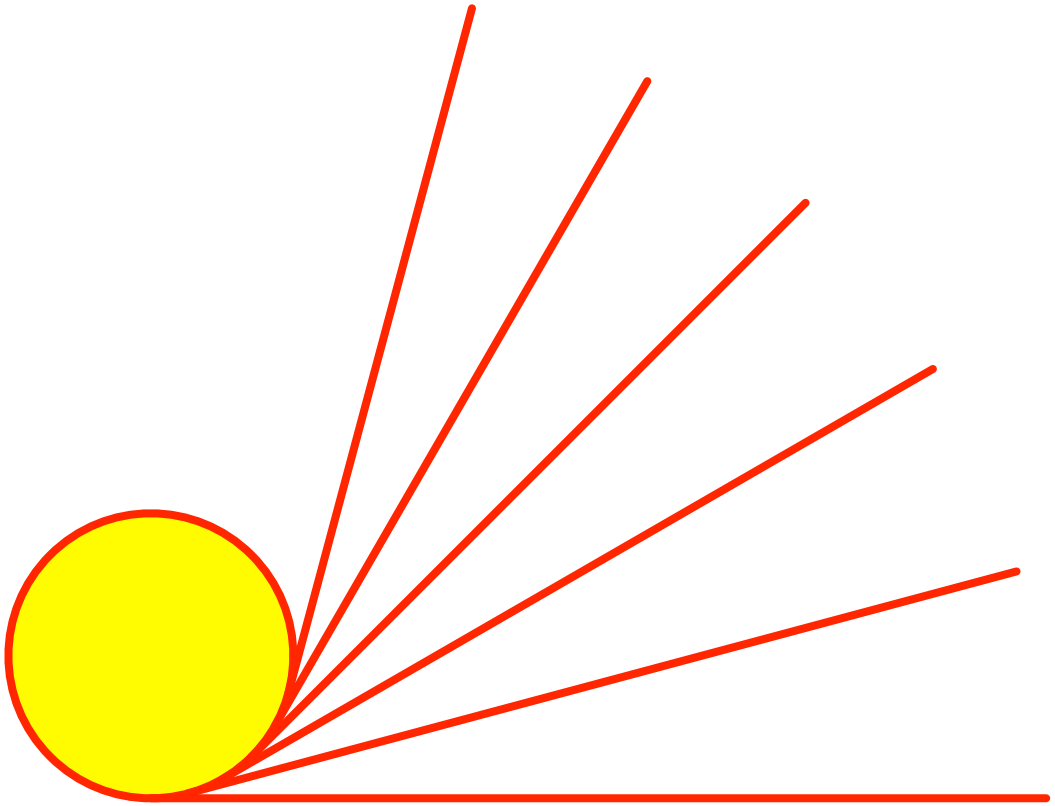

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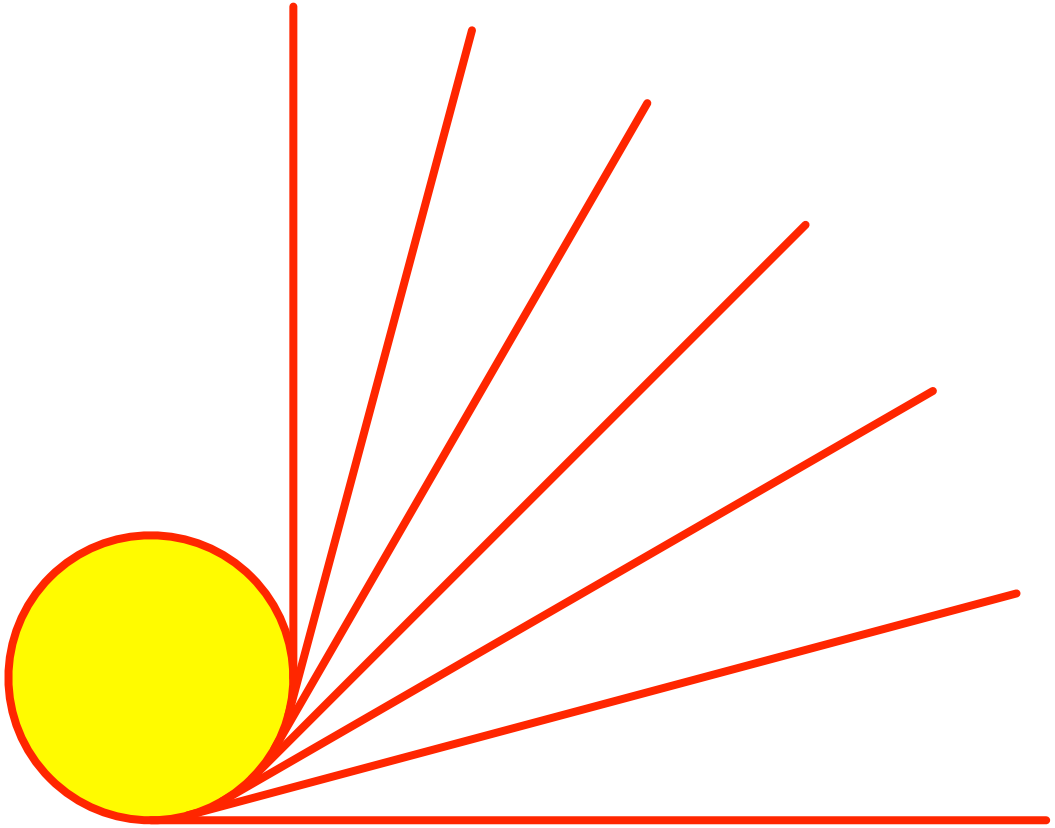

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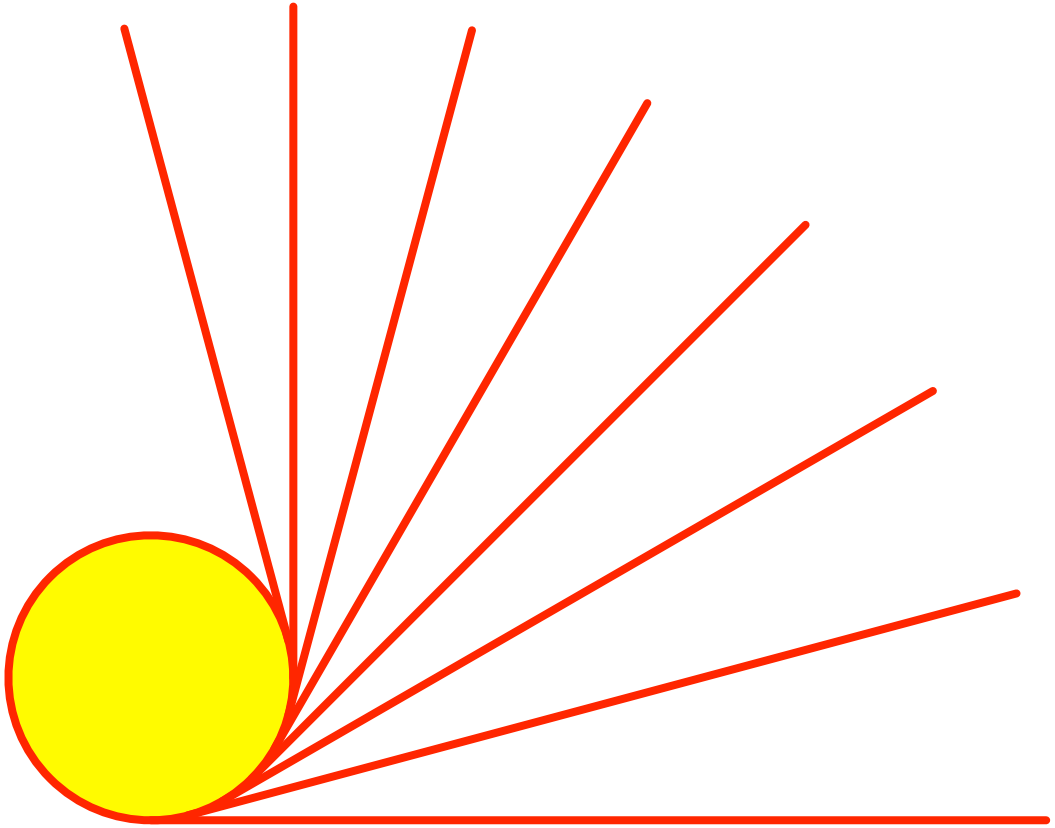

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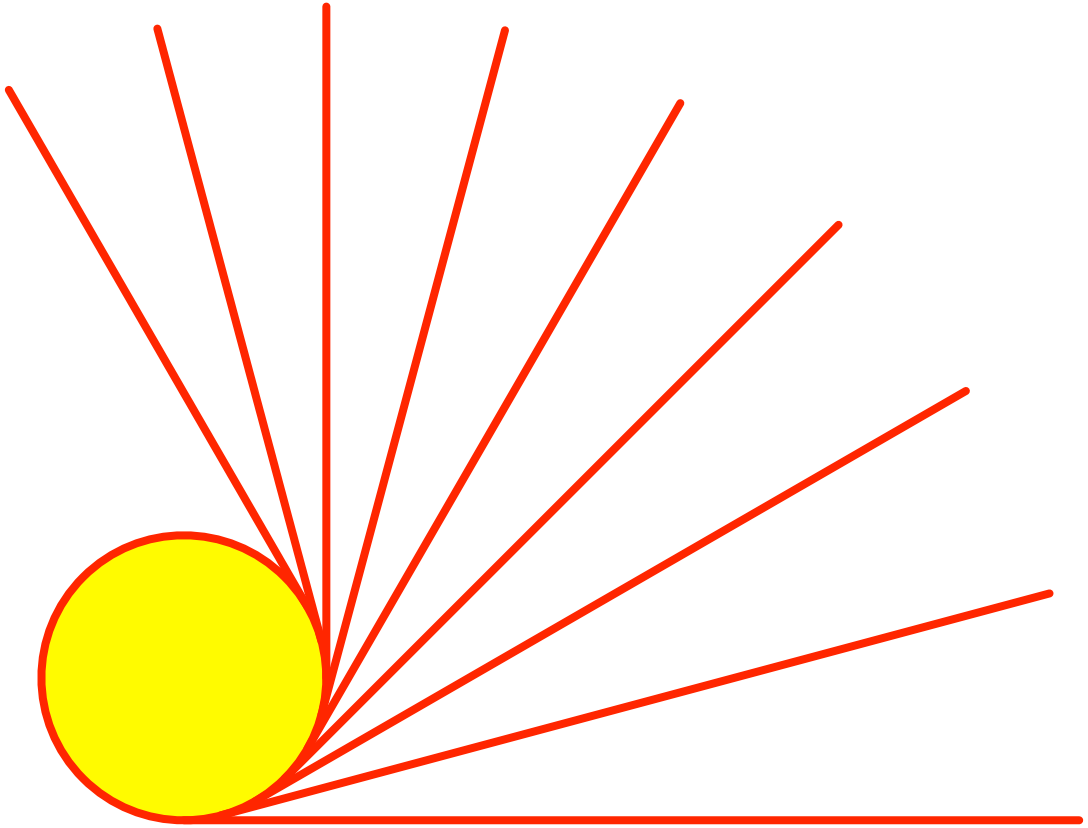

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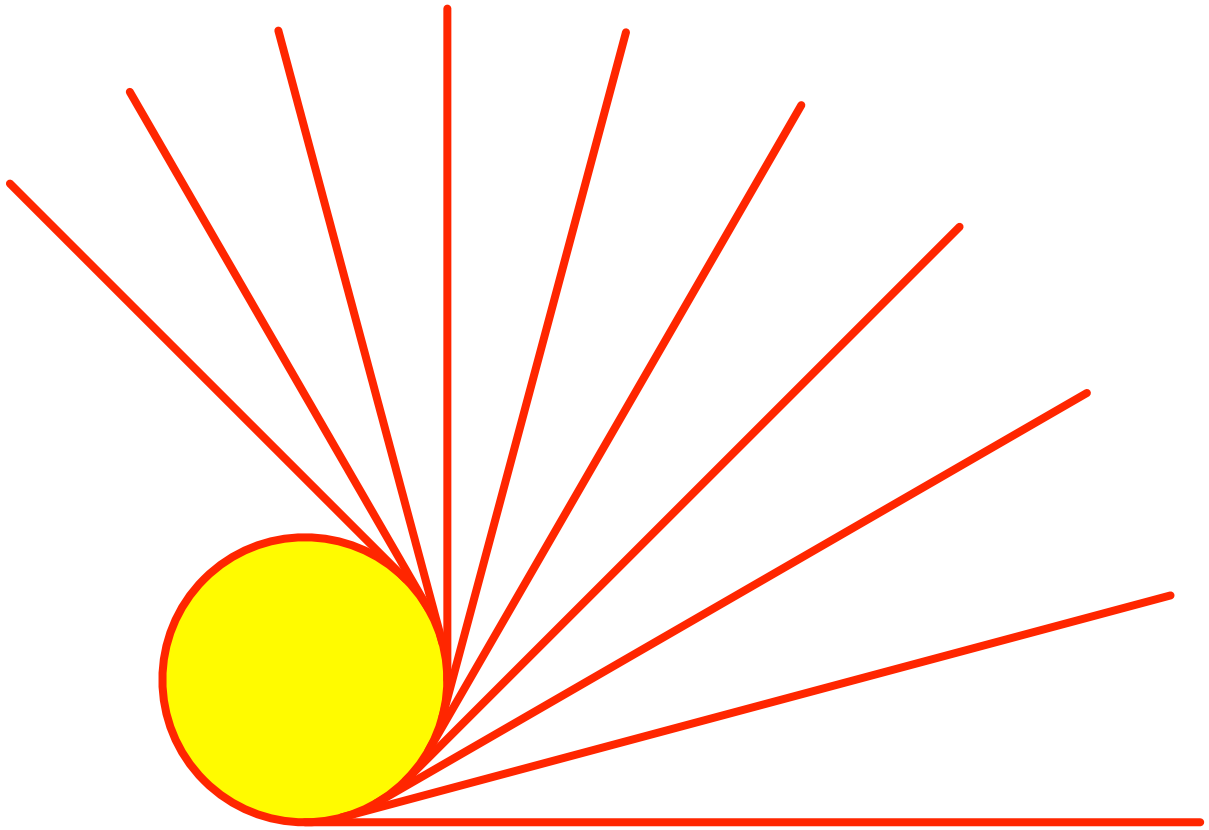
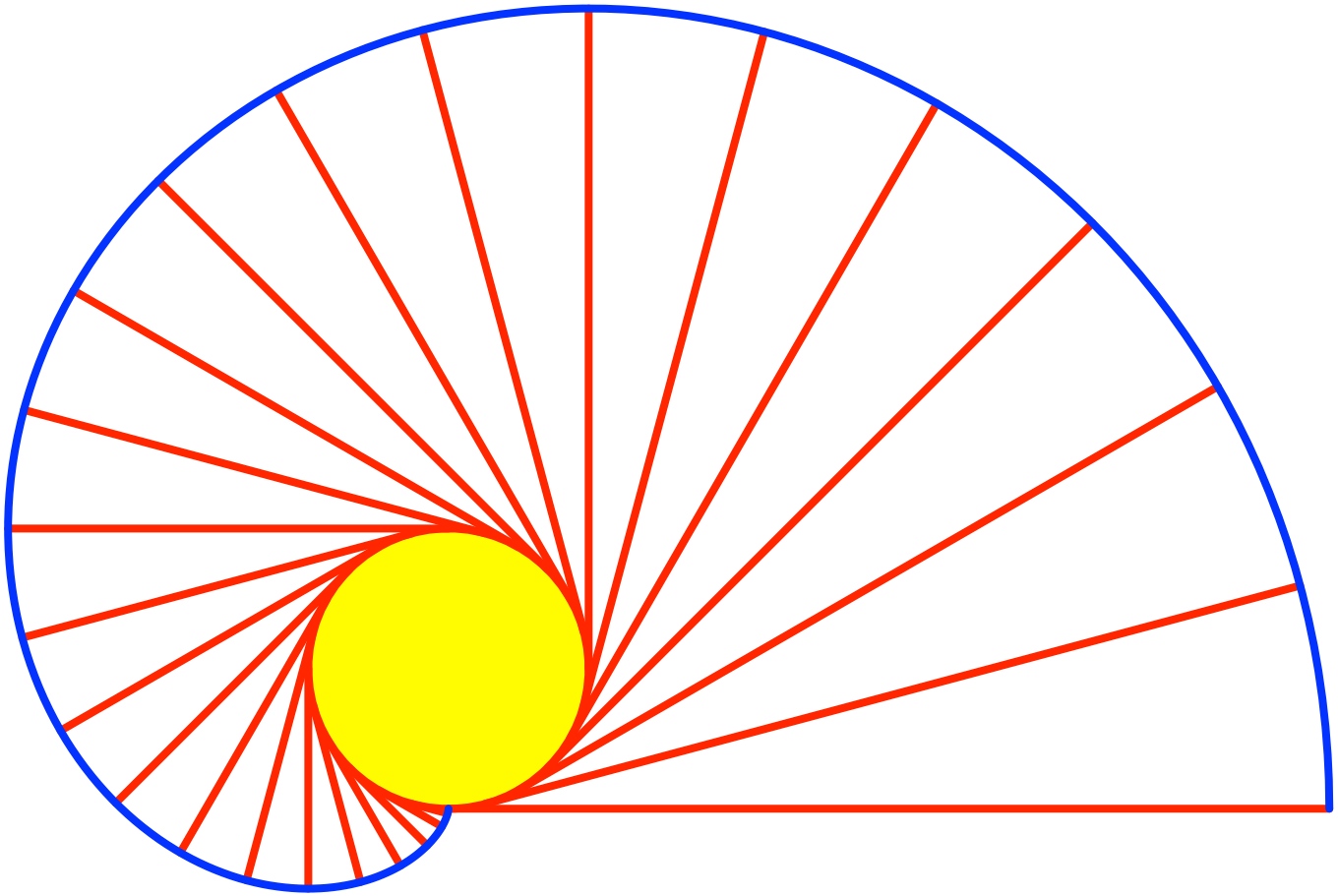
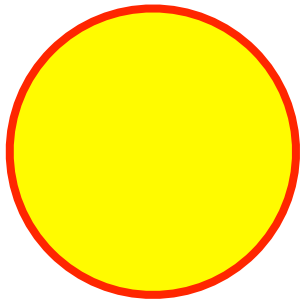


Bahnkurve des Endpunktes: Evolvente



Aufwickeln zum Kreis



Straff gespannt

Gleichmäßig gekrümmt

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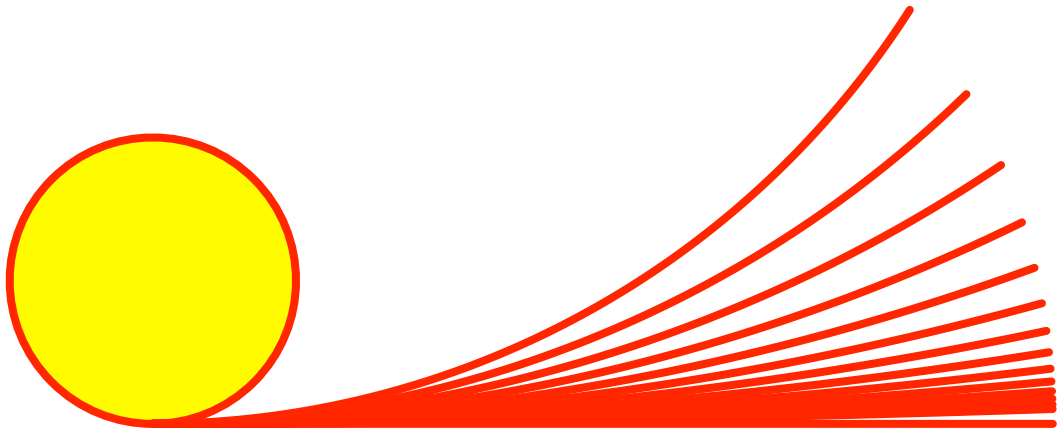

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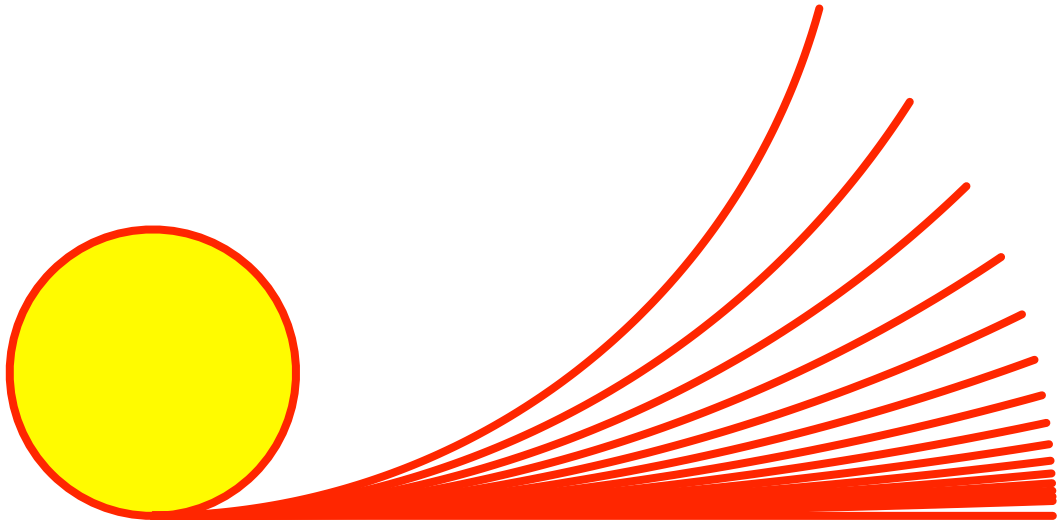

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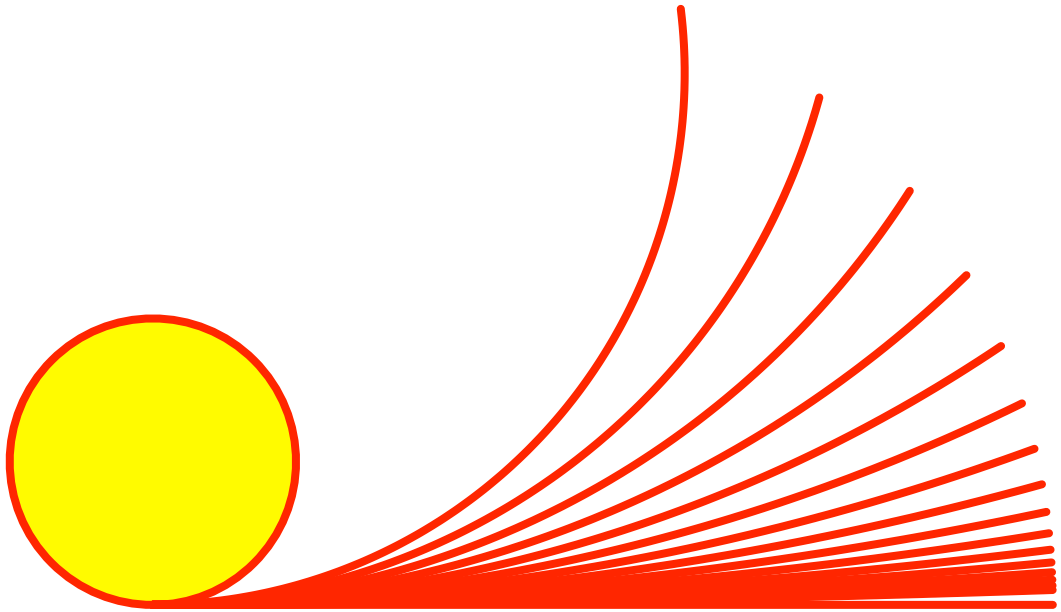

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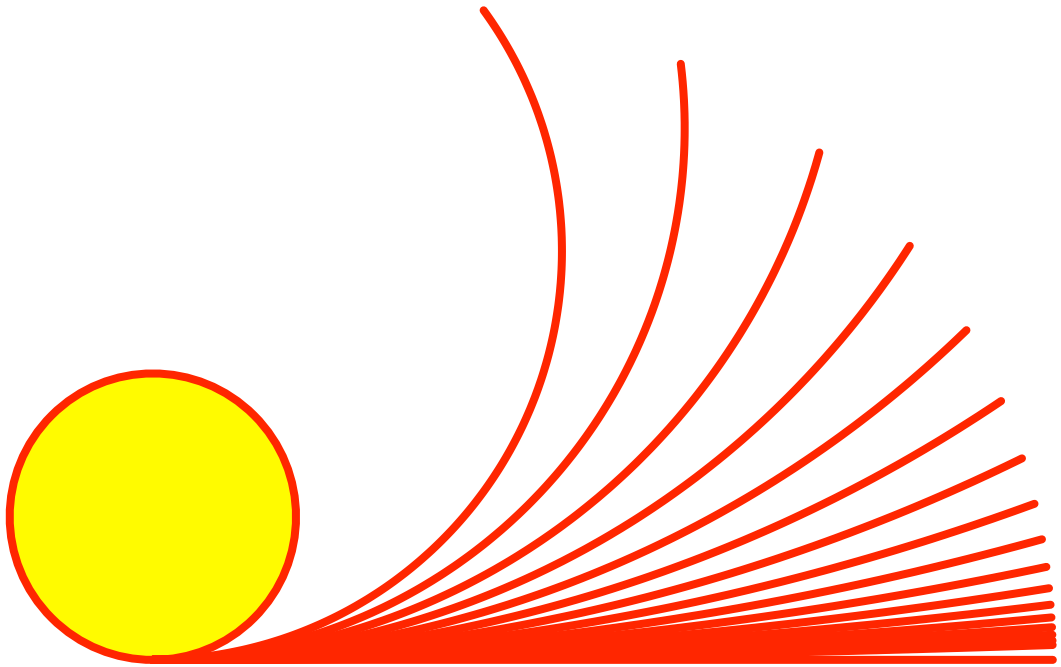

