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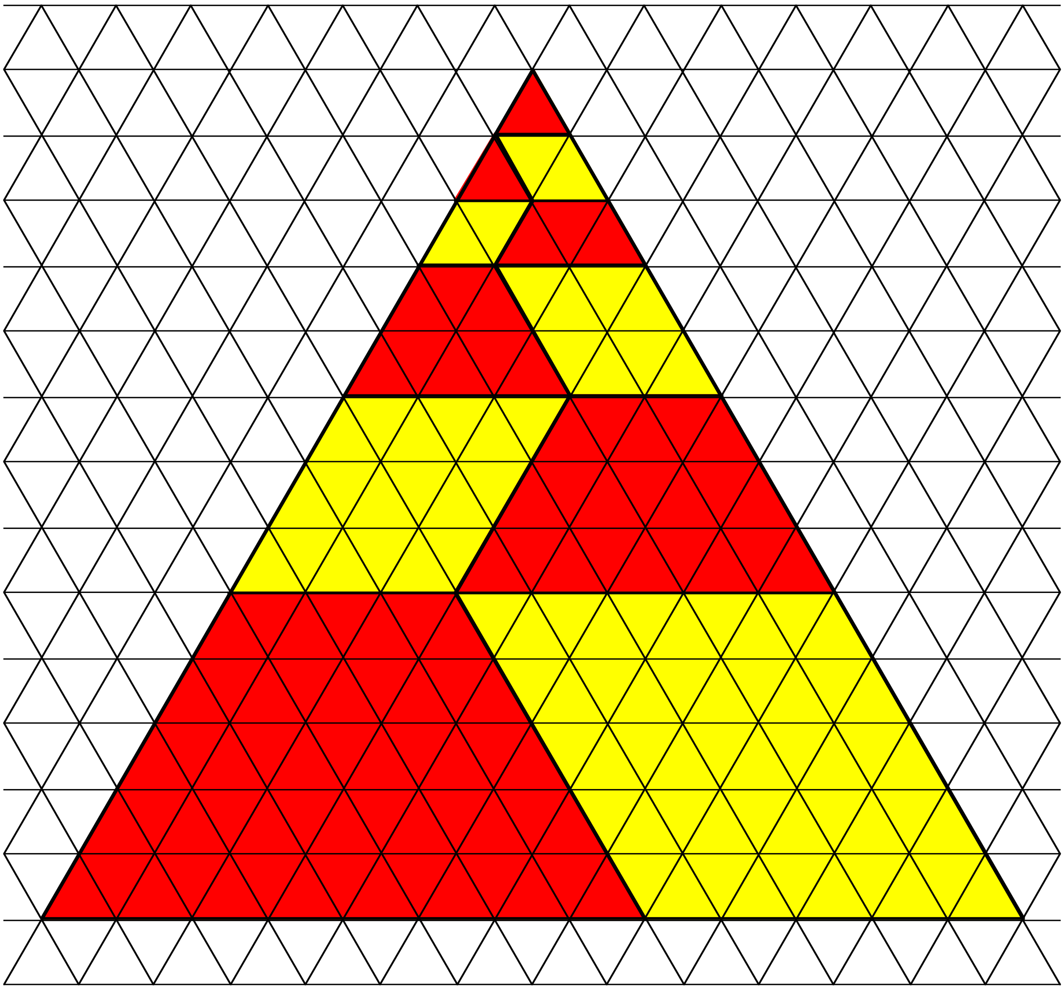
Fibonacci in the Triangular Lattice

1 The Fibonacci Triangle

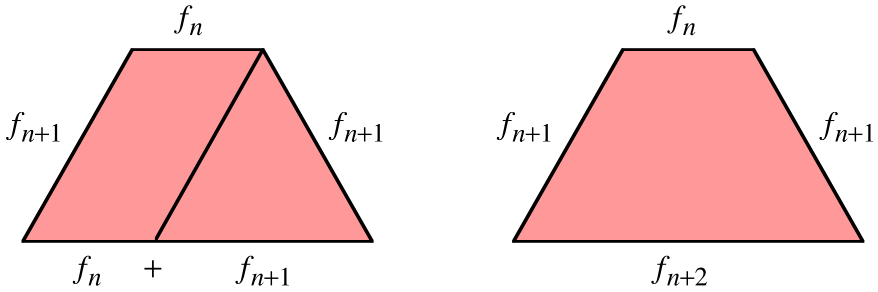
In a regular triangular lattice we draw on top a red regular unit triangle, underneath a yellow rhombus and beneath a second red triangle. Under the red triangle an other yellow rhombus and beneath a red isosceles trapezium. And now always under the red trapezium a yellow rhombus and under the yellow rhombus a red trapezium.