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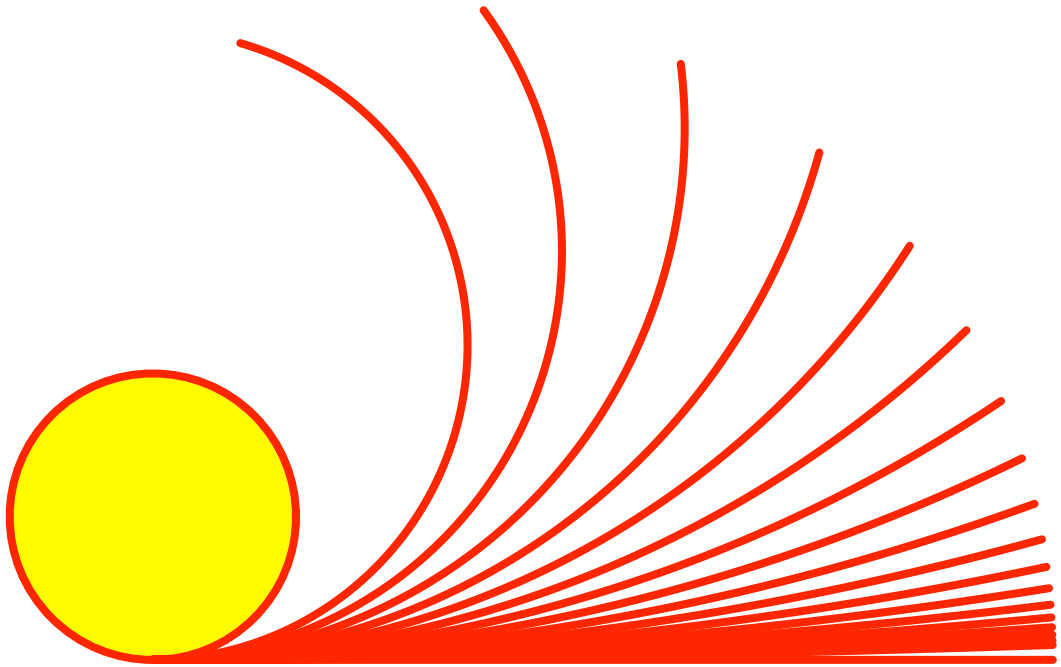

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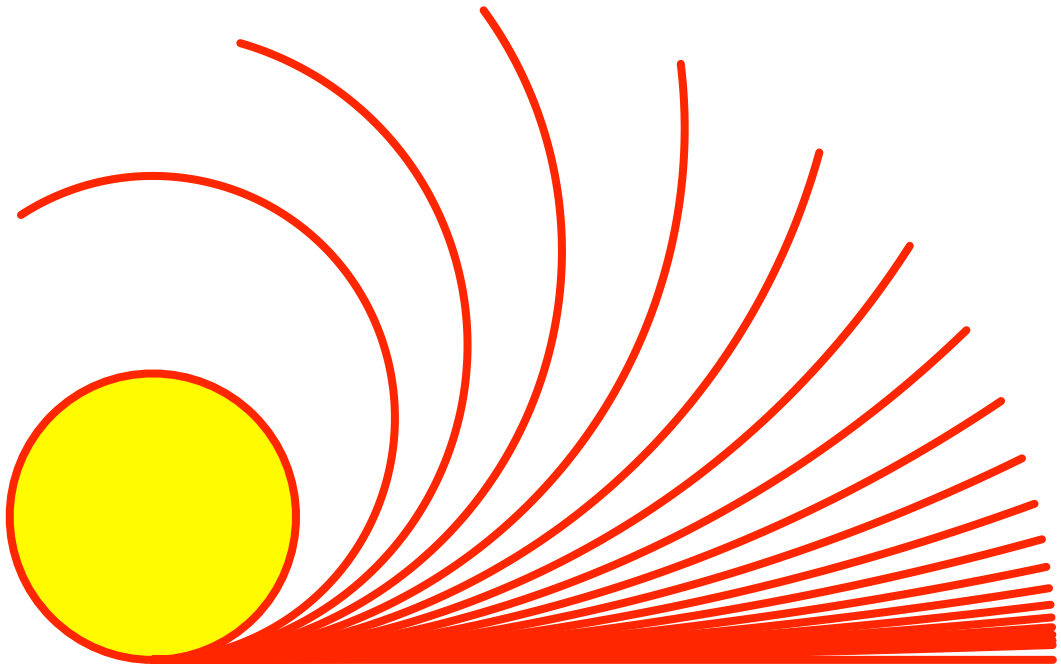

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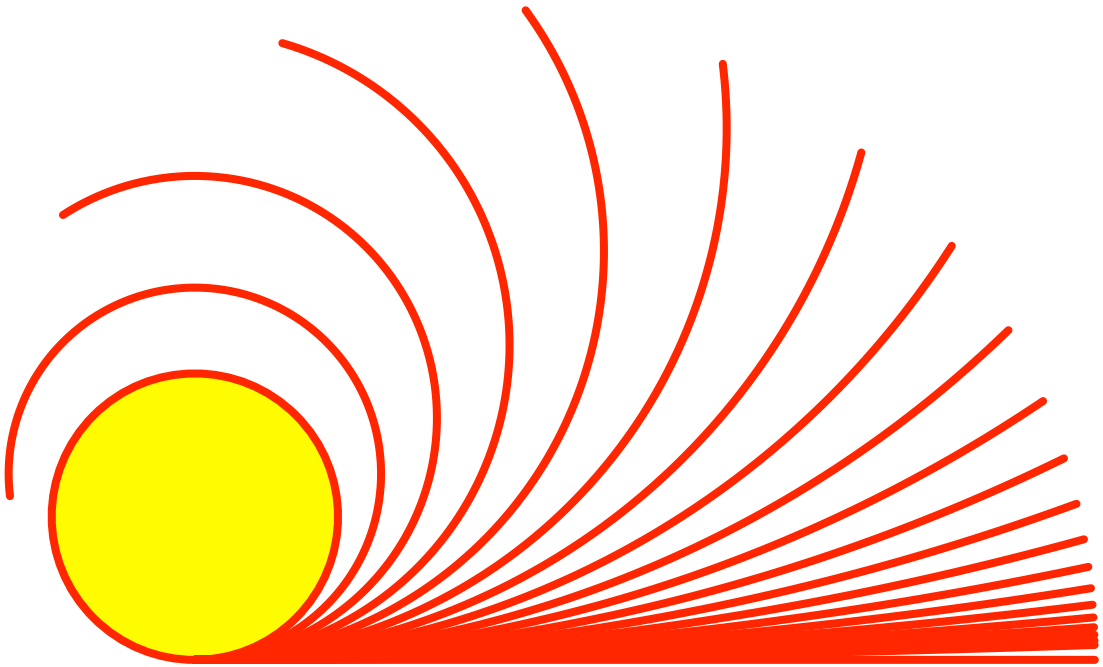
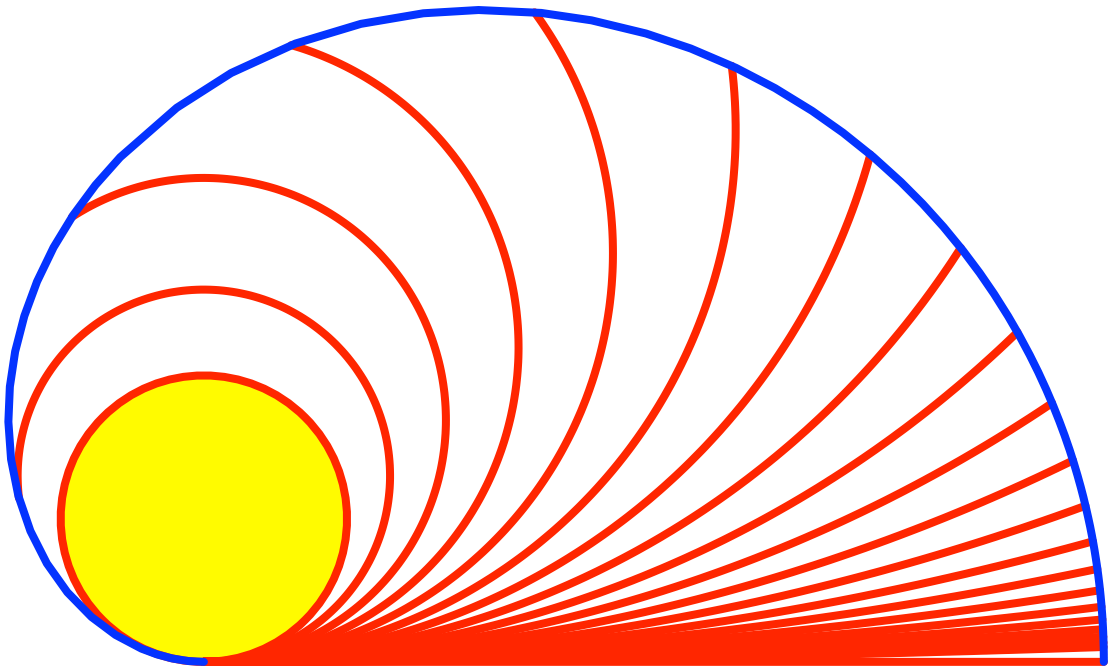
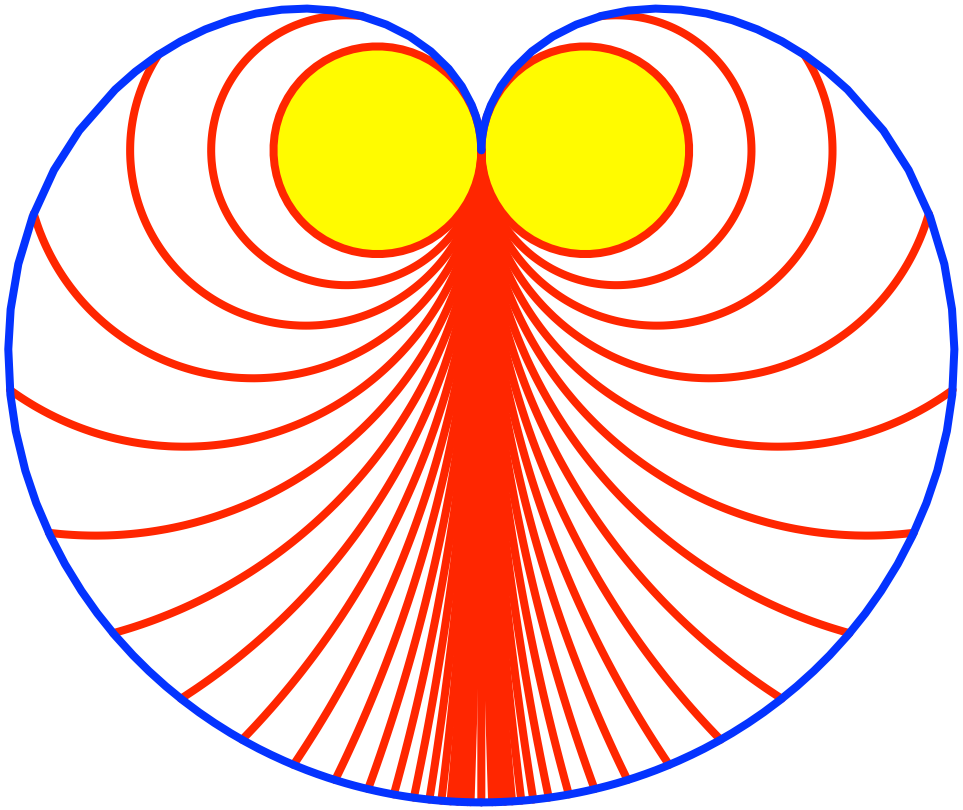


Bahnkurve des Endpunktes



$$r(\varphi) = \frac{1}{\varphi}\sin(\varphi)$$

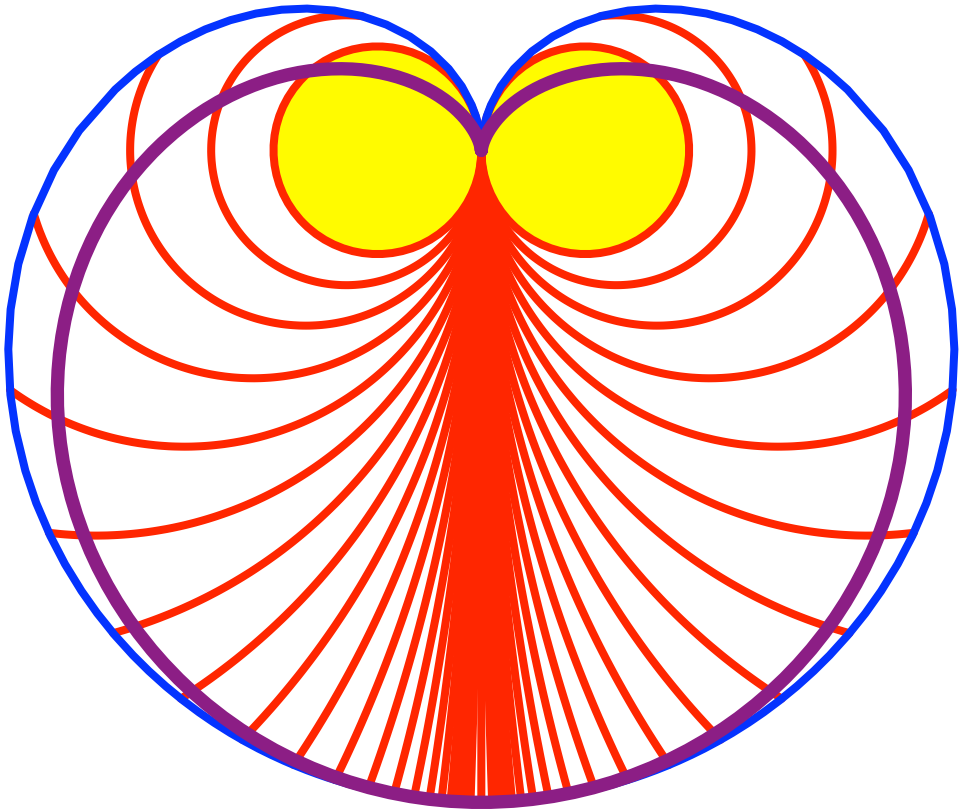
Bahnkurve des Endpunktes: Herzkurve



www.walser-h-m.ch/hans/Miniaturen/H/Herzkurve4/Herzkurve4.htm

Bahnkurve des Endpunktes: Herzkurve

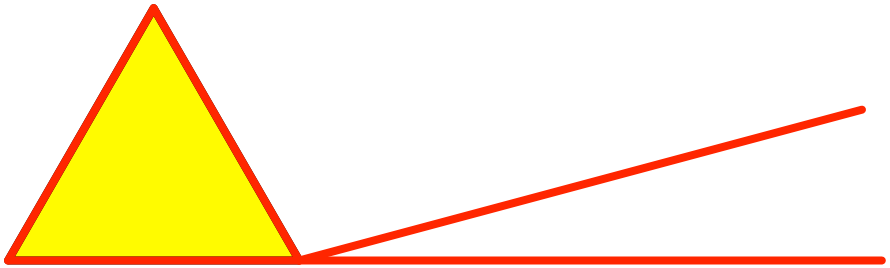
Kardioide



Aufwickeln zum Dreieck

Straff gespannt

Straff gespannt



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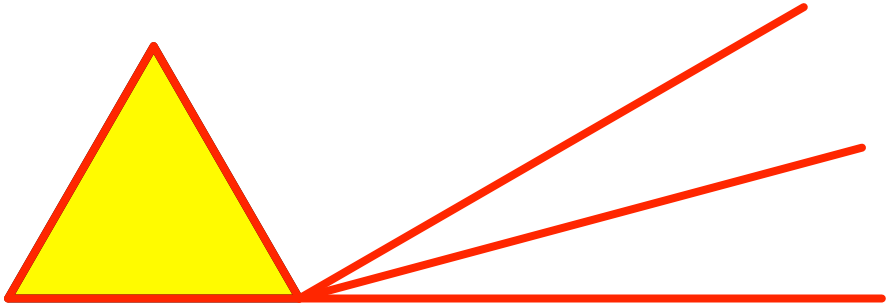

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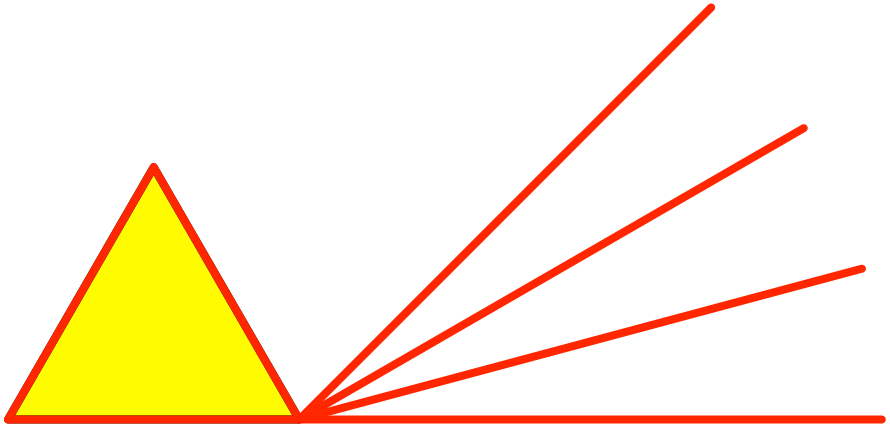

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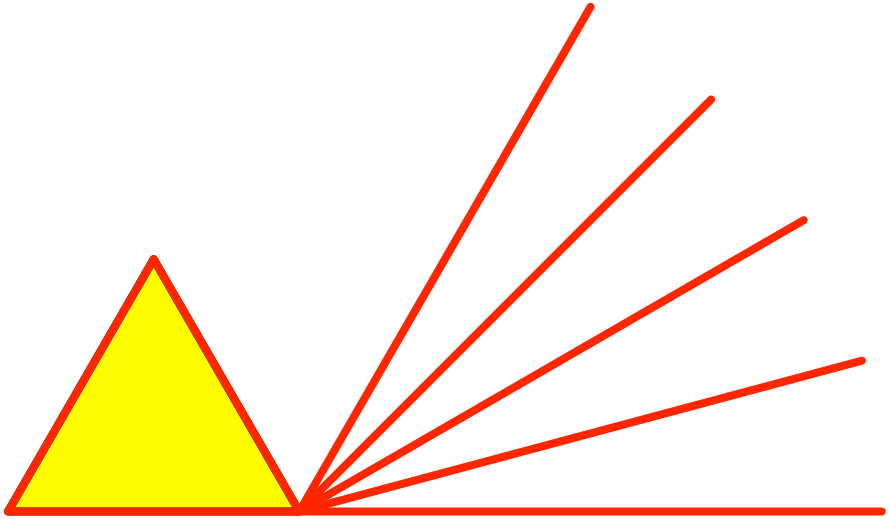

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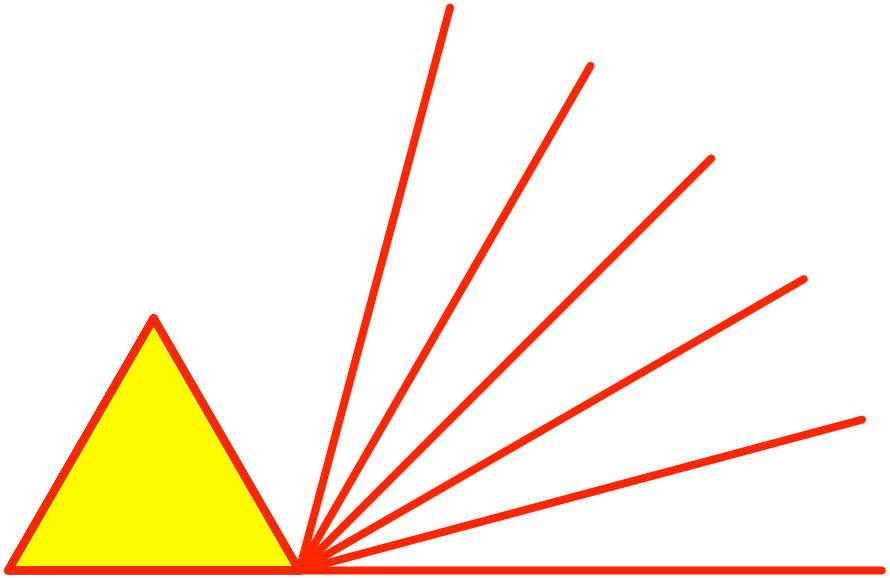

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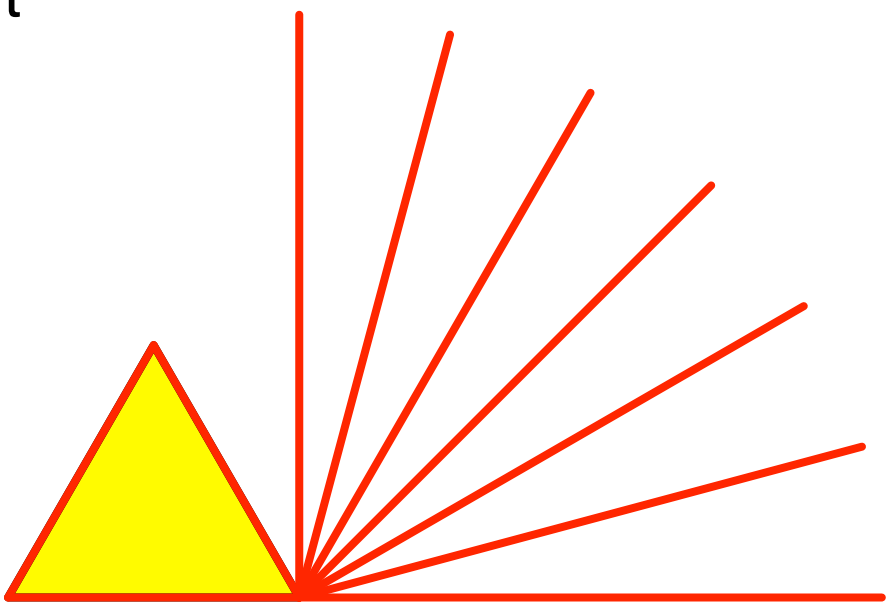

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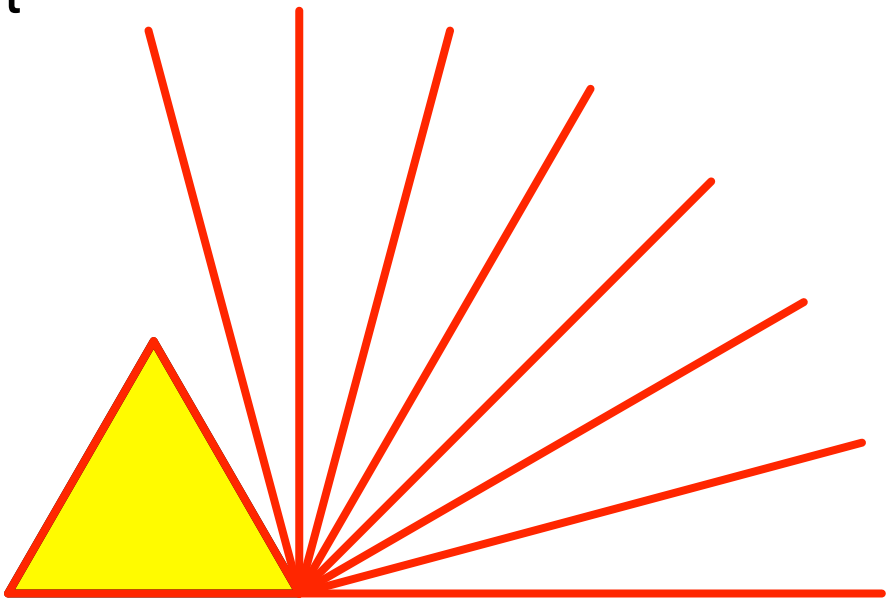

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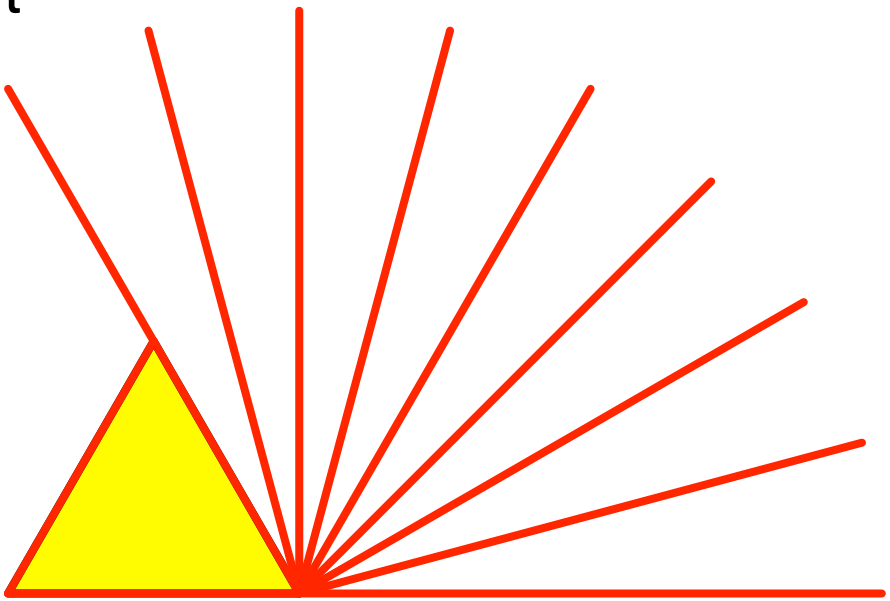

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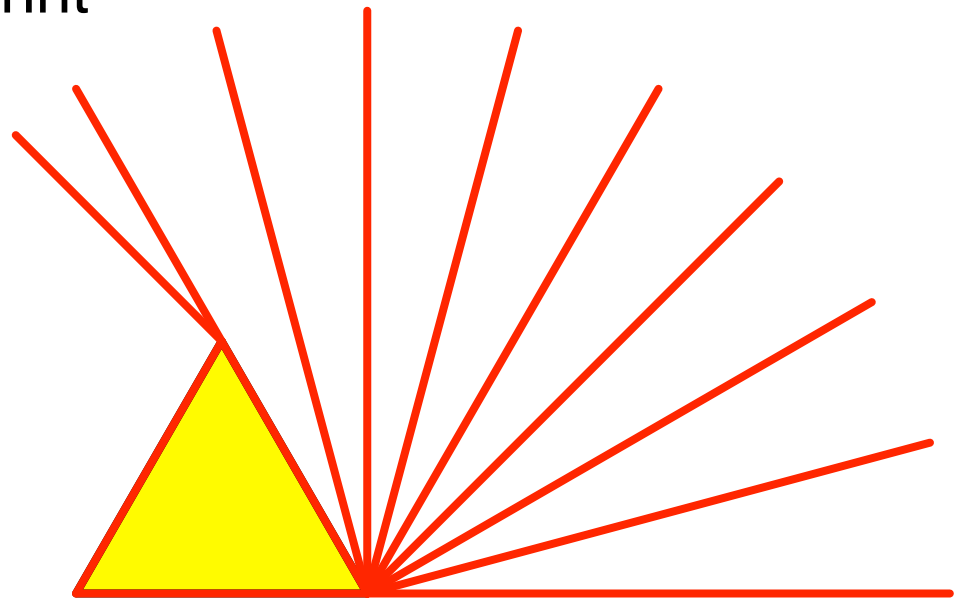

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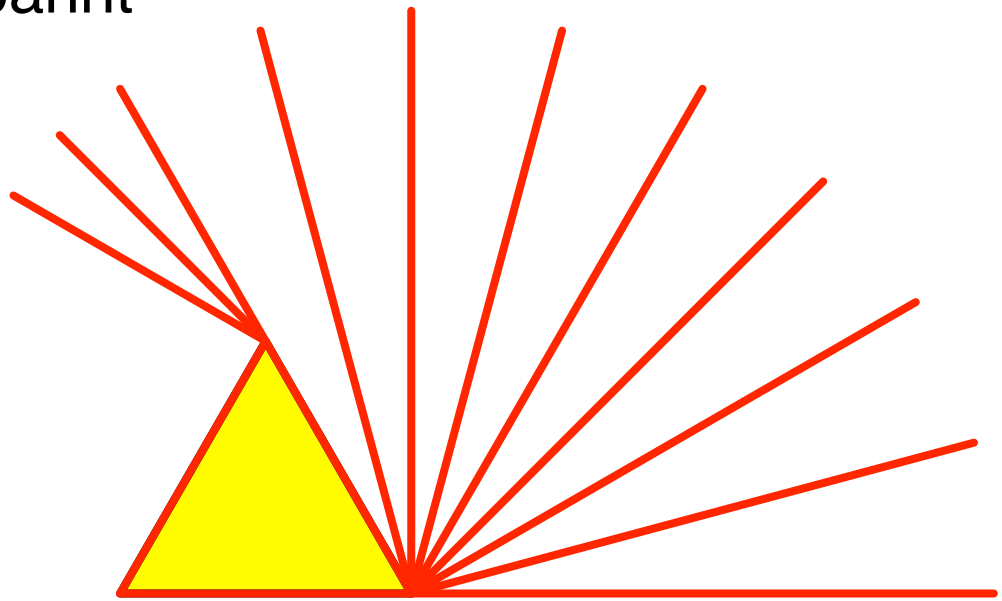

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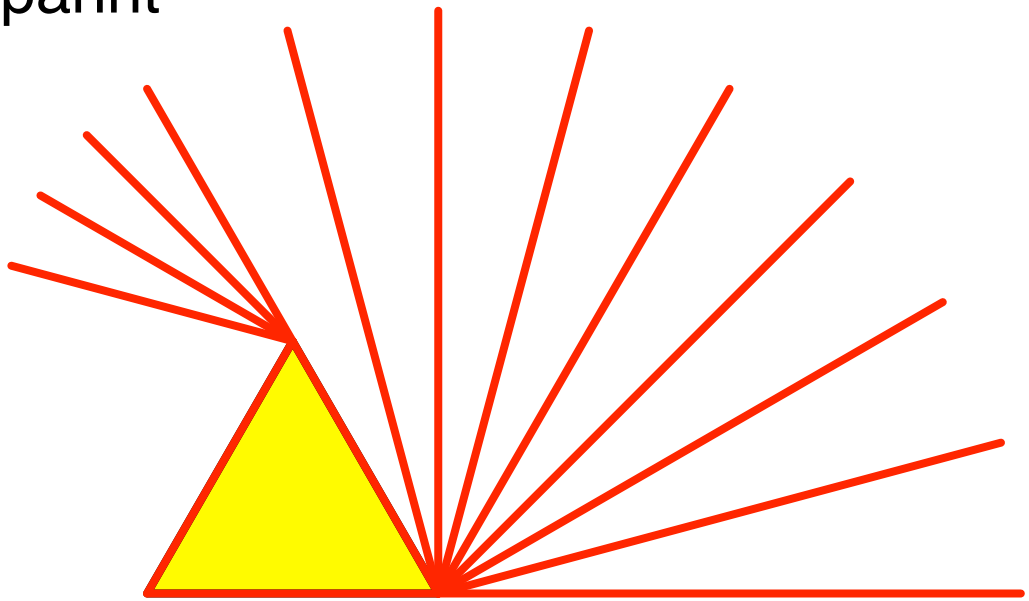
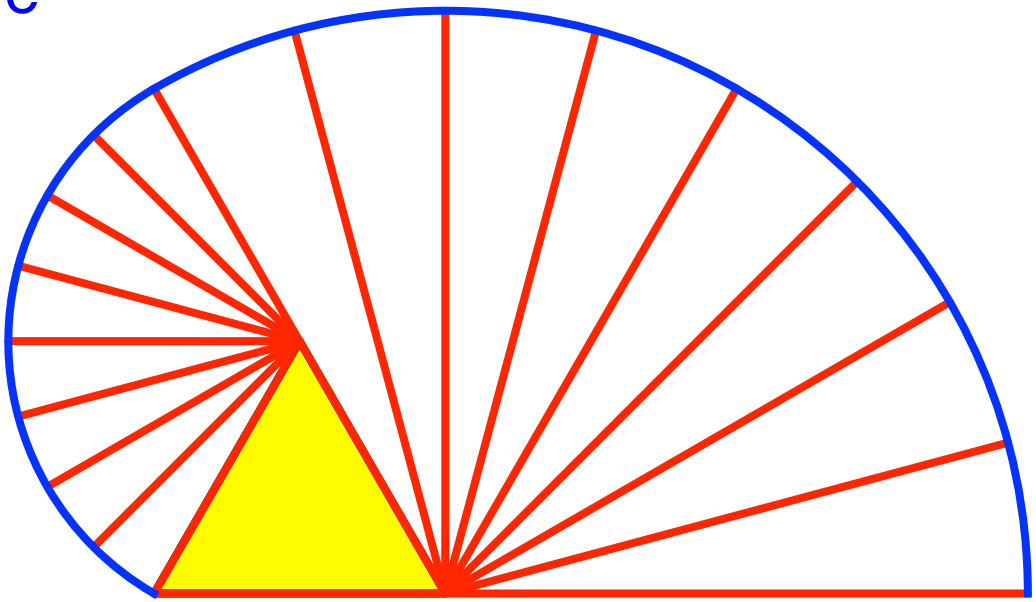
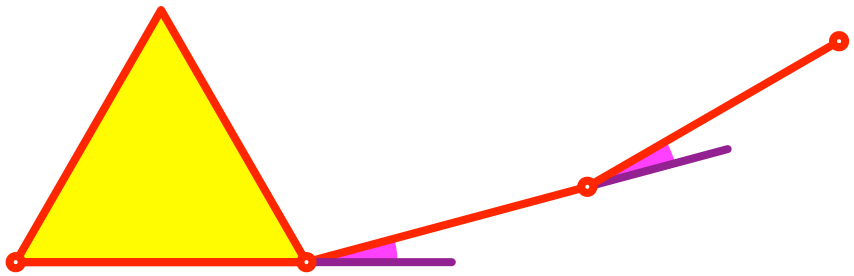


Bahnkurve



Gleichmäßig geknickt



Schließen der flachen Hand zur Faust

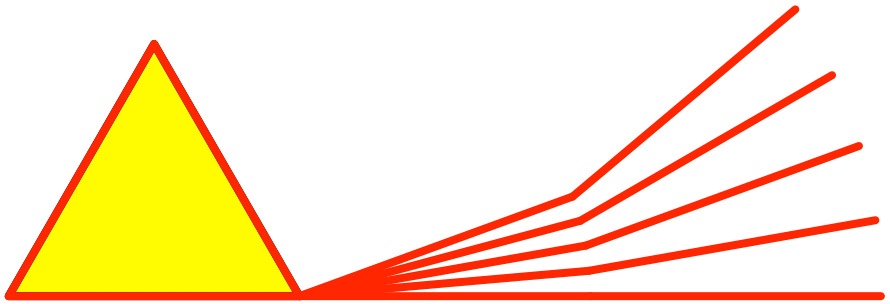
Gleichmäßig geknickt

Gleichmäßig geknickt

Gleichmäßig geknickt

Gleichmäßig geknickt

Gleichmäßig geknickt



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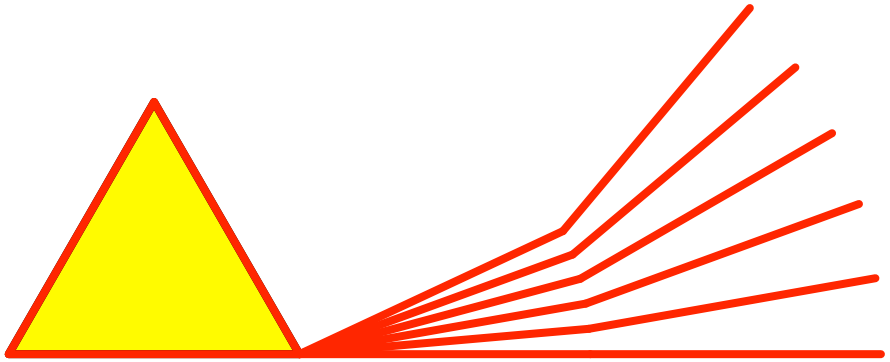

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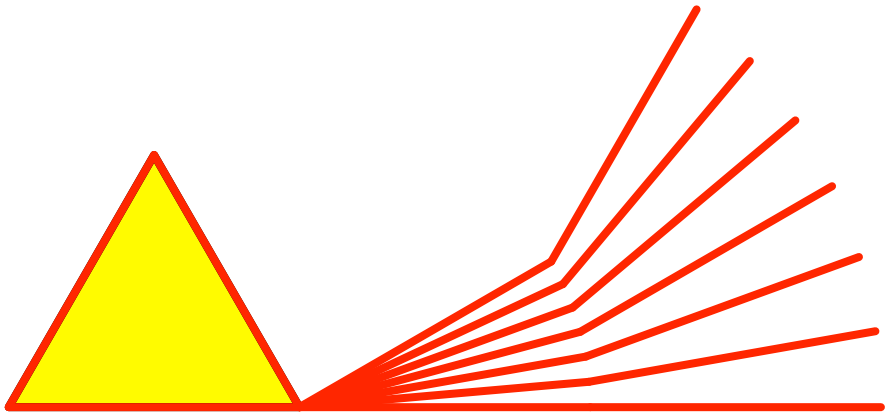
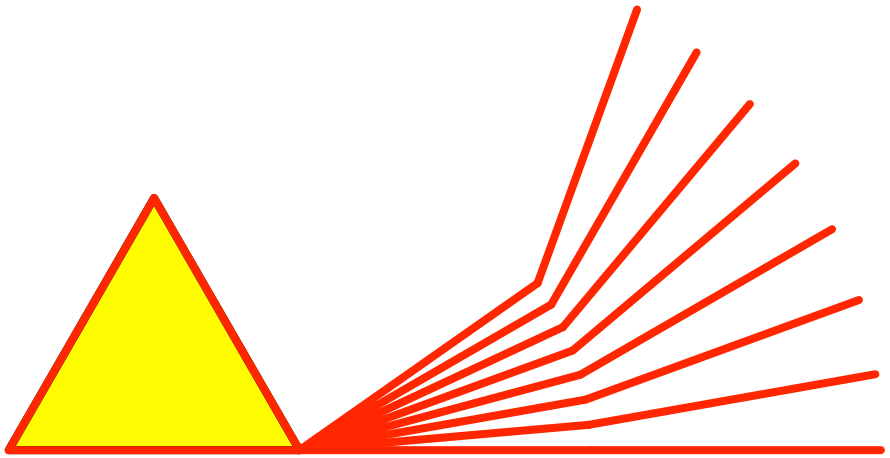
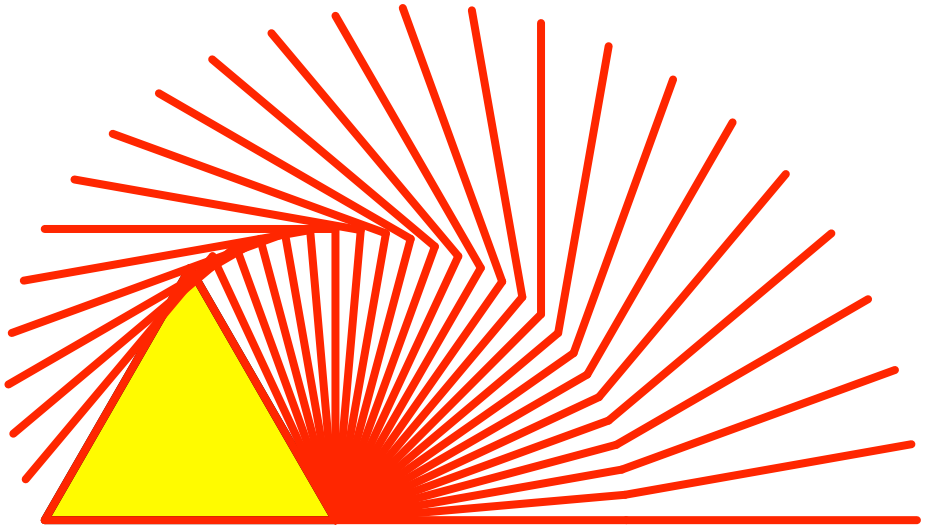


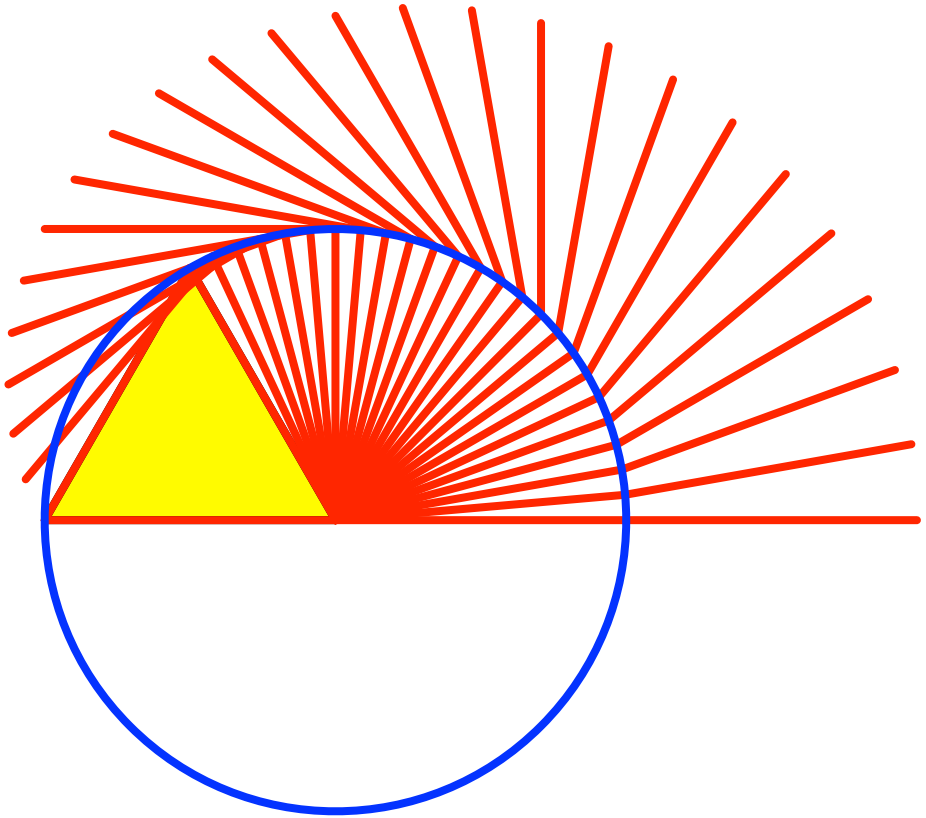
Gleichmäßig geknickt



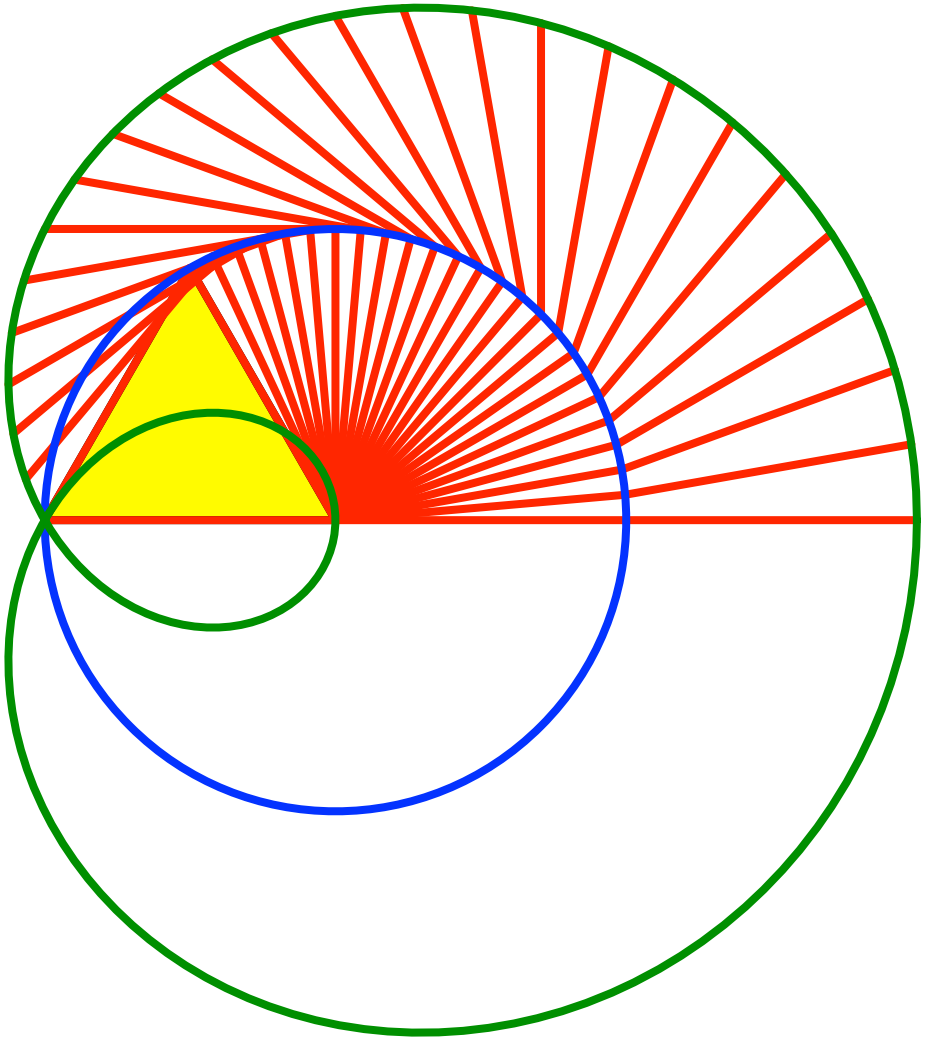
Gleichmäßig geknickt



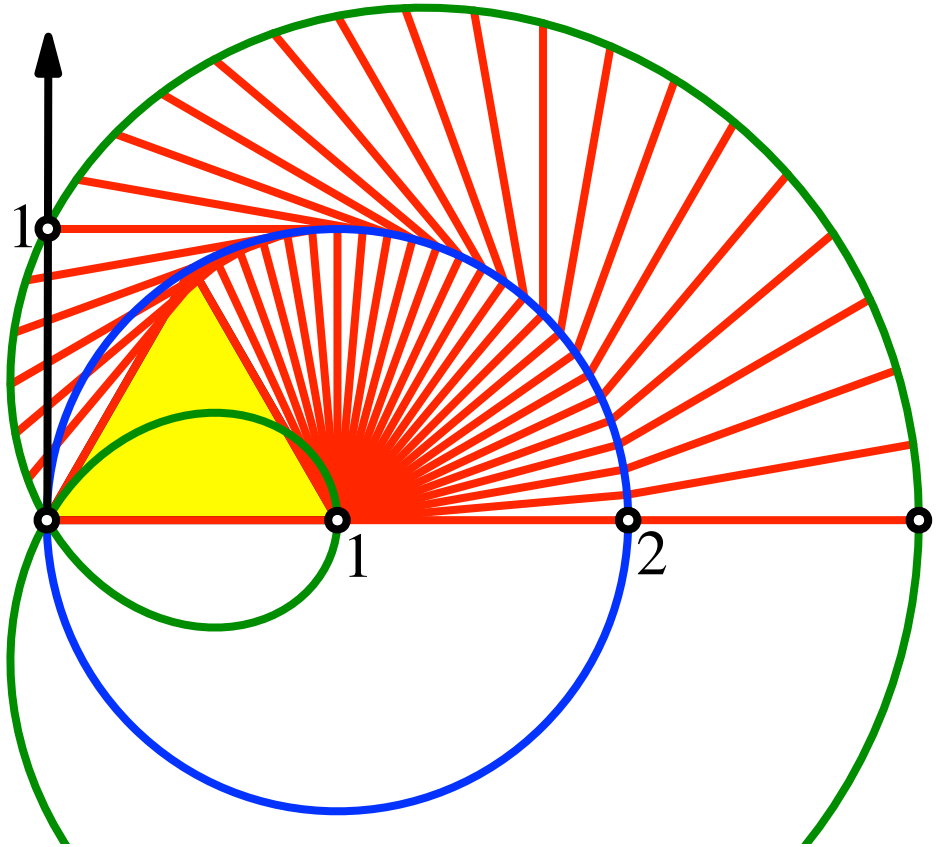
Kreis



Kreisüberlagerung

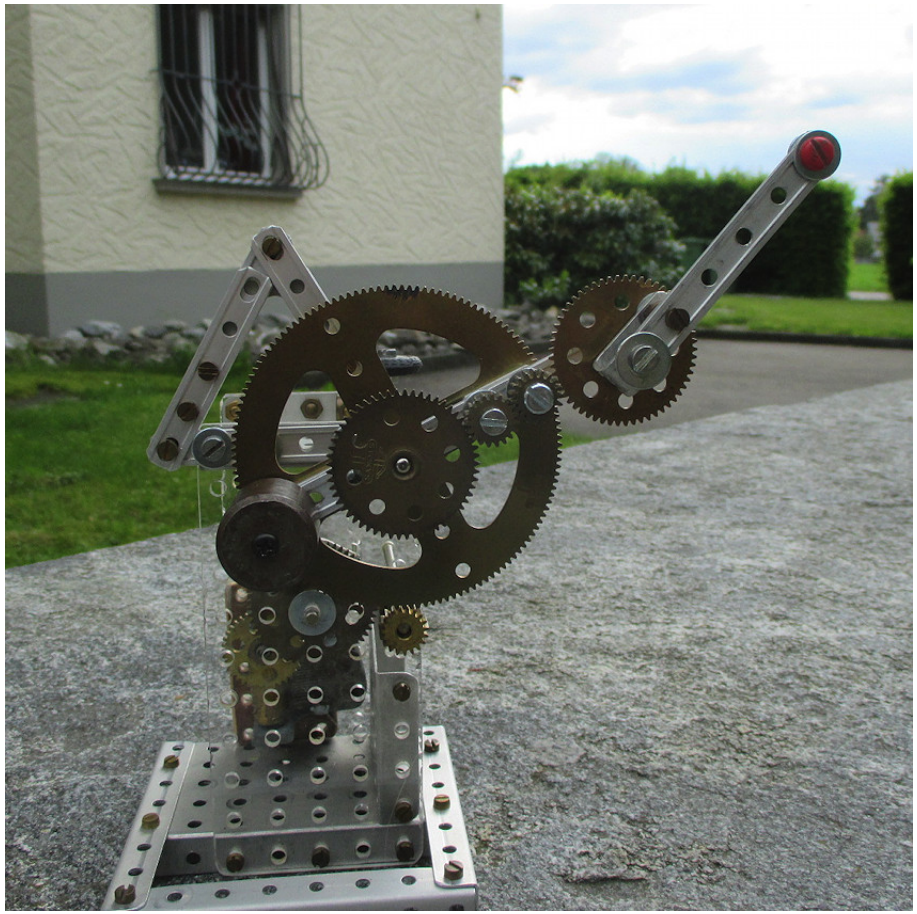


Kreisüberlagerung. Fourier-Entwicklung



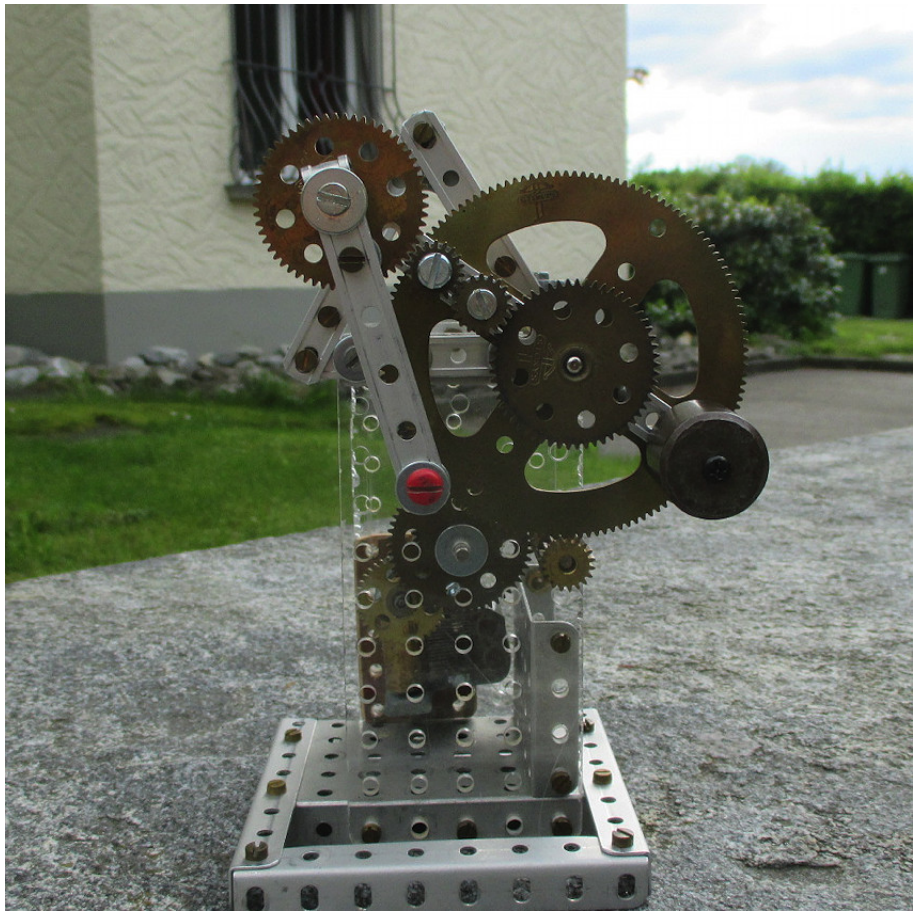
$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}, \quad t \in [0,\ldots,2\pi]$$

Gleichmäßig geknickt



Mechanisches Modell

Gleichmäßig geknickt

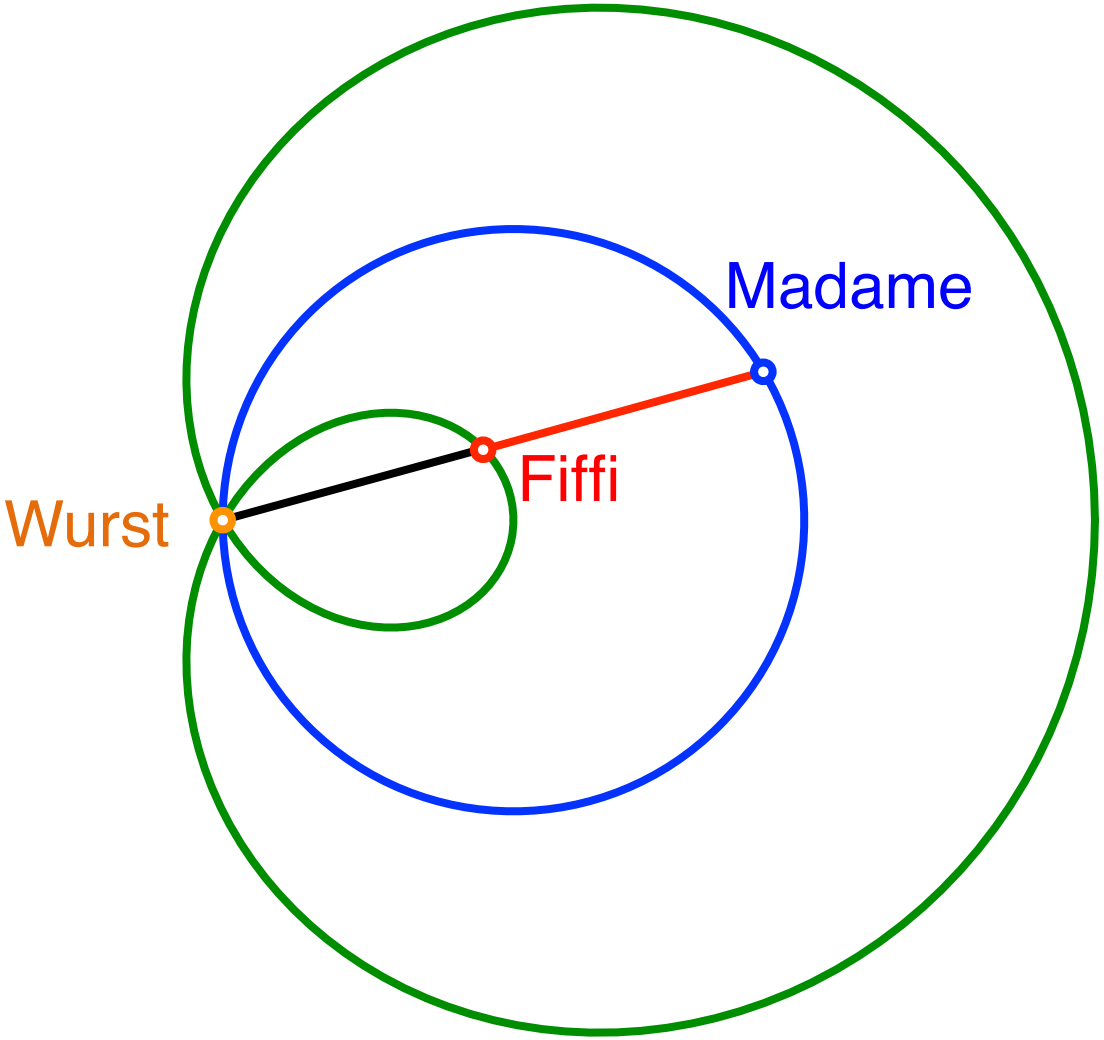


A for Art

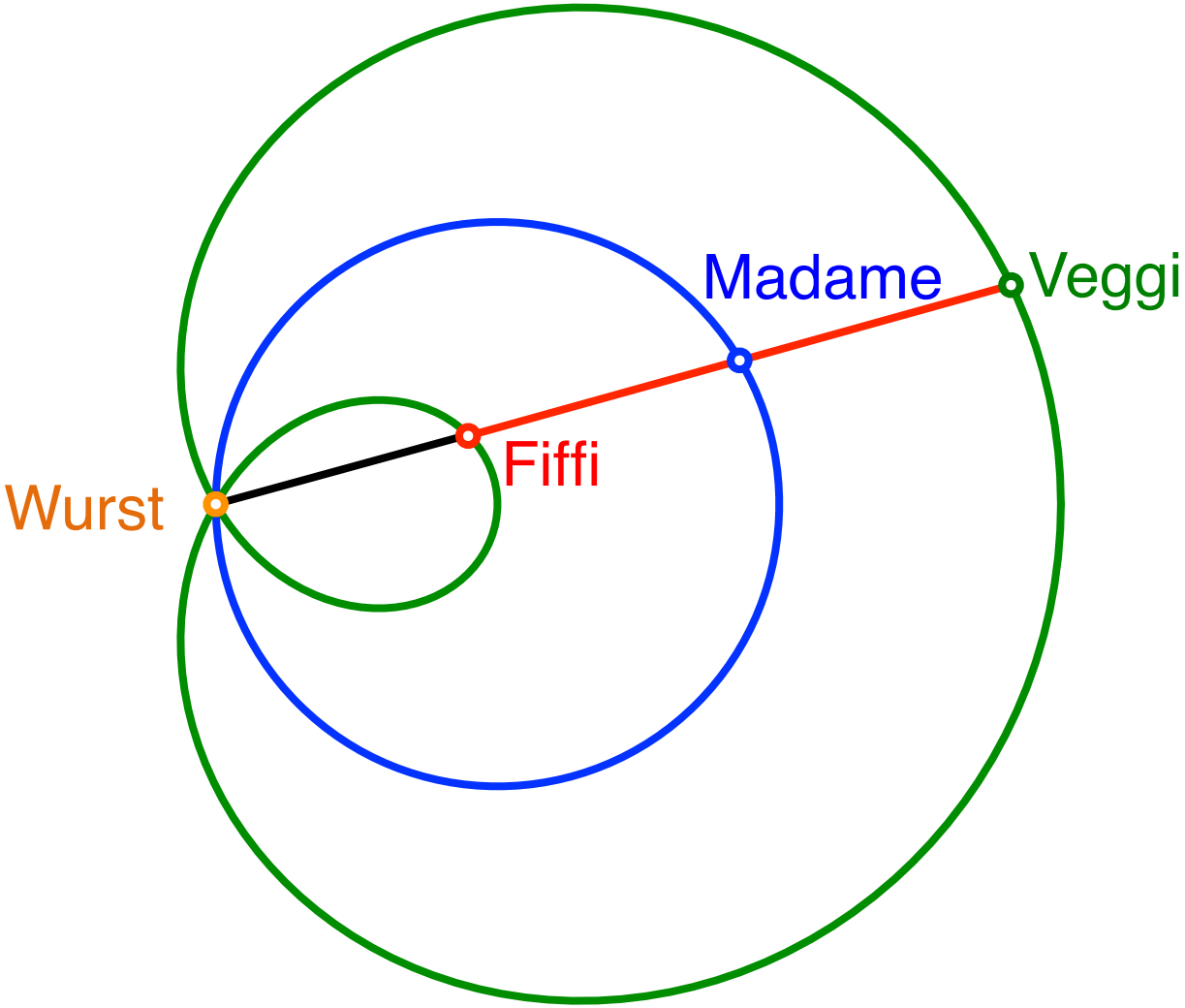
↓

STEM ➔ STEAM

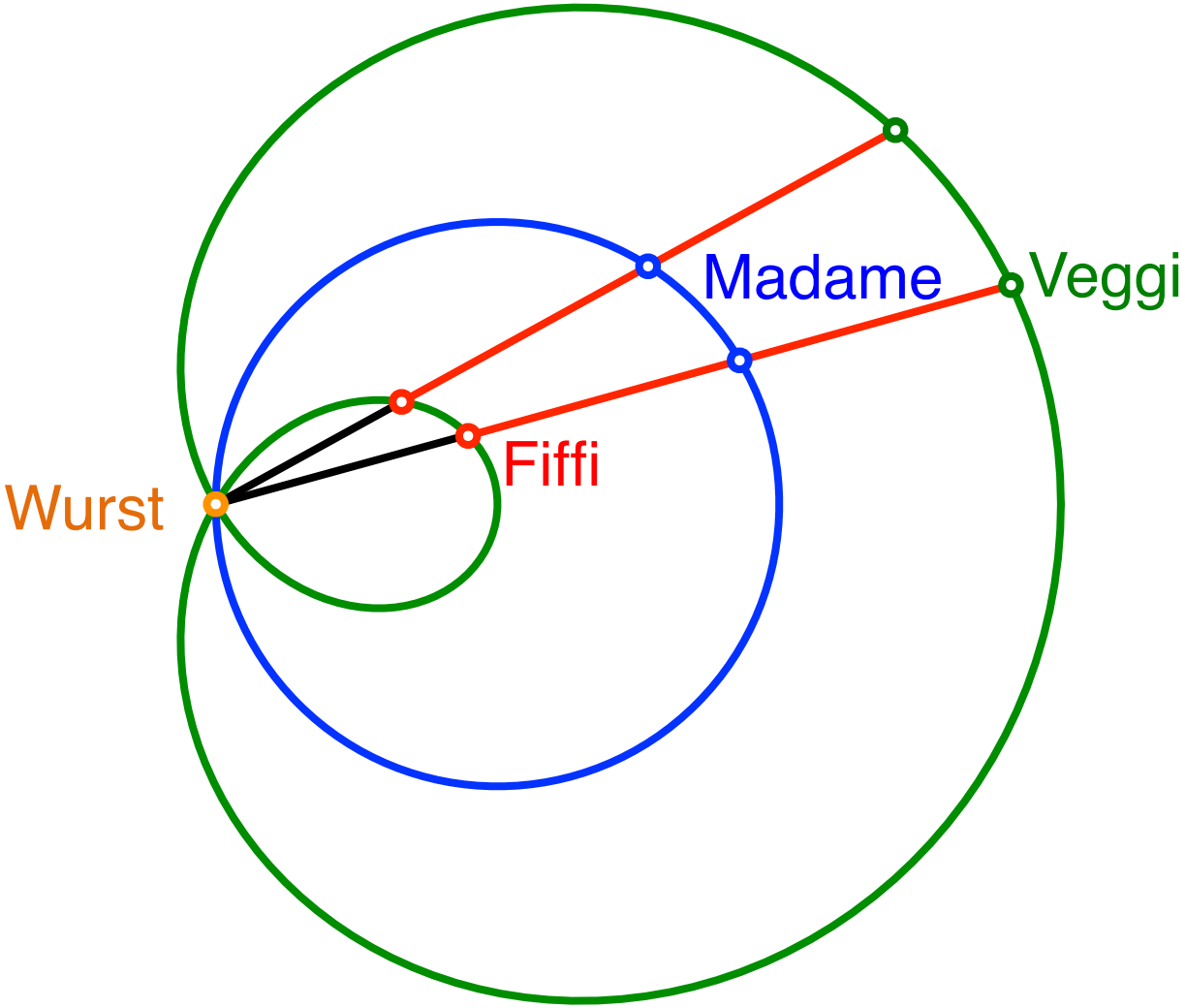
Hundekurve (curva canis)



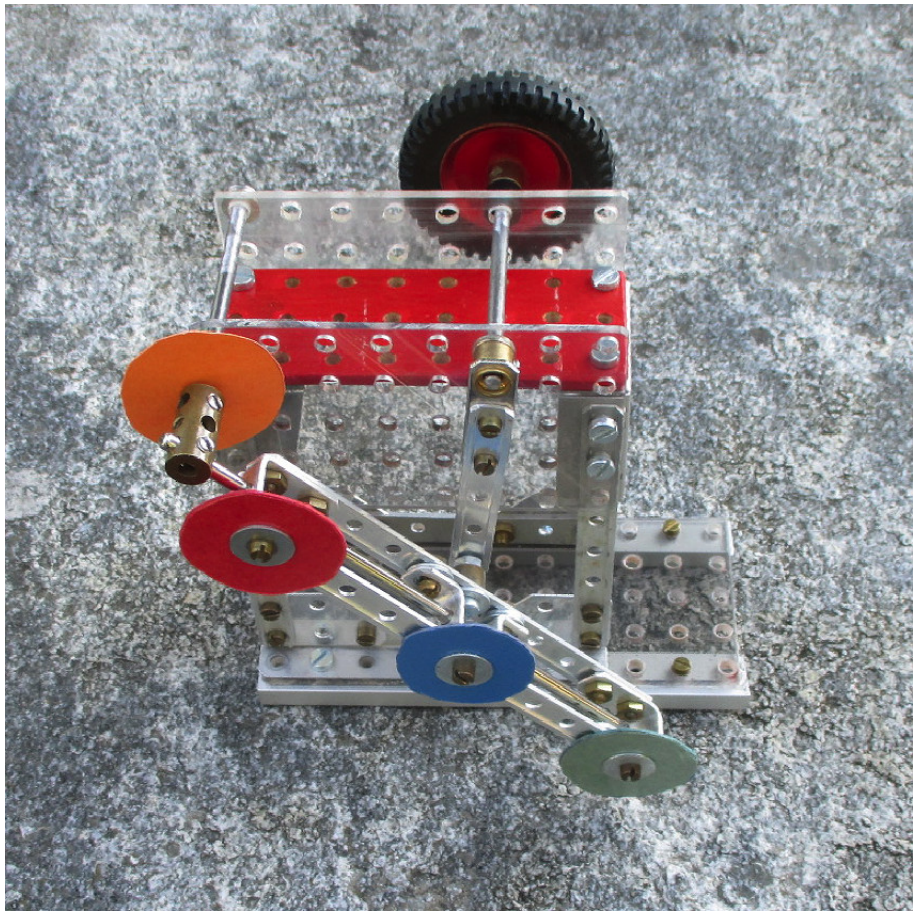
Hundekurve (curva canis)



Hundekurve (curva canis)

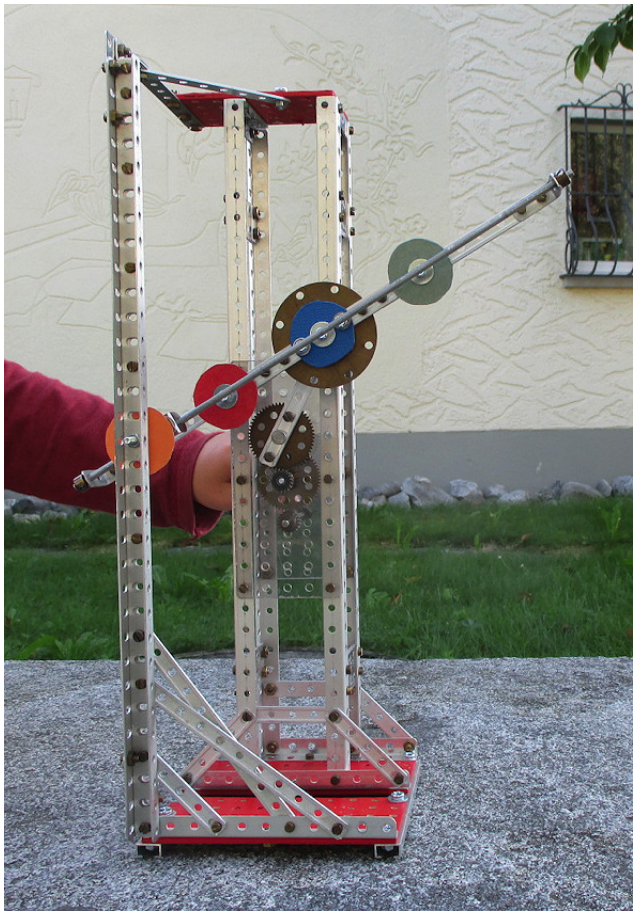


Hundekurve (curva canis)



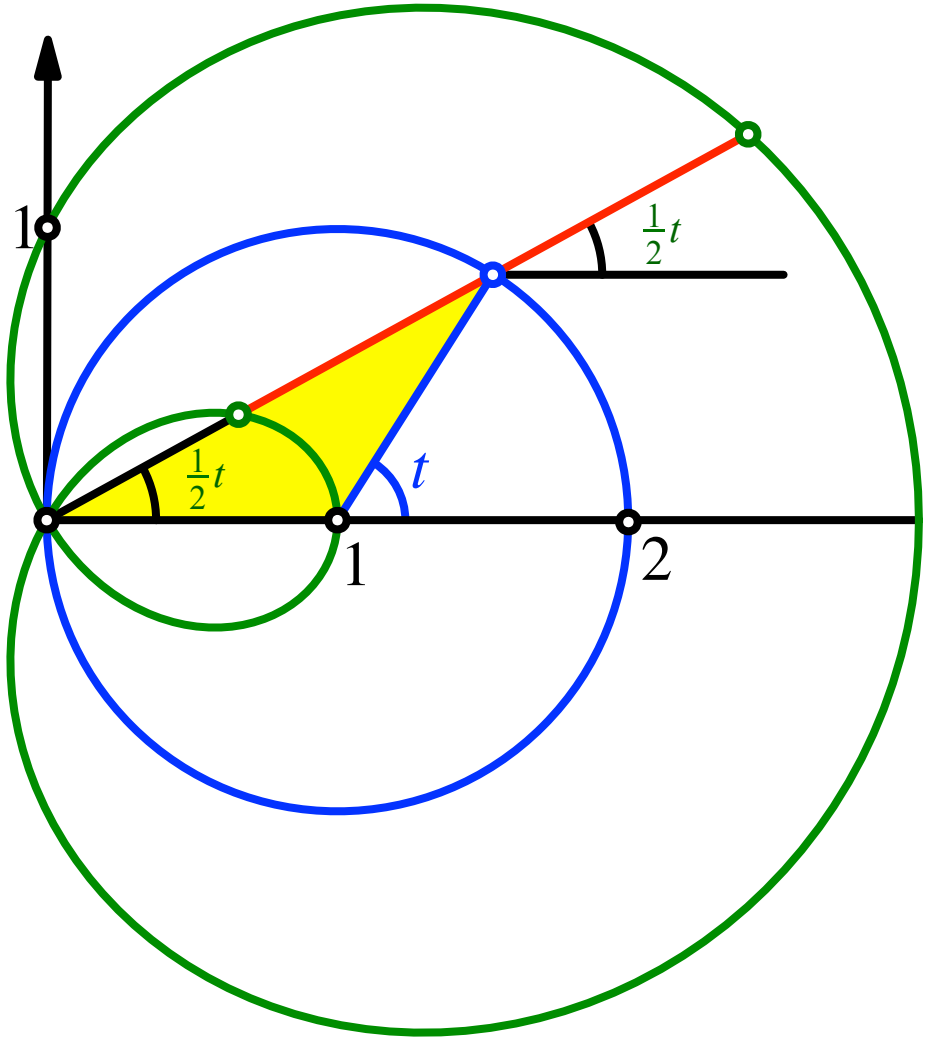
Mechanische Modelle

Hundekurve (curva canis)

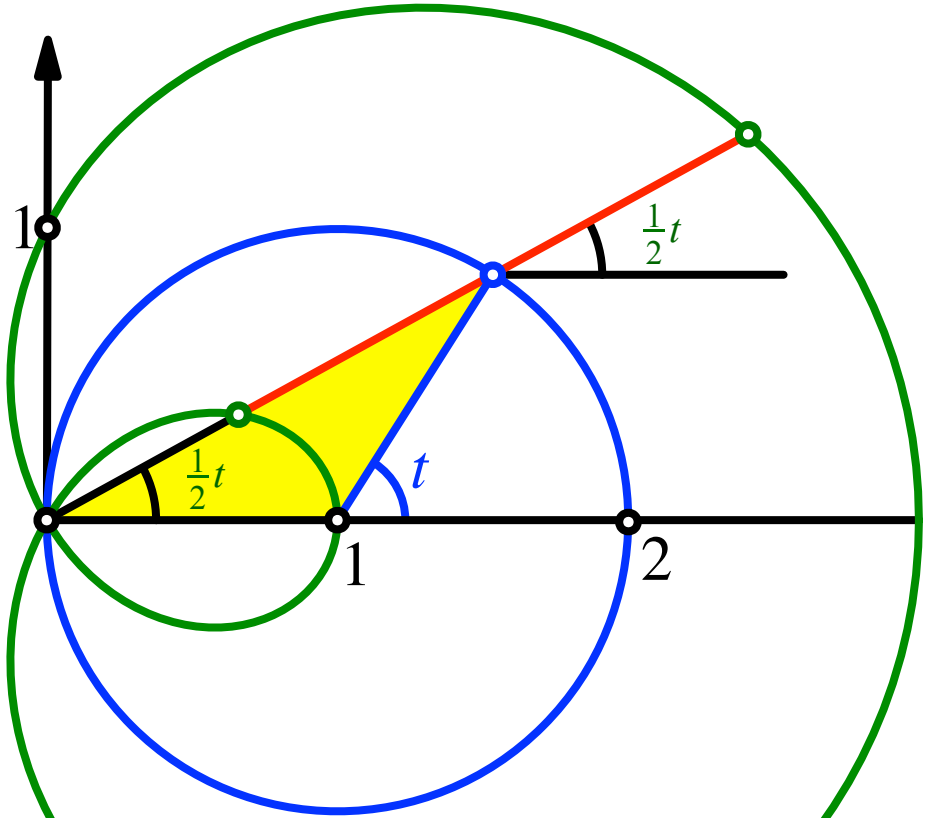


Mechanische Modelle

Hundekurve (curva canis)

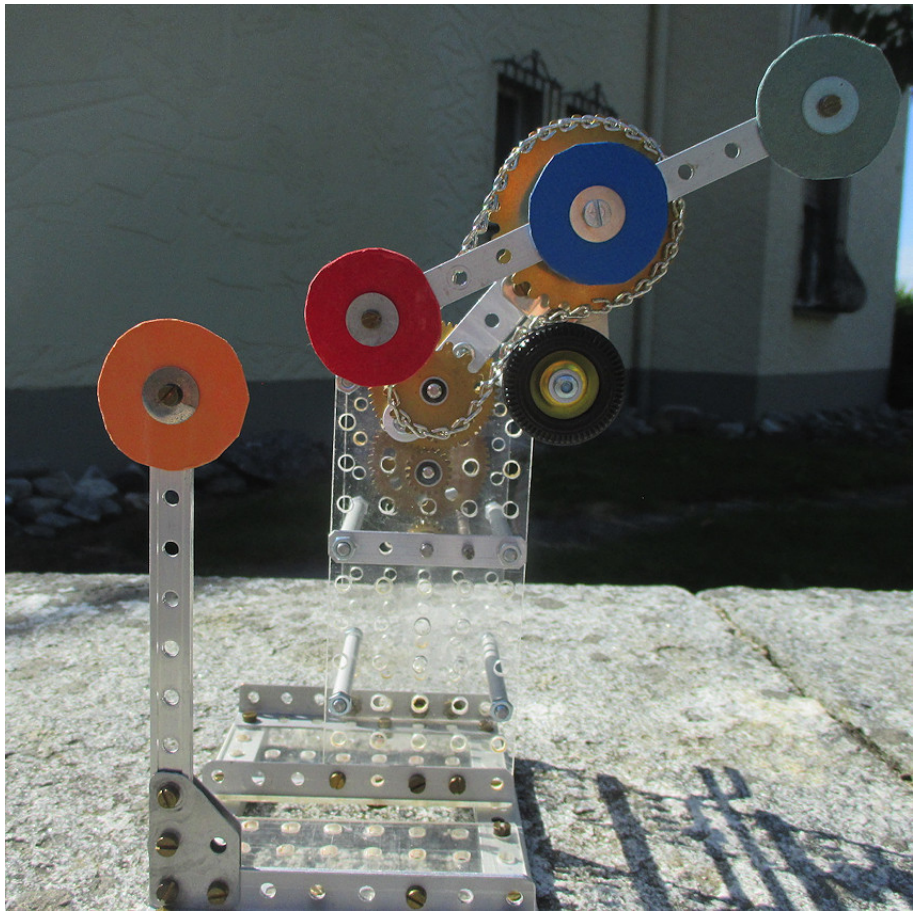


Hundekurve (curva canis)



$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{1}{2}t\right) \\ \sin\left(\frac{1}{2}t\right) \end{bmatrix}, \quad t \in [0,\ldots,4\pi]$$

Hundekurve (Pascalsche Schnecke)



Mechanisches Modell

Kreisüberlagerung. Fourier-Entwicklung

$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}, \quad t \in [0,\ldots,2\pi]$$

Dieselbe Kurve?

Hundekurve

$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{1}{2}t\right) \\ \sin\left(\frac{1}{2}t\right) \end{bmatrix}, \quad t \in [0,\ldots,4\pi]$$

Kreisüberlagerung. Fourier-Entwicklung

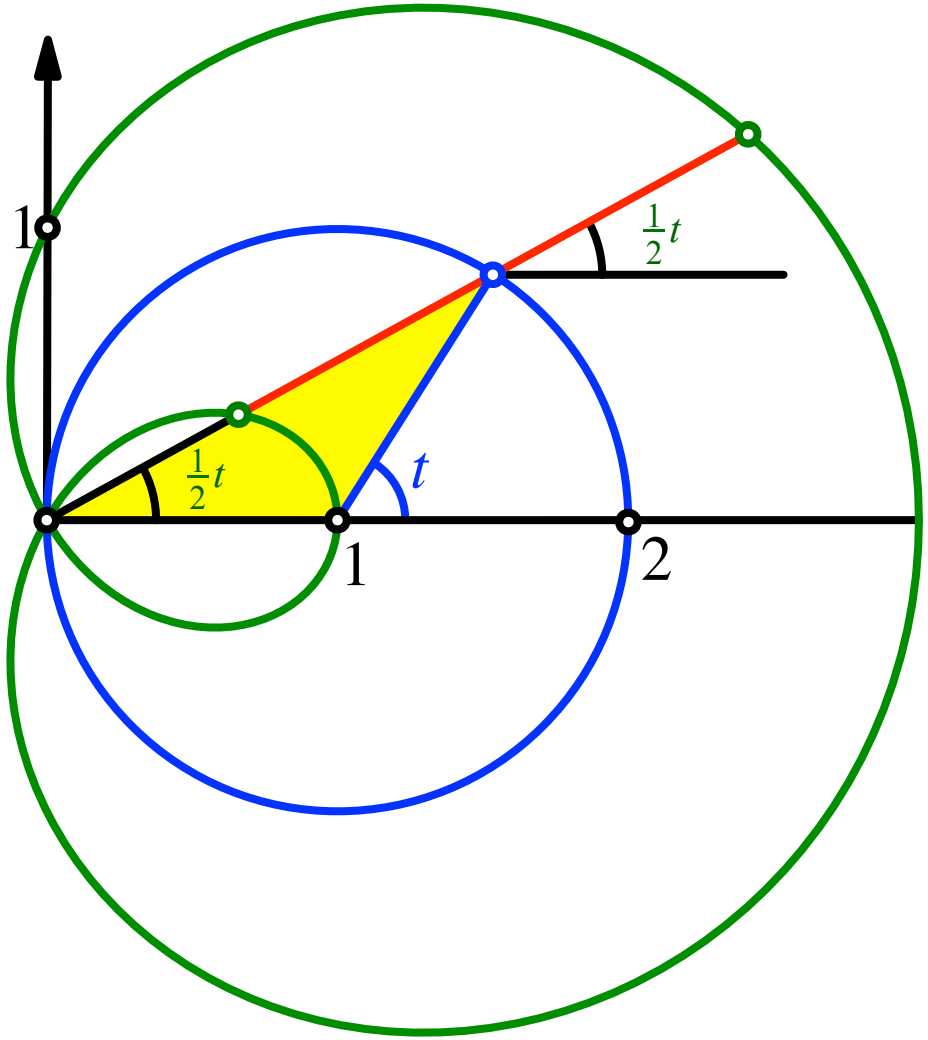
$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}, \quad t \in [0,\ldots,2\pi]$$

Parametertransformation

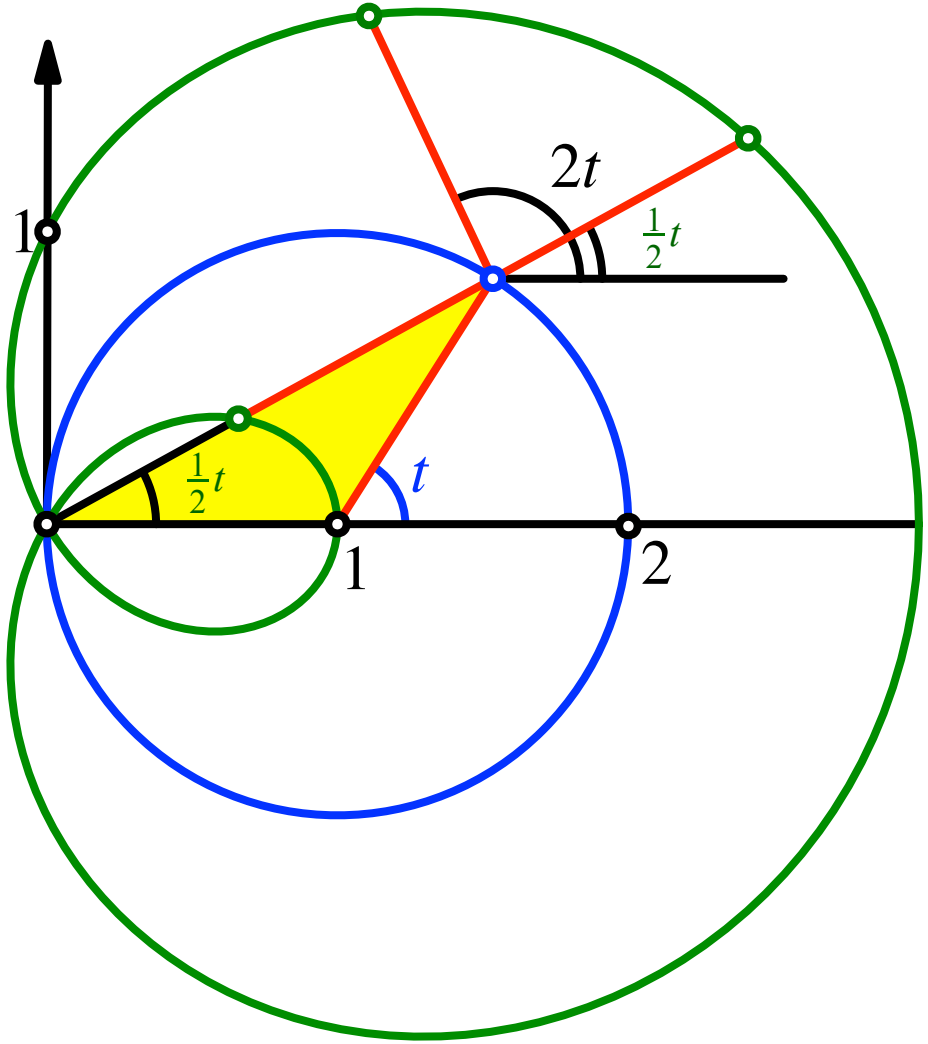
Hundekurve

$$\vec{x}(t) = \begin{bmatrix} \cos(0t) \\ \sin(0t) \end{bmatrix} + \begin{bmatrix} \cos(1t) \\ \sin(1t) \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{1}{2}t\right) \\ \sin\left(\frac{1}{2}t\right) \end{bmatrix}, \quad t \in [0,\ldots,4\pi]$$

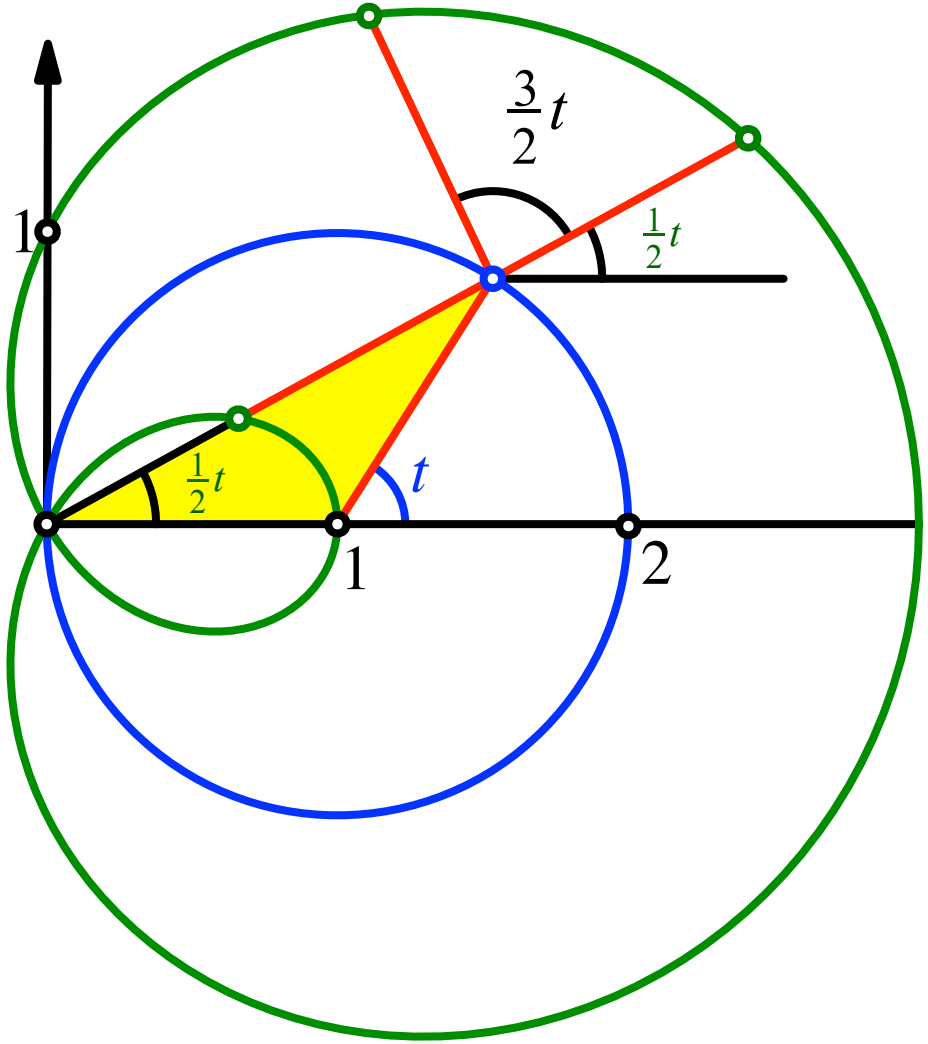
Hundekurve (Pascalsche Schnecke)



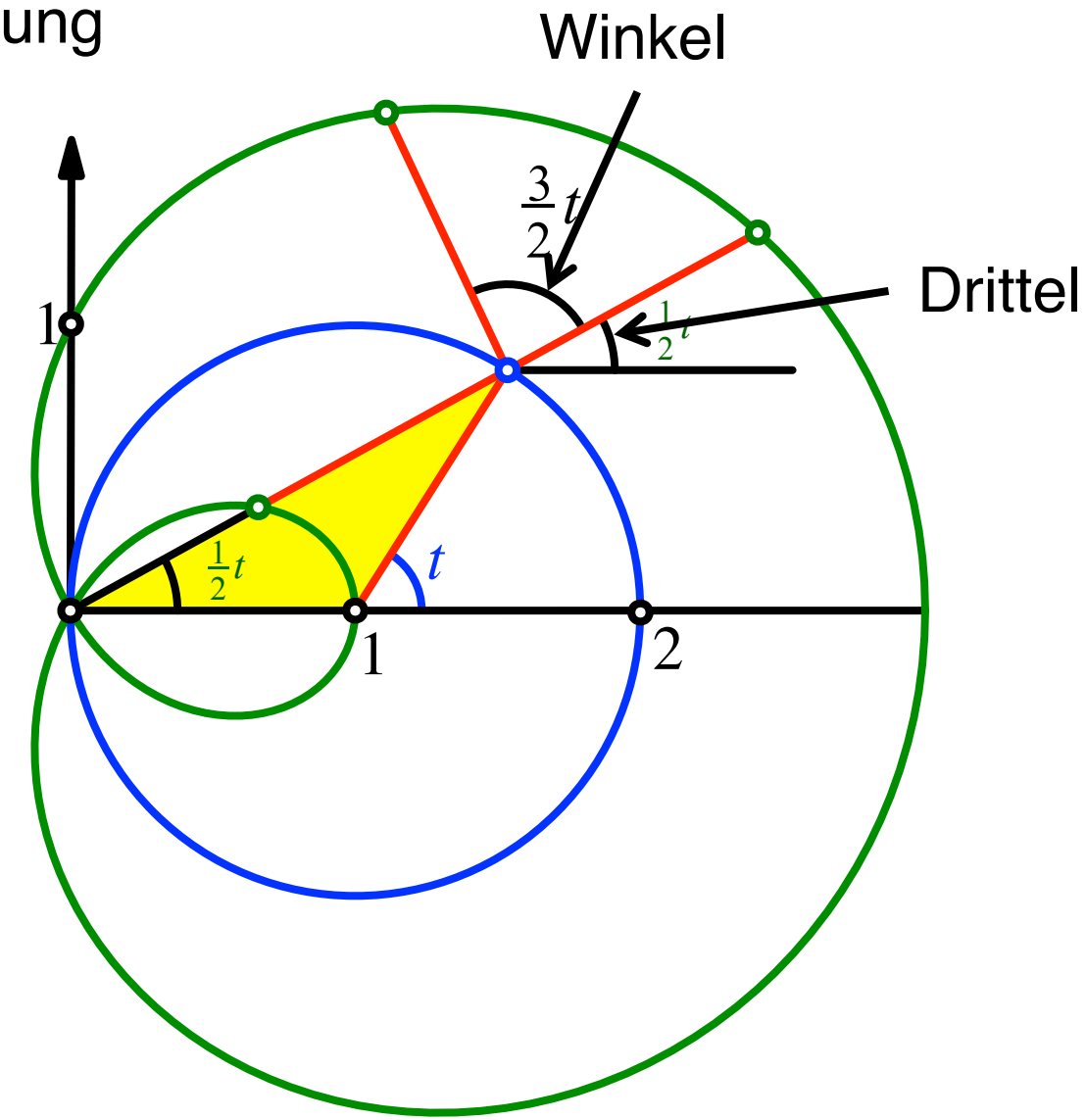
Hundekurve (Pascalsche Schnecke)



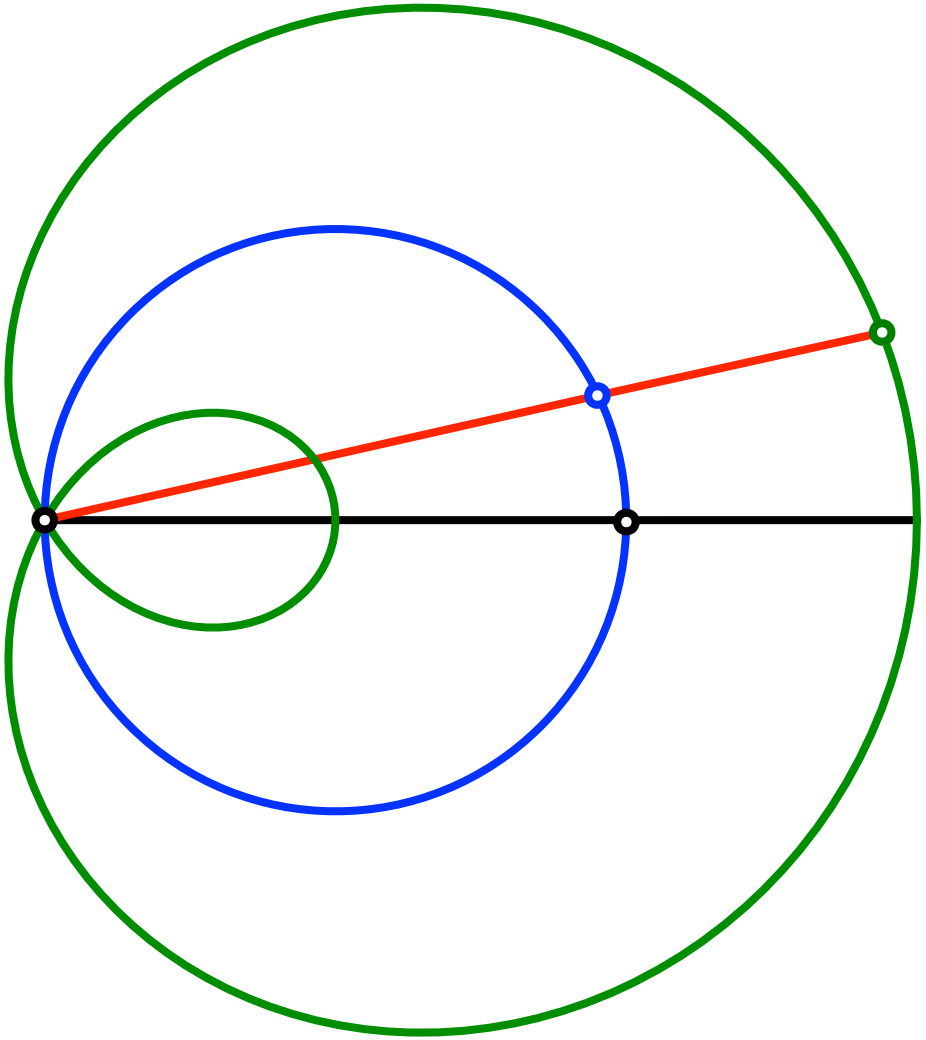
Hundekurve (Pascalsche Schnecke)



Winkeldrittelung

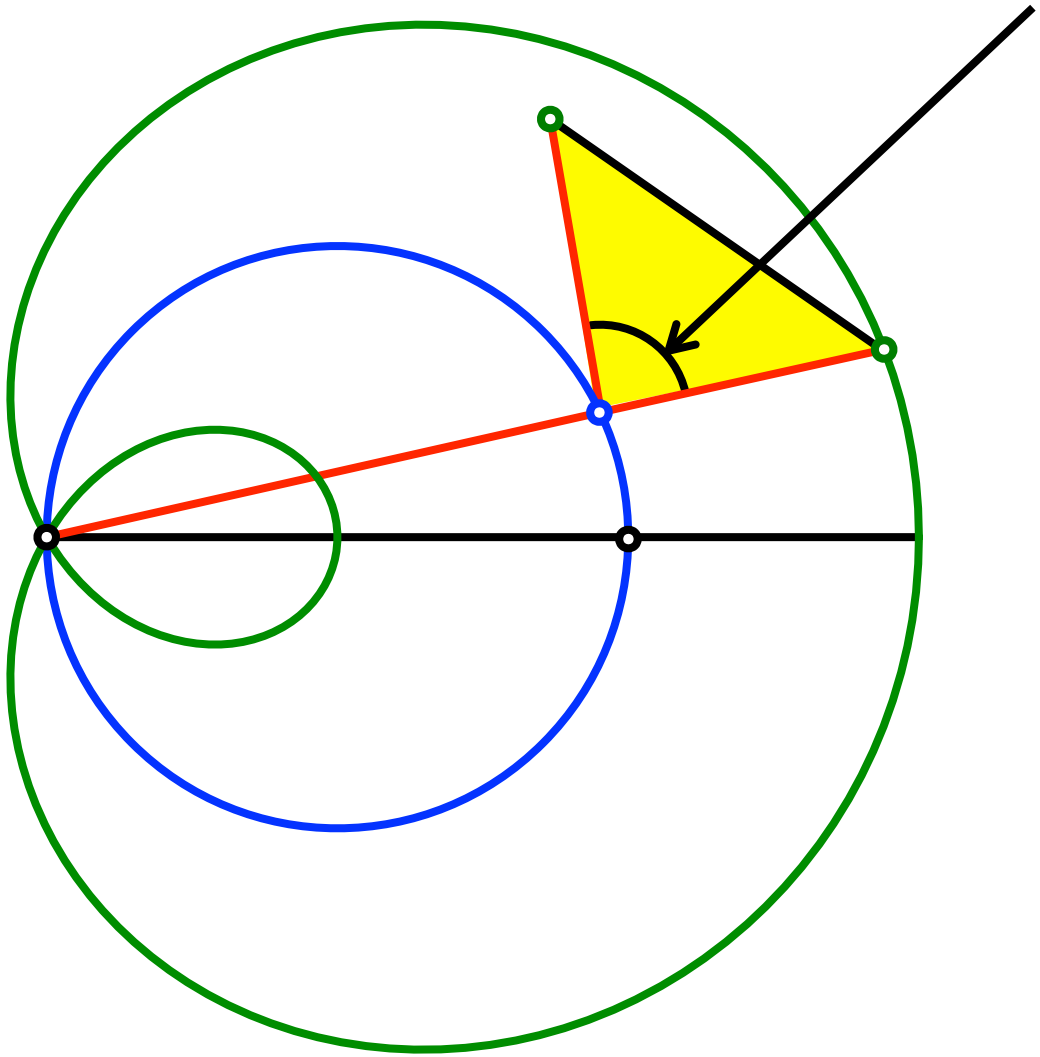


Winkeldrittelung



Winkeldrittelung

Gegebener Winkel



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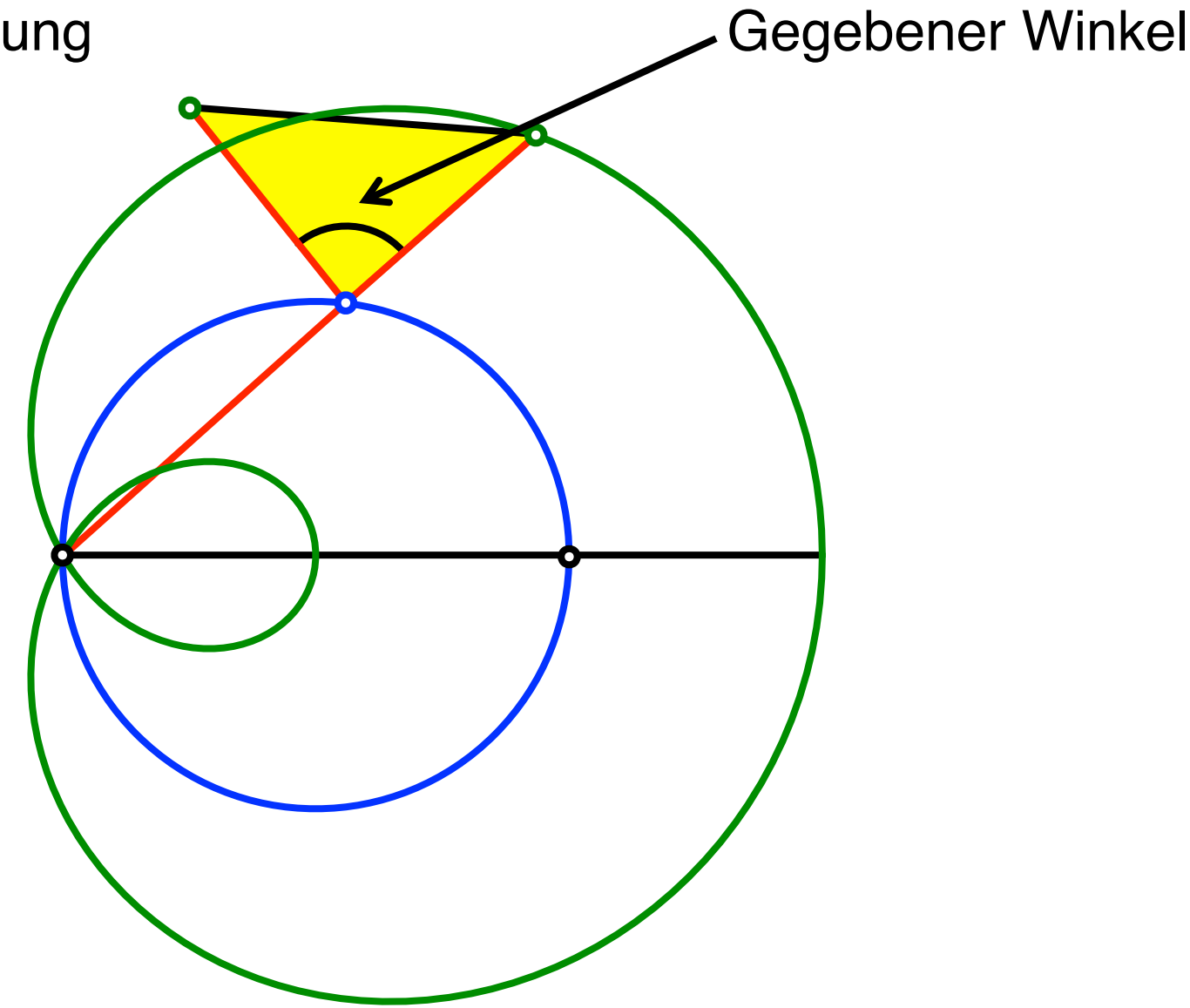
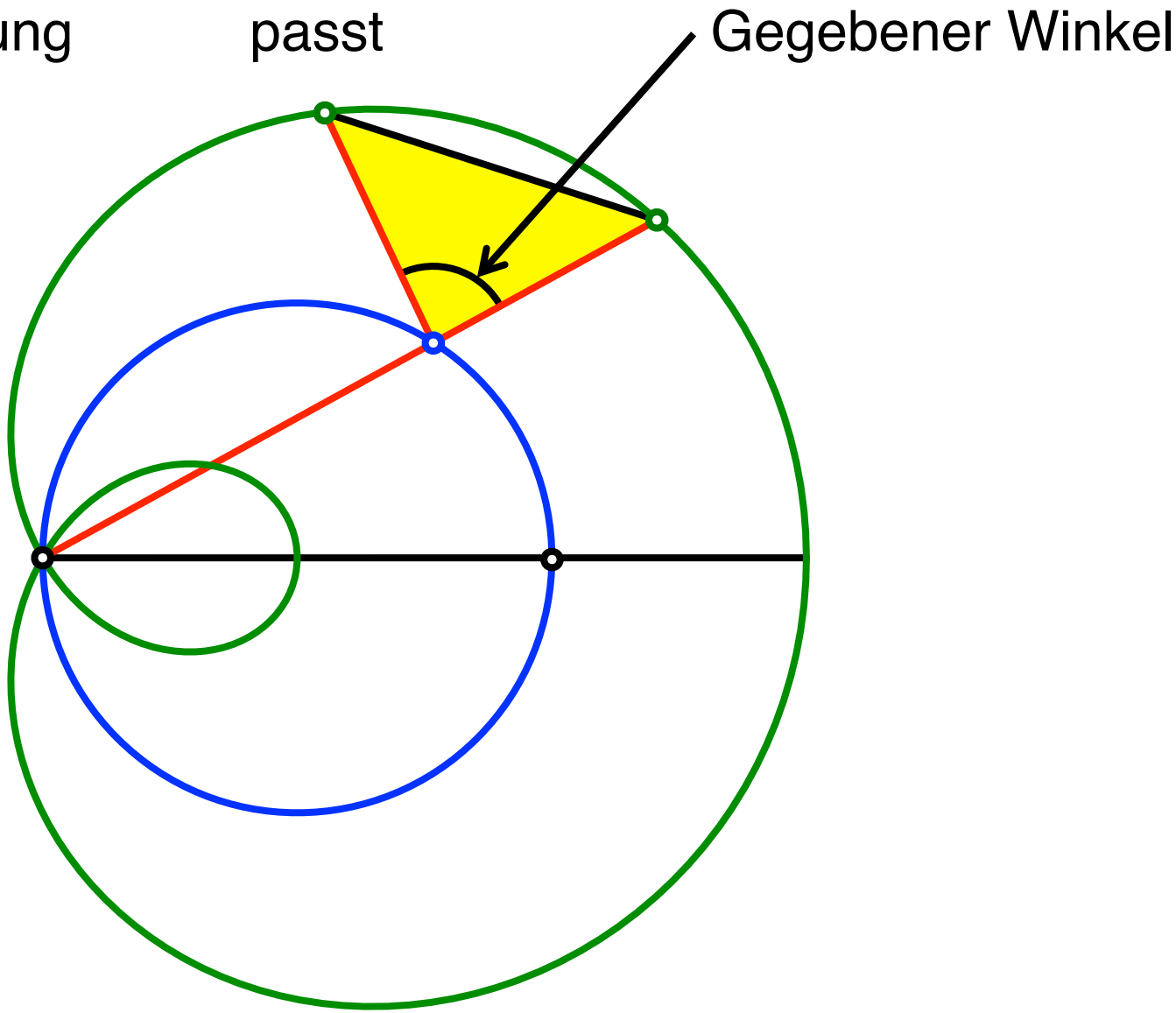
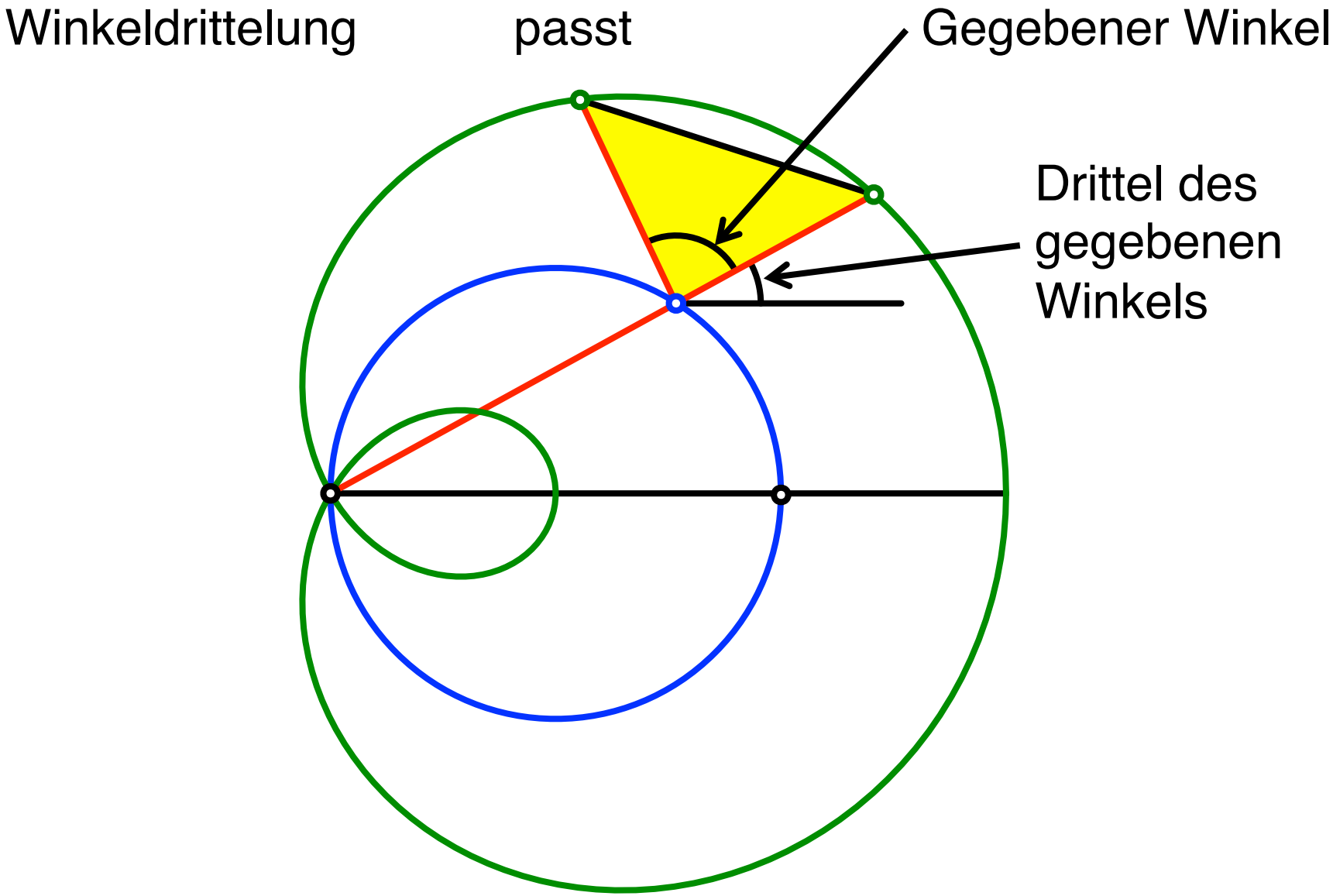
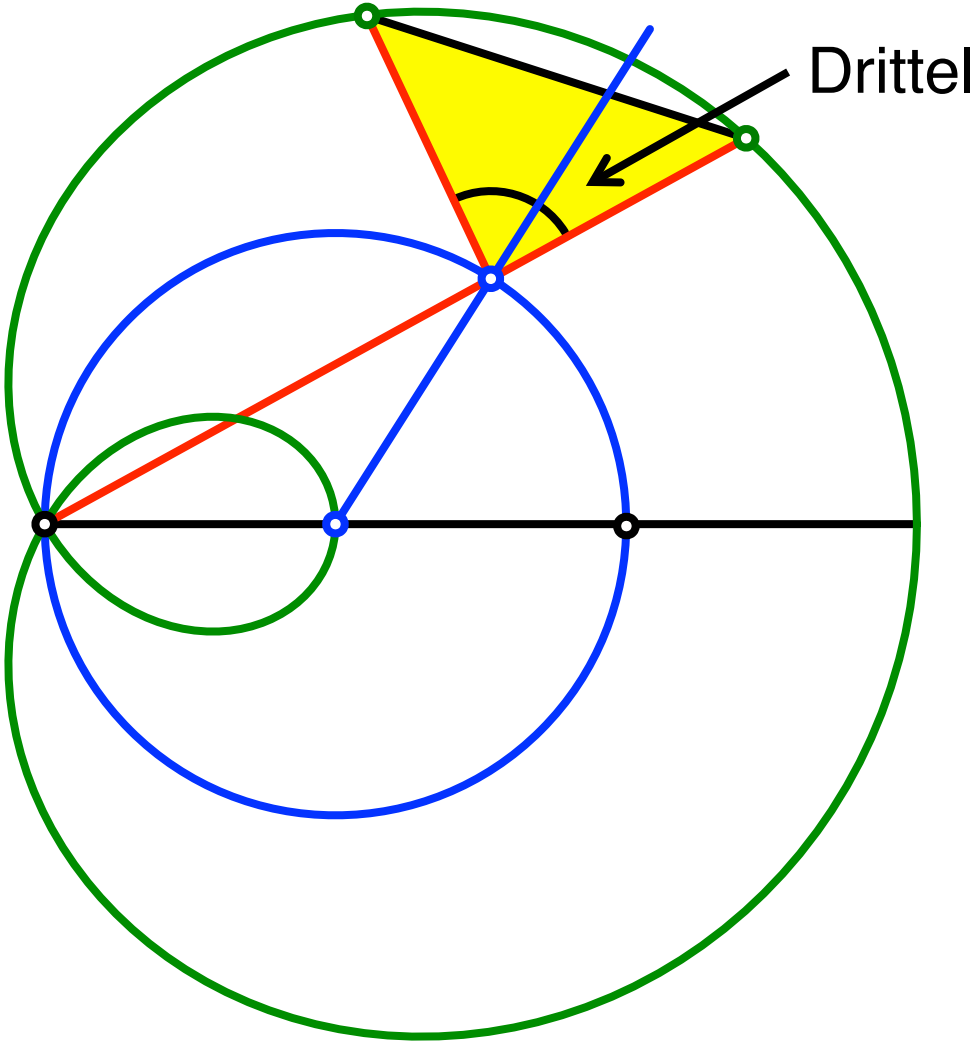


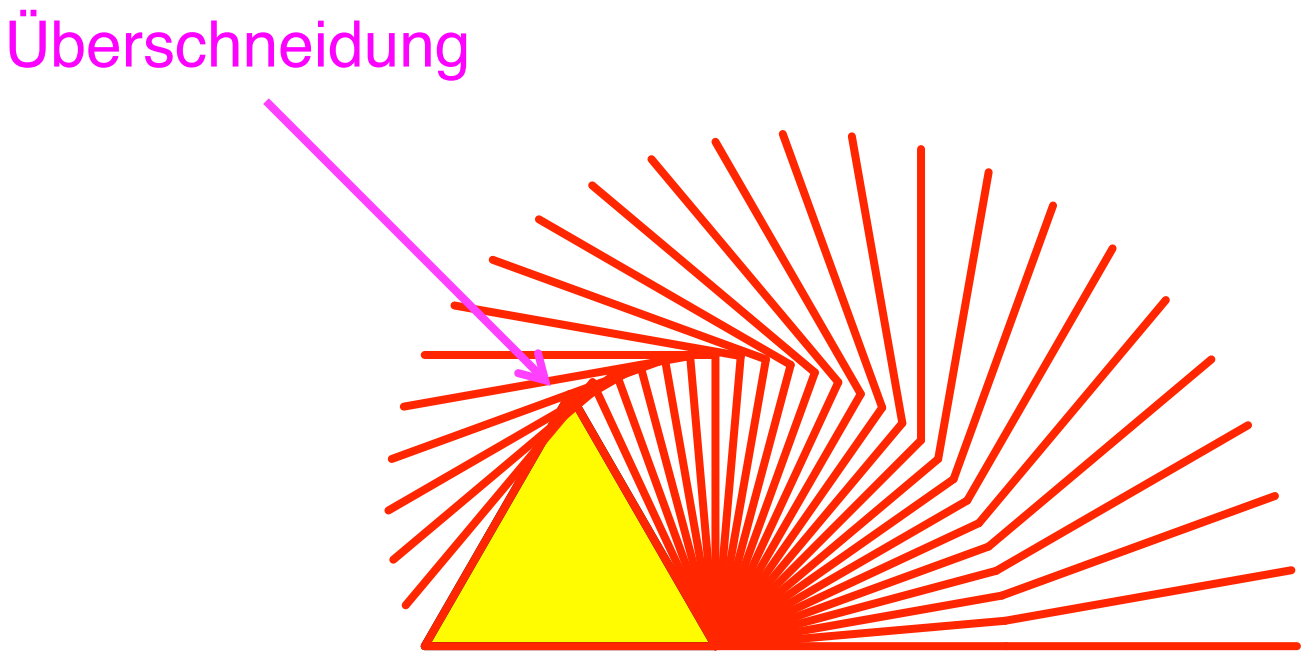
Winkeldrittelung passt



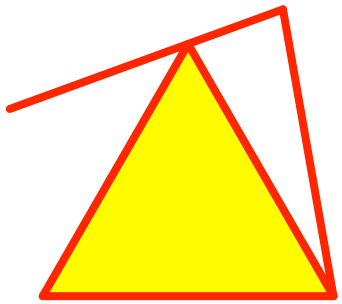

Winkeldrittelung
passt
Gegebener Winkel
Drittel des
gegebenen
Winkels

Winkeldrittelung passt Gegebener Winkel




Überschneidung

Kollision



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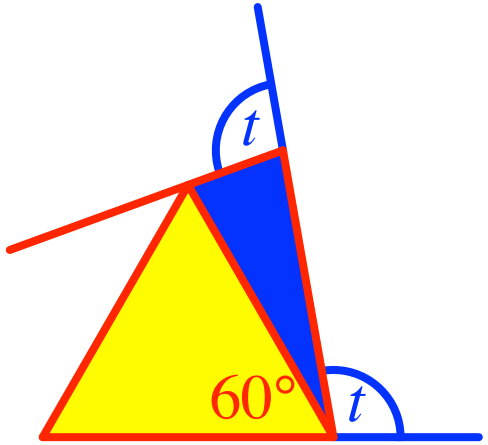

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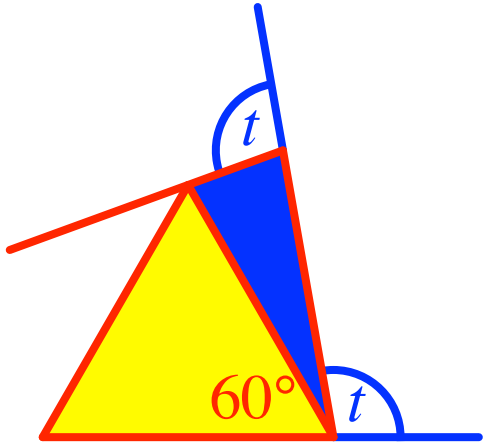
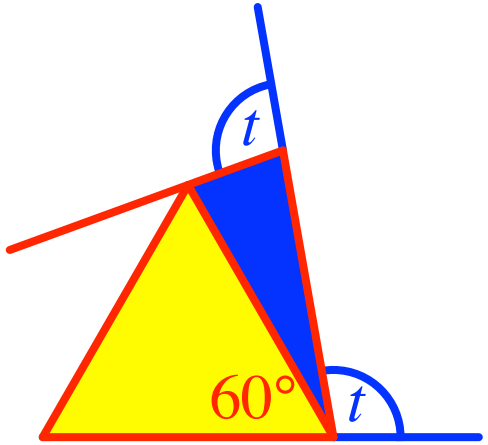


Gleichschenkliges Dreieck

Außenwinkel

Basis:	t
Basis:	t
Spitze:	$t + 60°$
Summe:	$3t + 60°$

Kollision



Gleichschenkliges Dreieck

Außenwinkel

Basis: t
Basis: t
Spitze: $t + 60°$

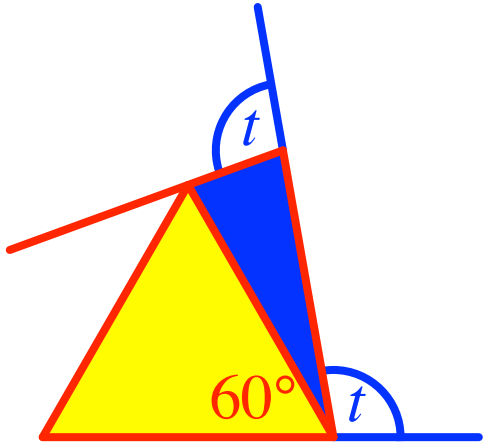
Summe: $3t + 60°$

Mathematik ist das Finden von Invarianten.

Heinz Hopf

Mathe ist, wenn im Stundenplan ein M steht.

Kollision



Gleichschenkliges Dreieck

Außenwinkel

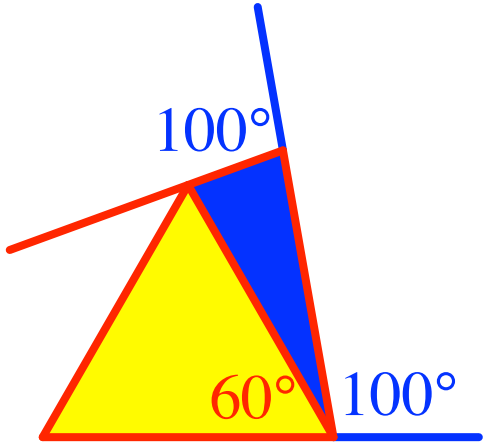
Basis: t
Basis: t
Spitze: $t + 60°$

Summe: $3t + 60°$

Außenwinkelsumme = $360°$

$$3t + 60° = 360°$$
$$t = 100°$$

Kollision



Gleichschenkliges Dreieck

Außenwinkel

Basis: t
Basis: t
Spitze: $t + 60°$

Summe: $3t + 60°$

Außenwinkelsumme = $360°$

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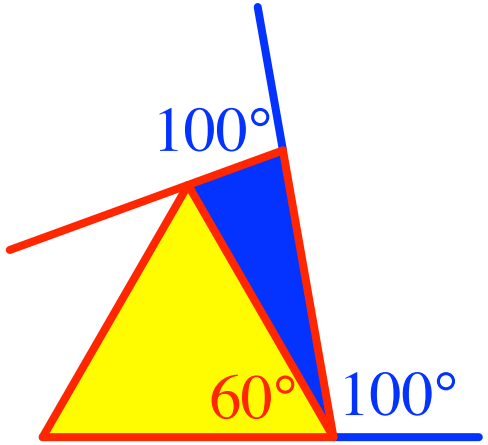
Kollision



$100°$ *nicht* mit
Zirkel und Lineal
konstruierbar

Gleichschenkliges Dreieck

Außenwinkel

Basis: t
Basis: t
Spitze: $t + 60°$

Summe: $3t + 60°$

Außenwinkelsumme = $360°$

$$3t + 60° = 360°$$
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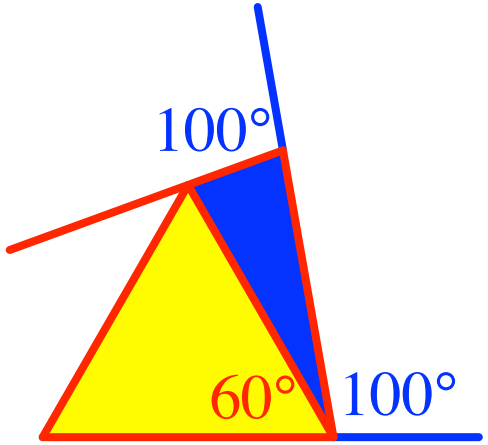
Kollision



$100°$ *nicht* mit
Zirkel und Lineal
konstruierbar

Kubische Gleichungen

Gleichschenkliges Dreieck

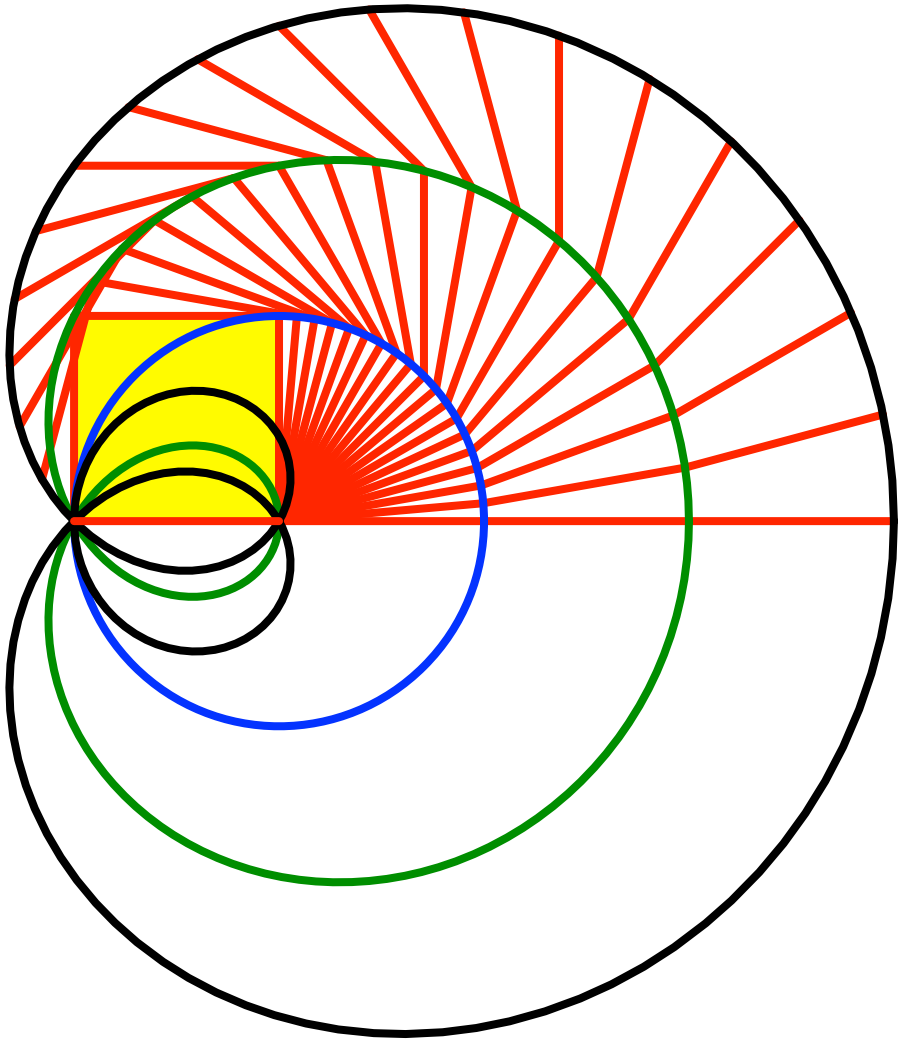
Außenwinkel

Basis:	t
Basis:	t
Spitze:	$t + 60°$
Summe:	$3t + 60°$

Außenwinkelsumme $= 360°$

$$3t + 60° = 360°$$
$$t = 100°$$

Aufwickeln zum Quadrat



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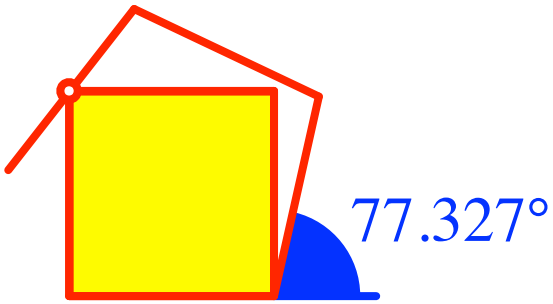
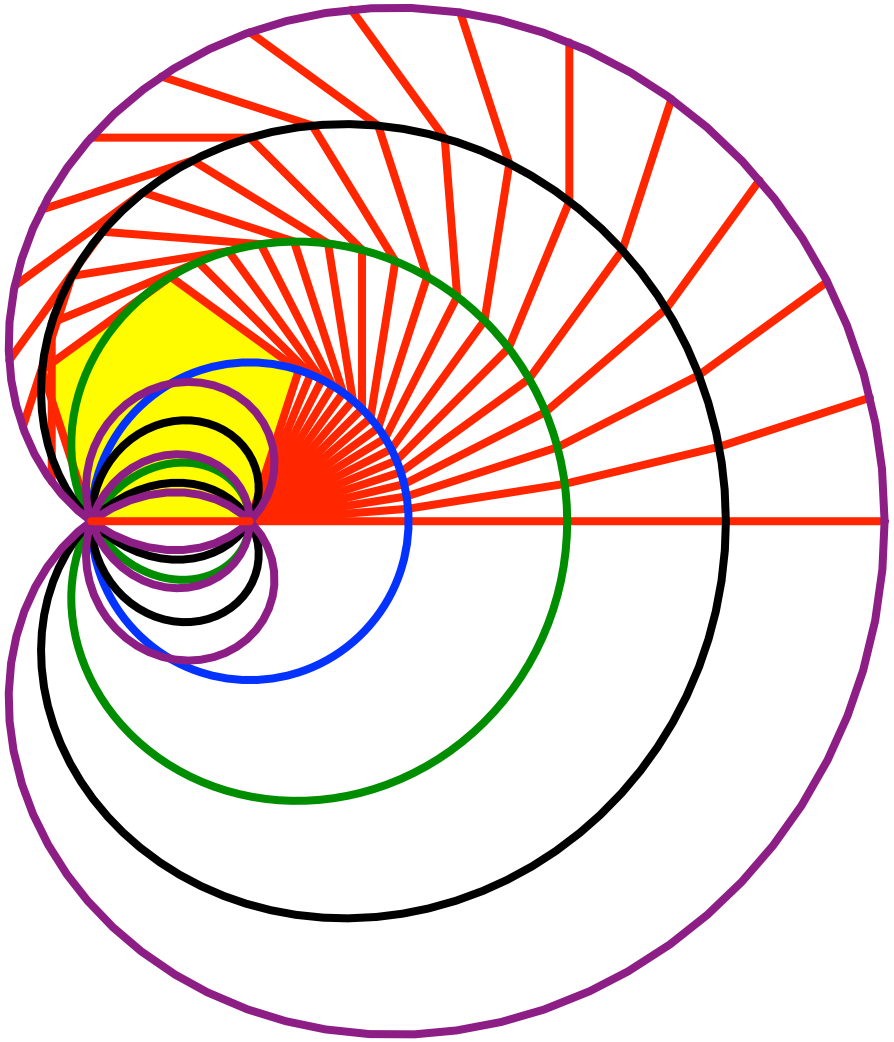


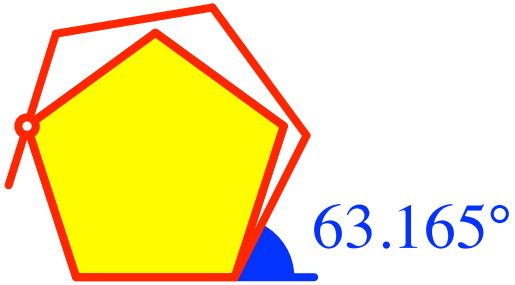
Gleichungen zwölften Grades

Aufwickeln zum Fünfeck



Von einem, der auszog, ...

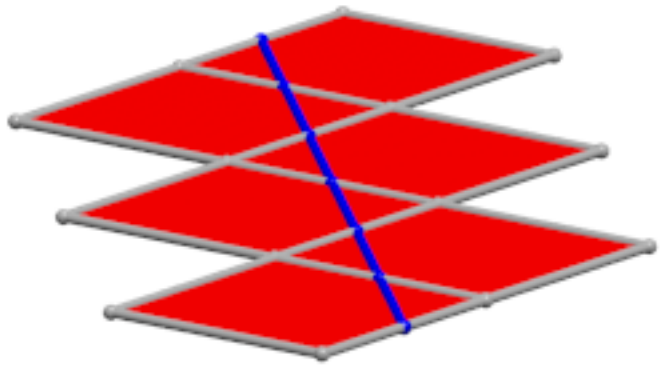
Kollision



Gleichungen vom Grad 56

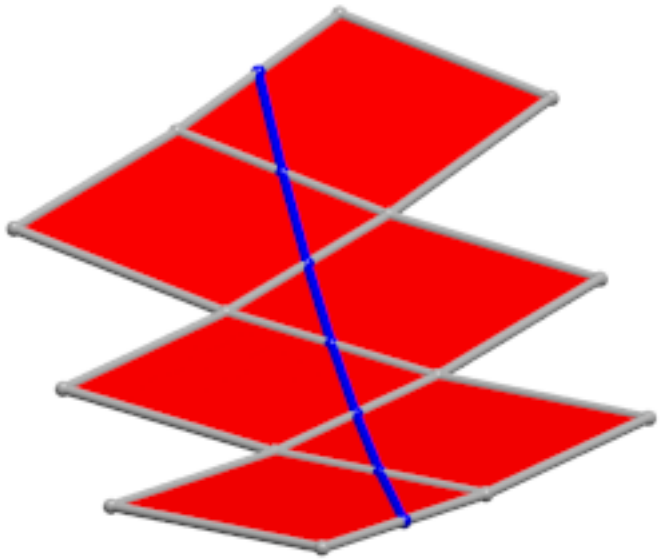
Von einem, der auszog, das Gruseln zu lernen

Aufwickeln zum Würfel



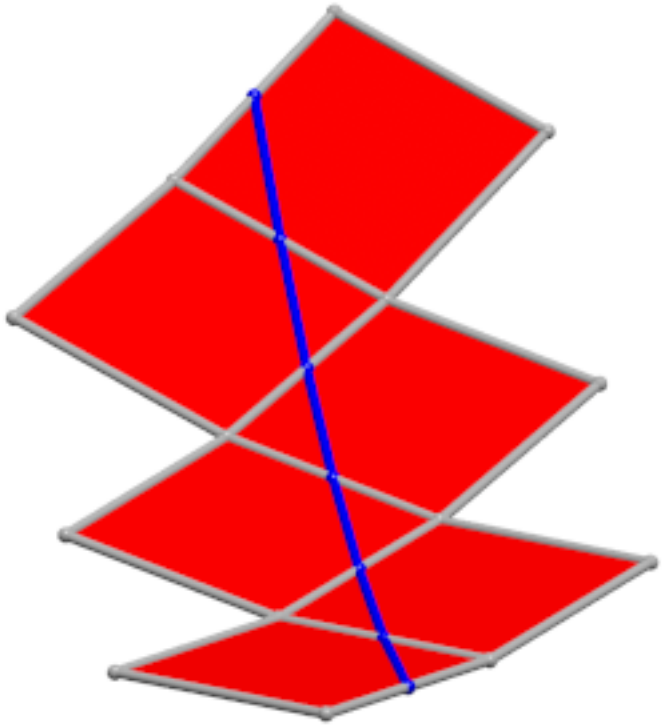
Roboter mit fünf Drehachsen

5°



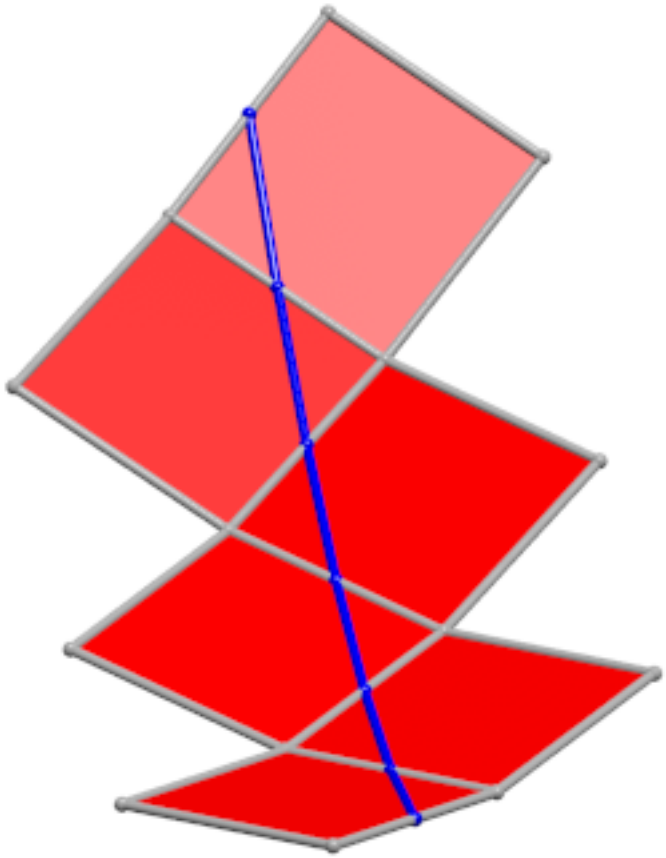
Roboter mit fünf Drehachsen

10°



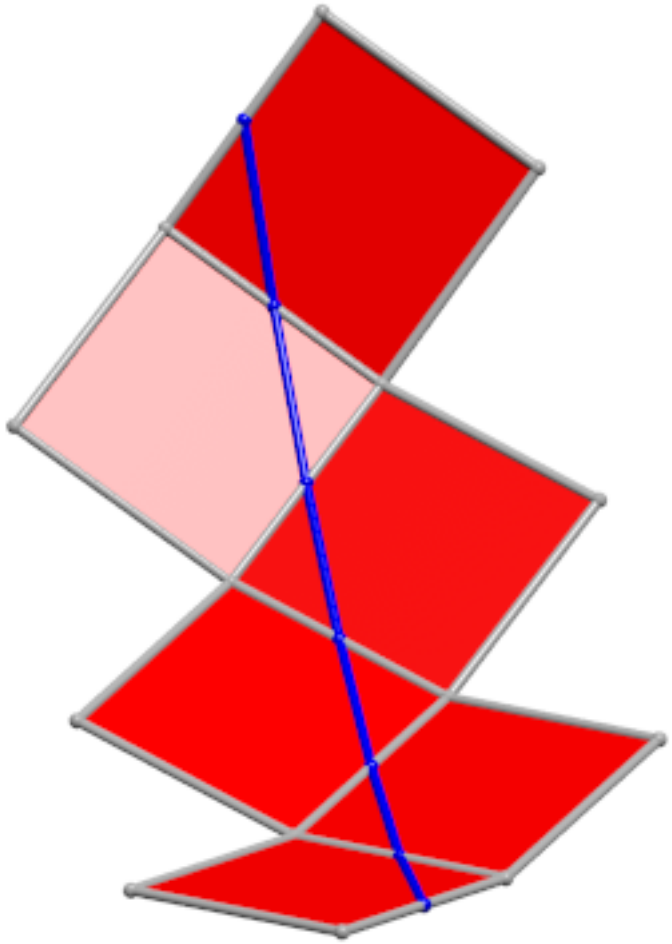
Roboter mit fünf Drehachsen

15°



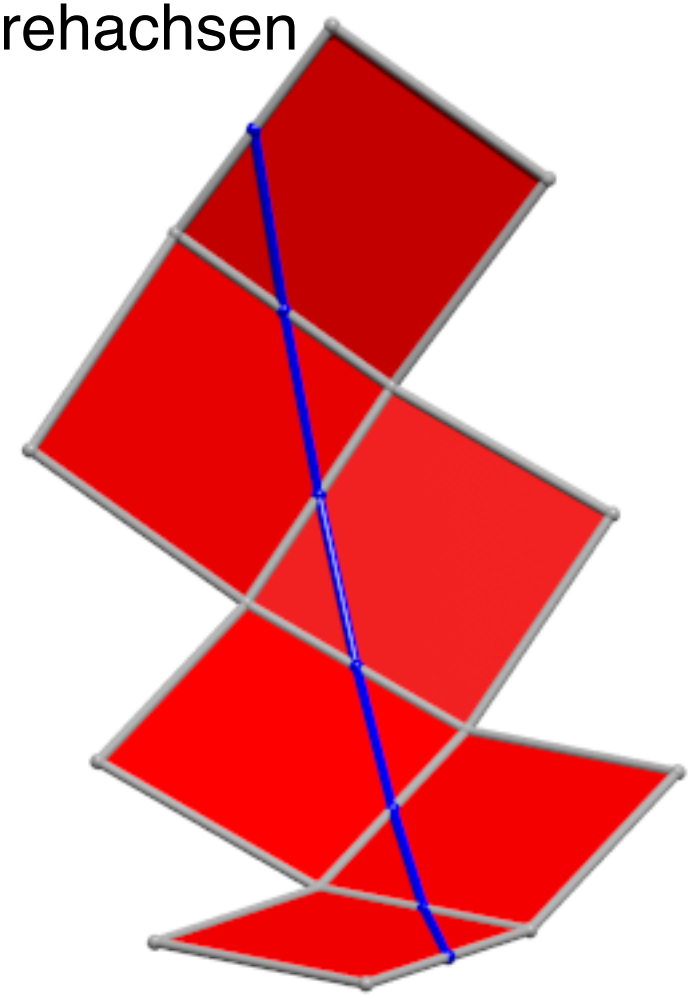
Roboter mit fünf Drehachsen

20°



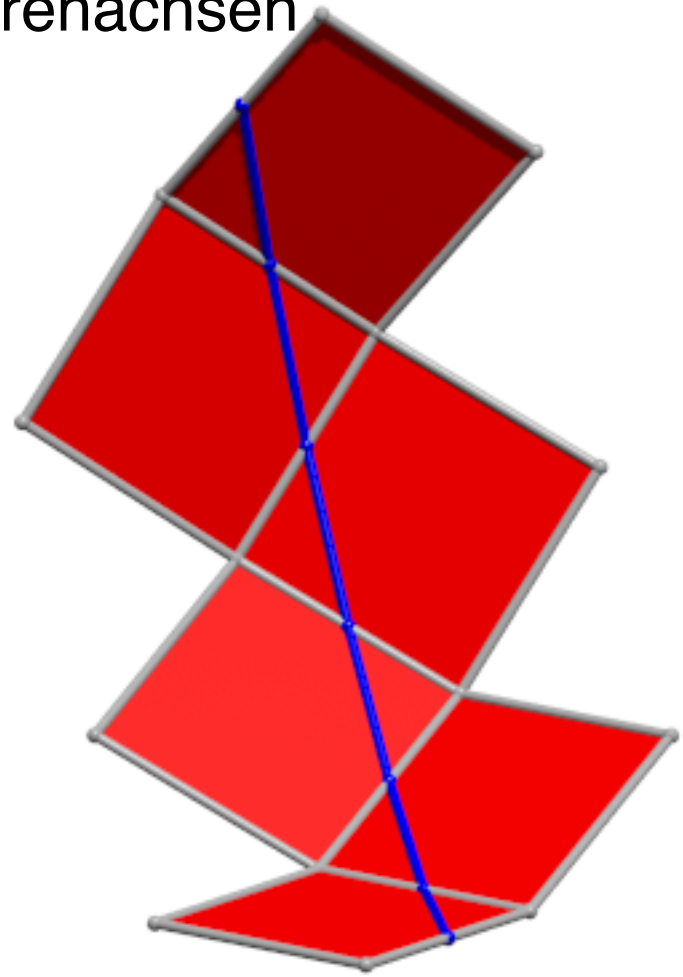
Roboter mit fünf Drehachsen

25°

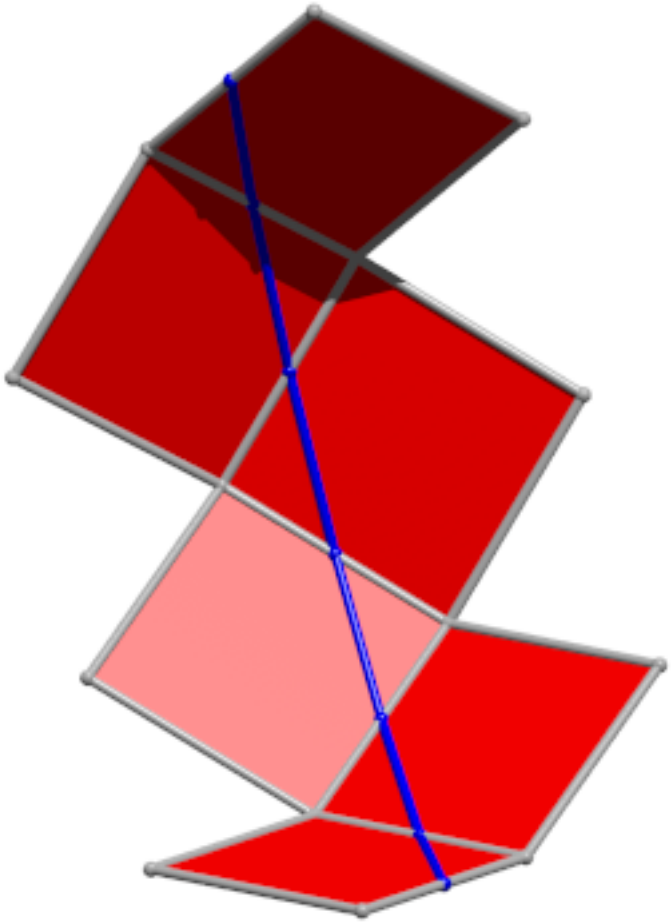


Roboter mit fünf Drehachsen

30°

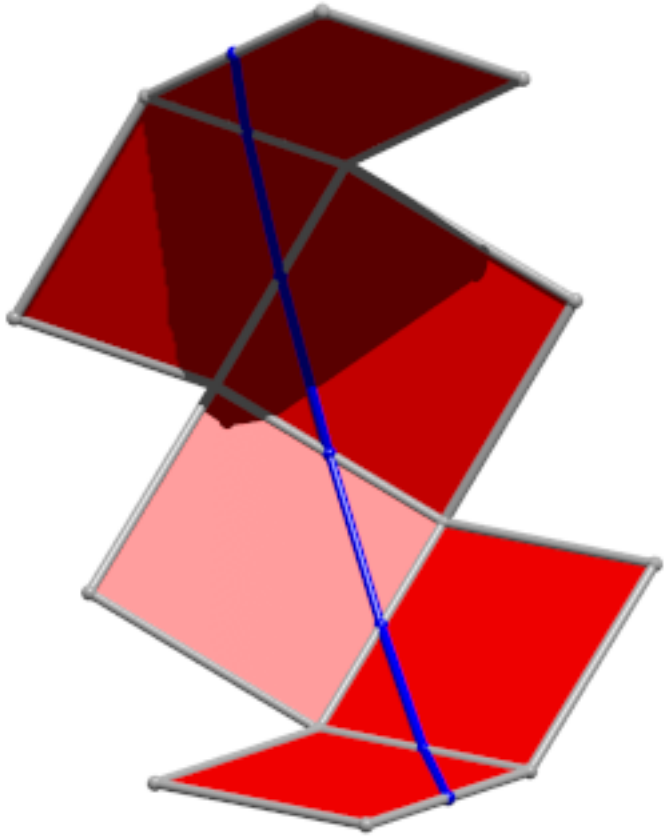


Roboter mit fünf Drehachsen 35°



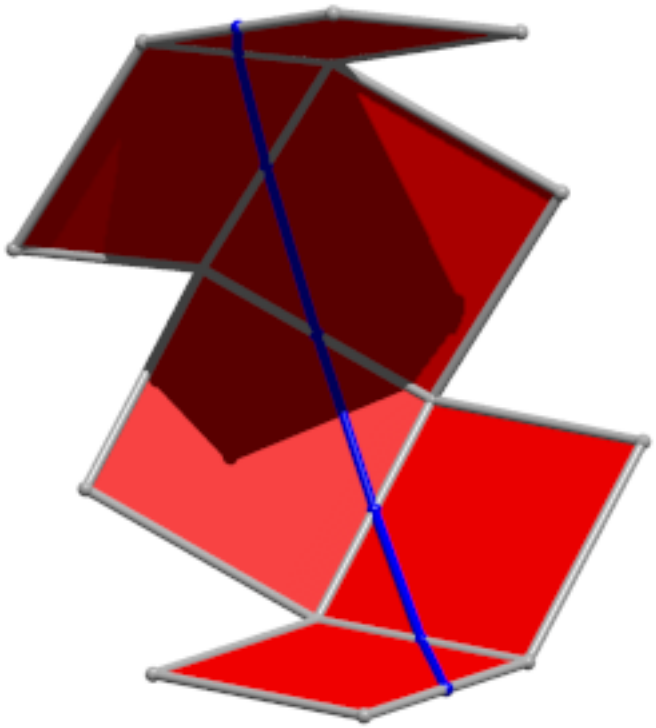
Roboter mit fünf Drehachsen

40°



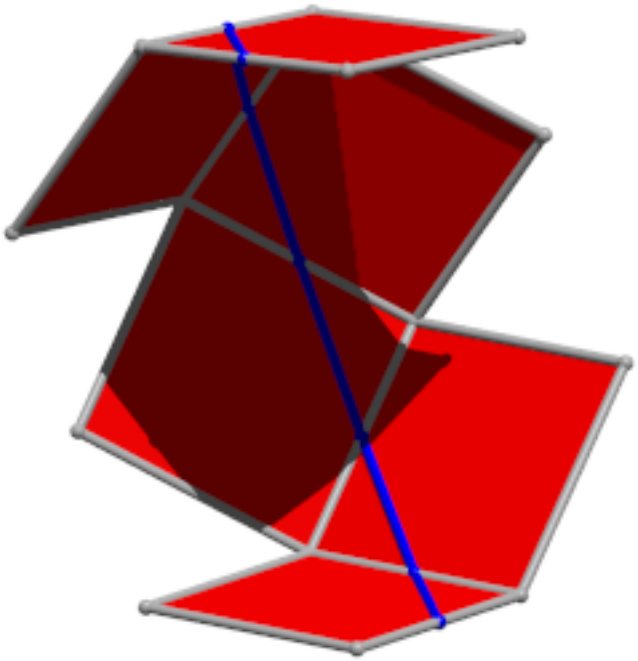
Roboter mit fünf Drehachsen

45°



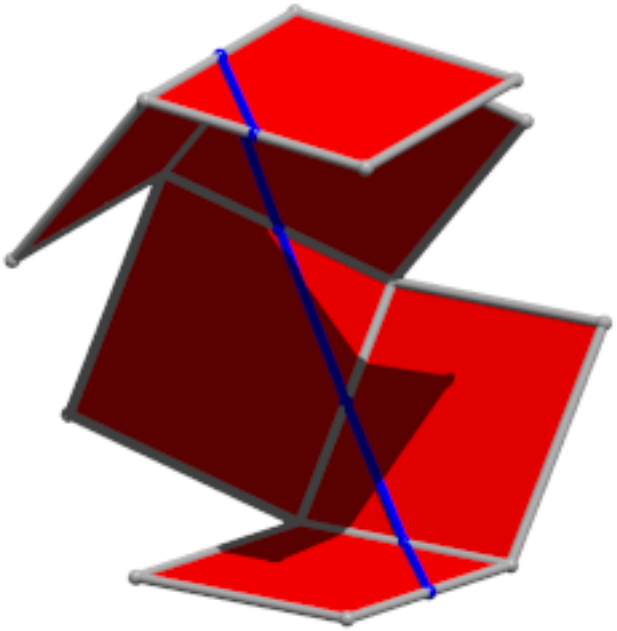
Roboter mit fünf Drehachsen

50°

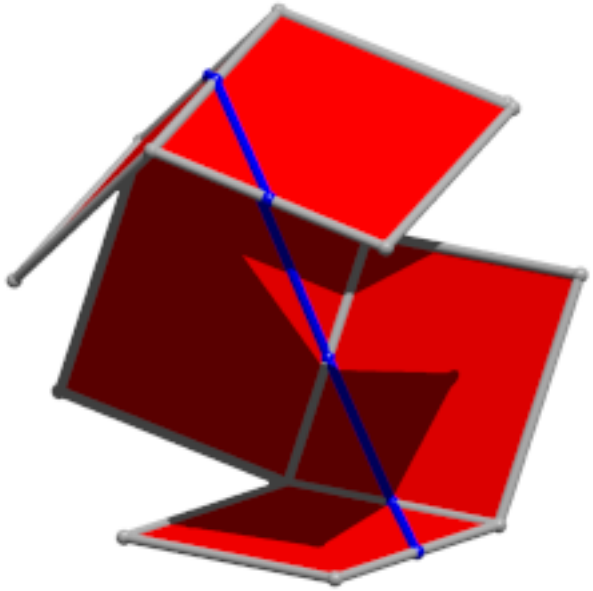


Roboter mit fünf Drehachsen

55°

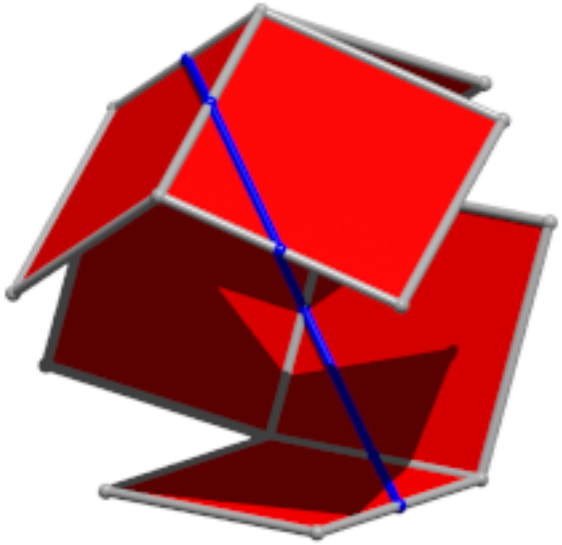


Roboter mit fünf Drehachsen 60°



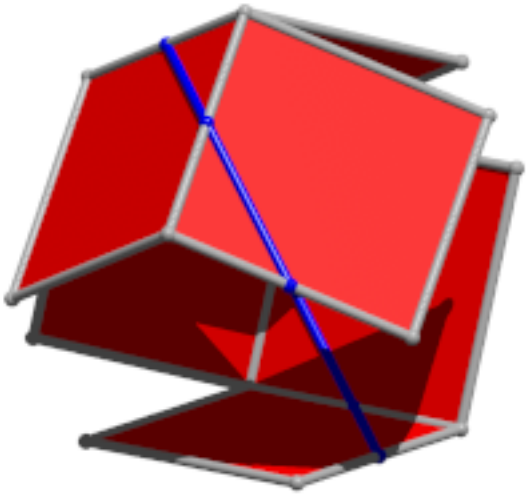
Roboter mit fünf Drehachsen

65°



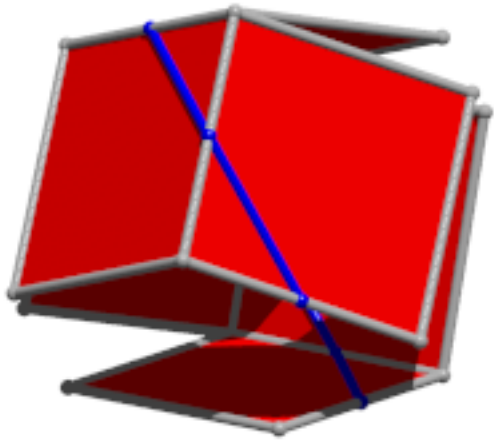
Roboter mit fünf Drehachsen

70°



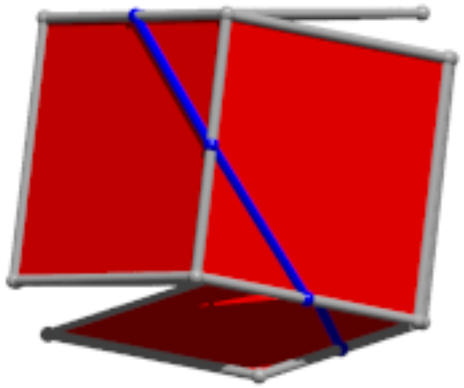
Roboter mit fünf Drehachsen

75°



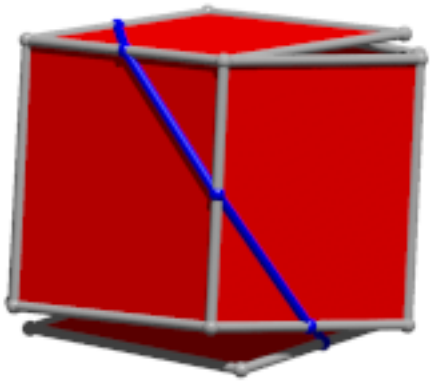
Roboter mit fünf Drehachsen

80°

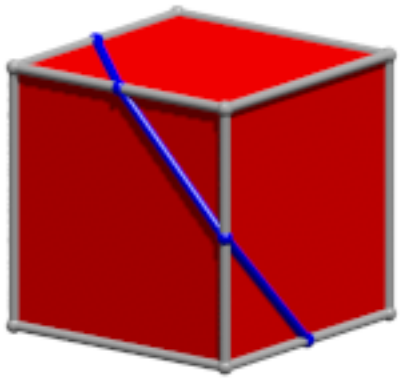


Roboter mit fünf Drehachsen

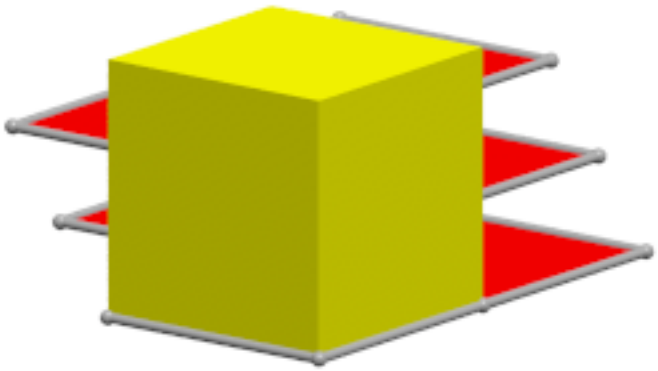
85°



Roboter mit fünf Drehachsen 90°

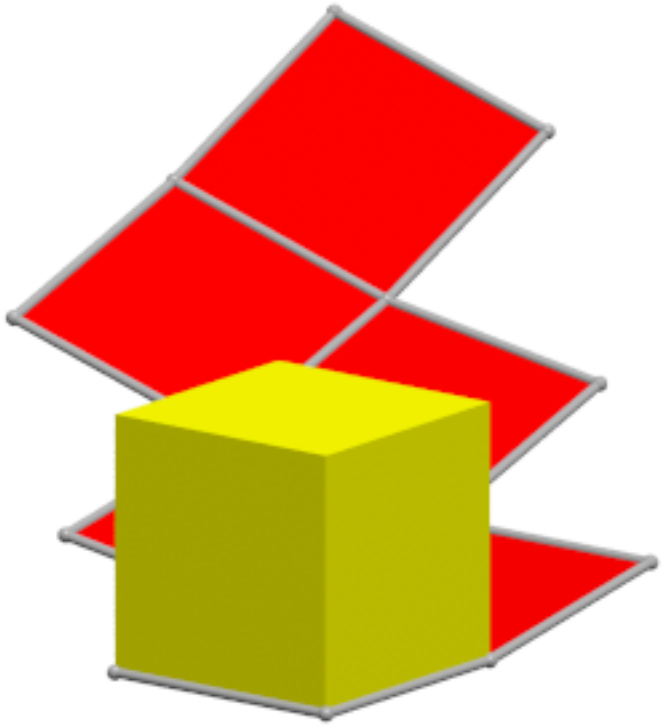


Kollision?



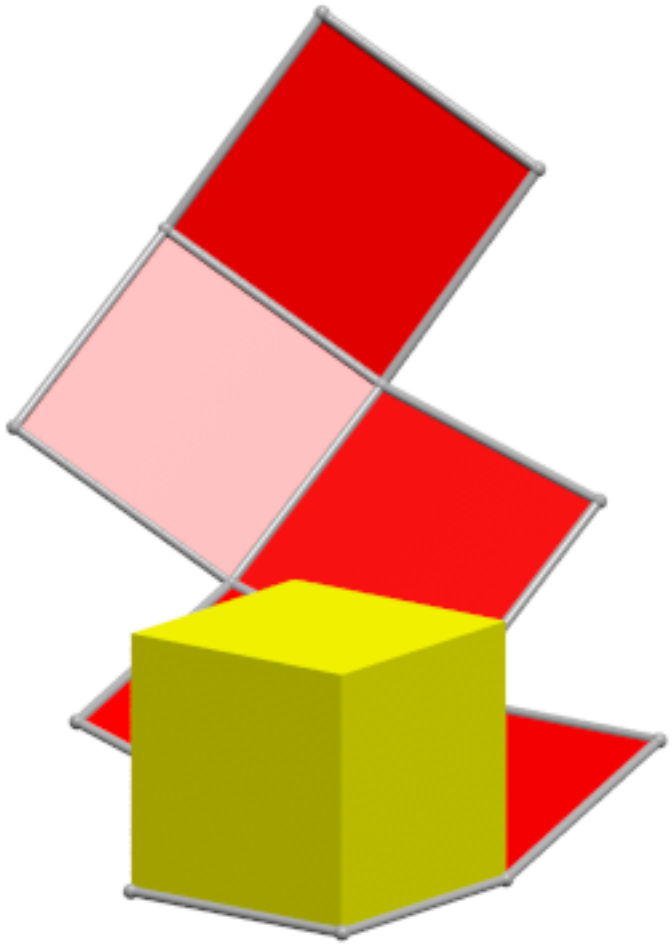
Kollision?

10°



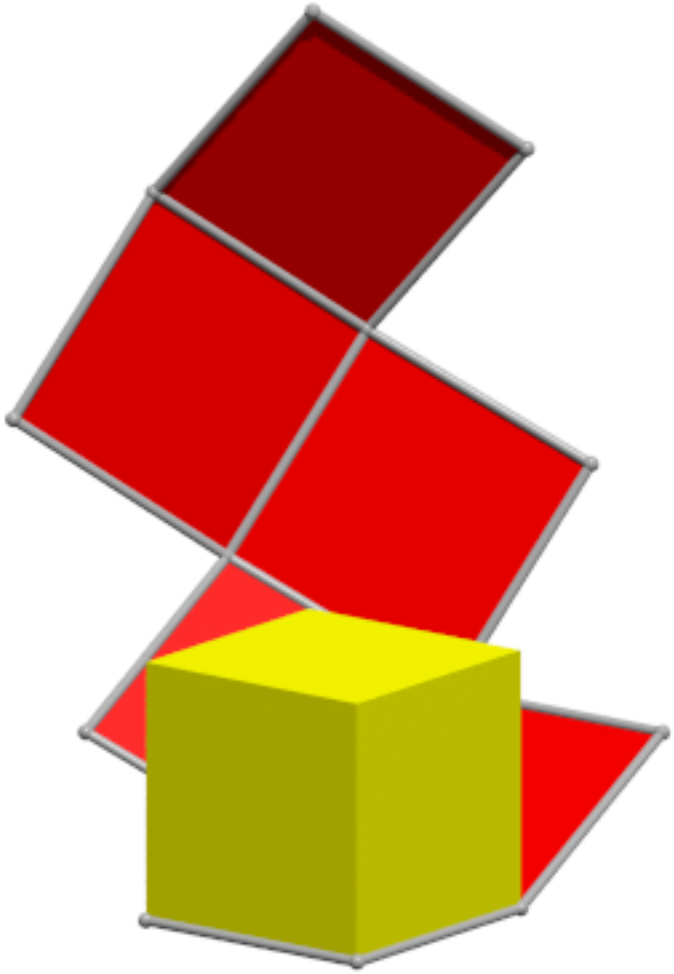
Kollision?

20°



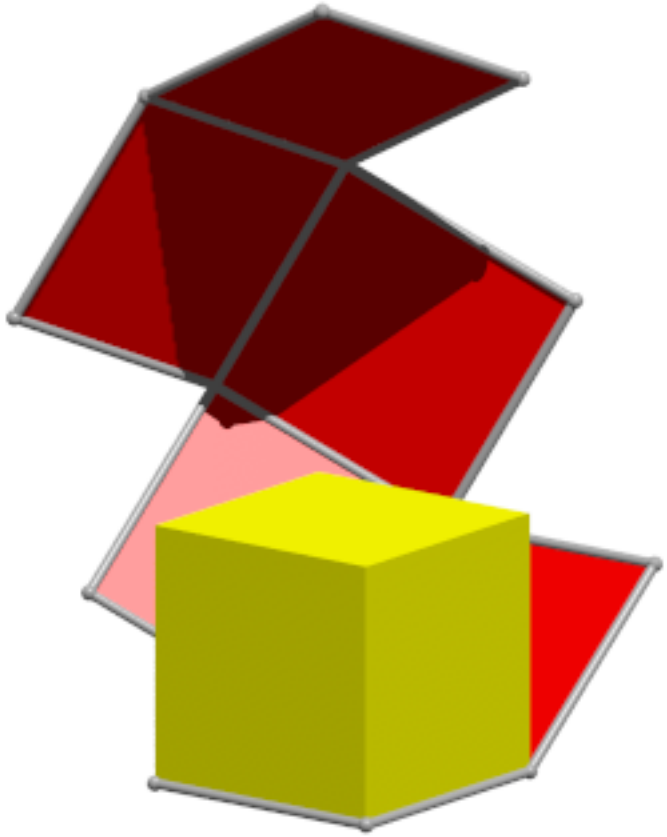
Kollision?

30°



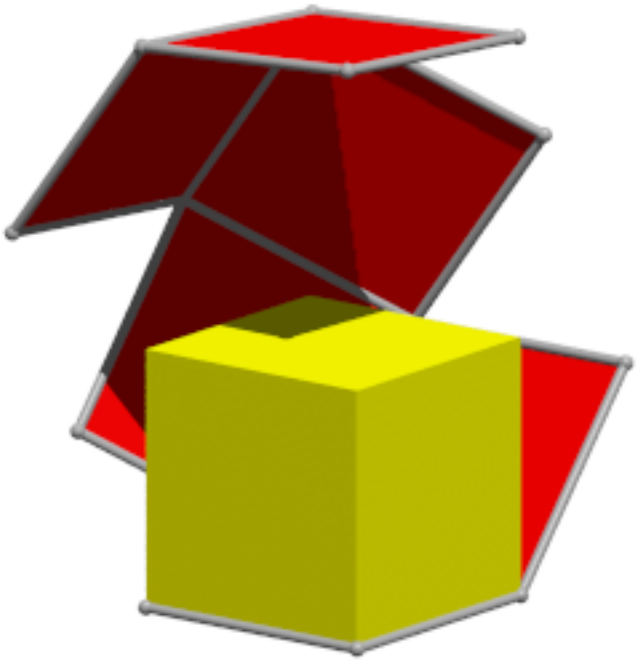
Kollision?

40°



Kollision?

50°

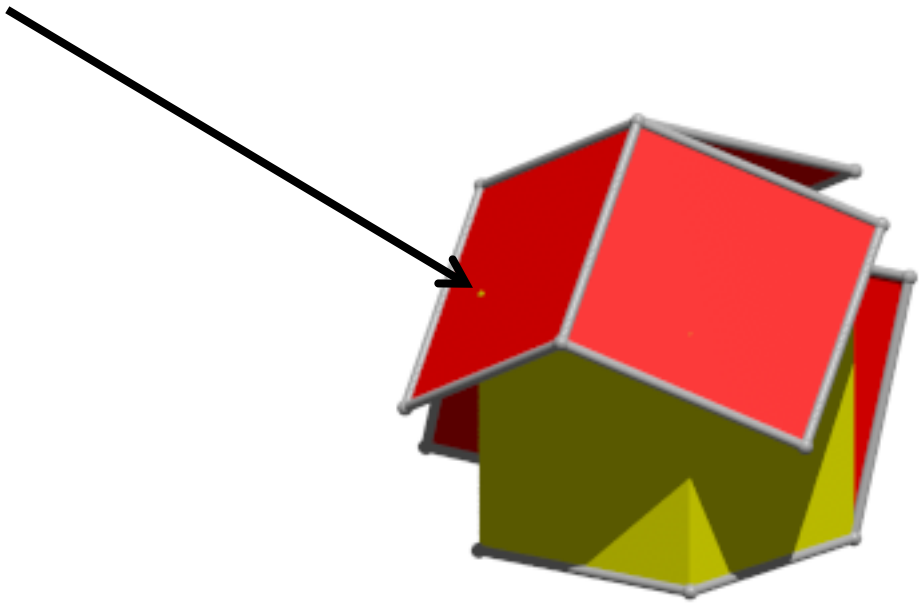


Kollision?

60°

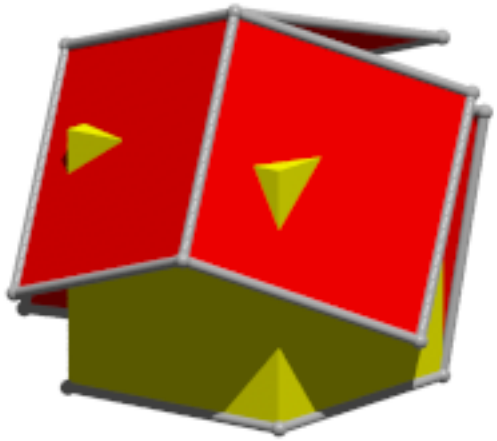
Kollision!

70°



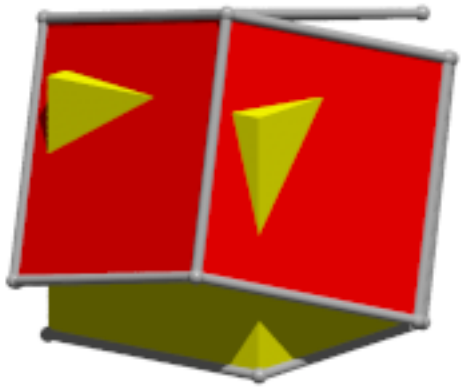
Verschneidung

75°



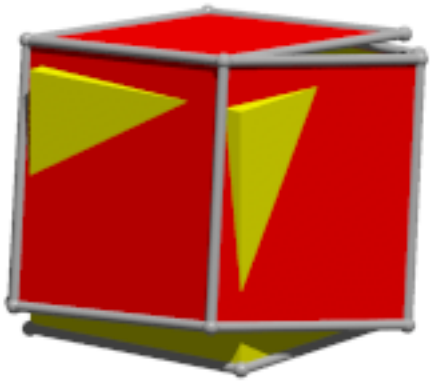
Verschneidung

80°



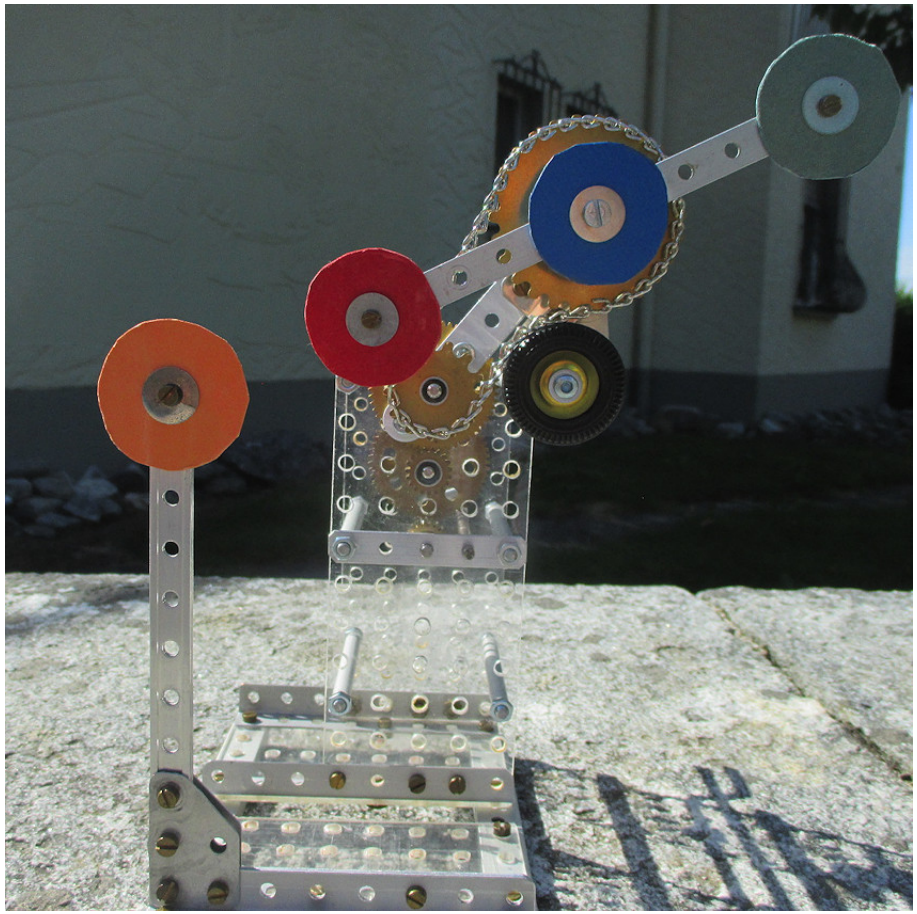
Verschneidung

85°



90°

Danke



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