Filling the triangle with trapeziums and rhombuses

Now the sides of the rhombuses are the Fibonacci numbers. The top side of a trapezium, the two isosceles sides and the base are three consecutive Fibonacci numbers.

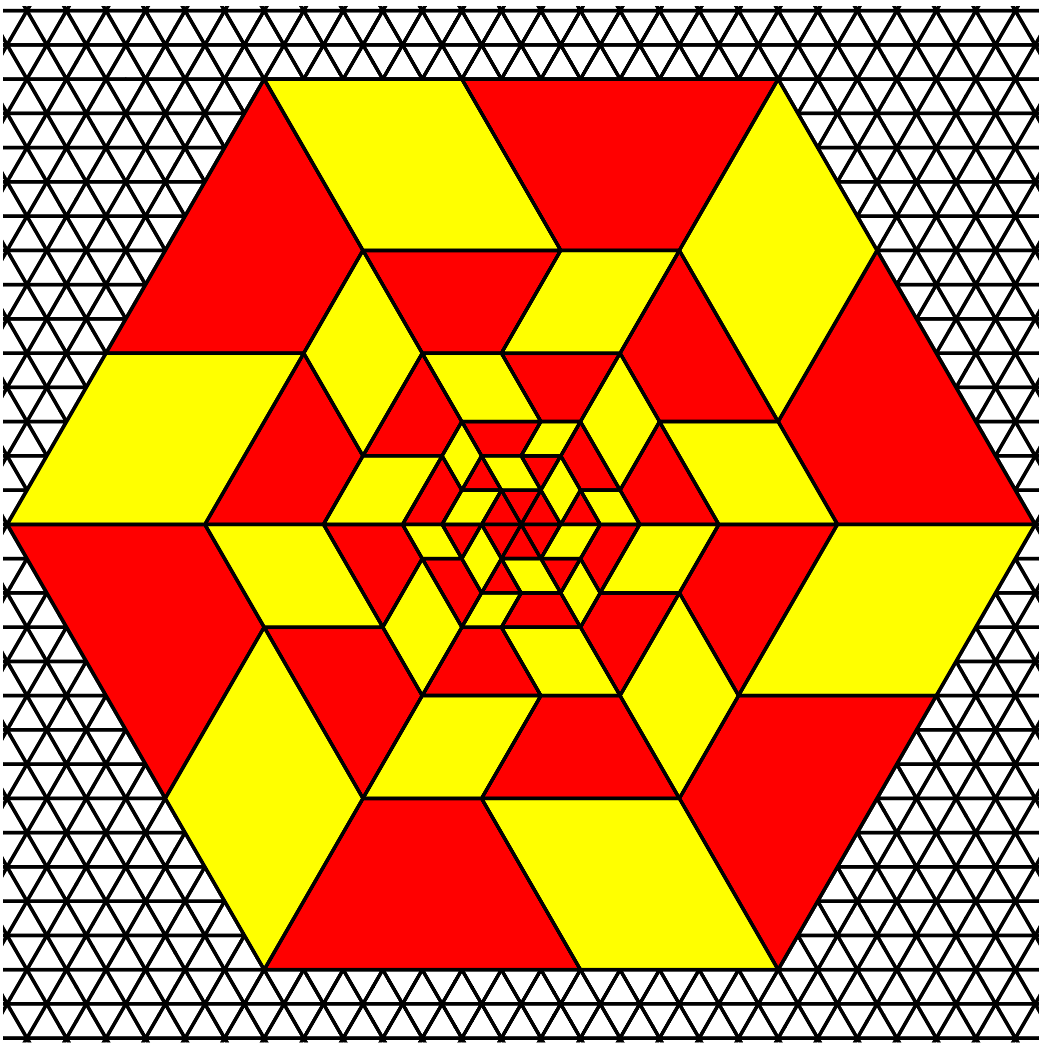
The proof is simple:



Proof

2 The Fibonacci Hexagon

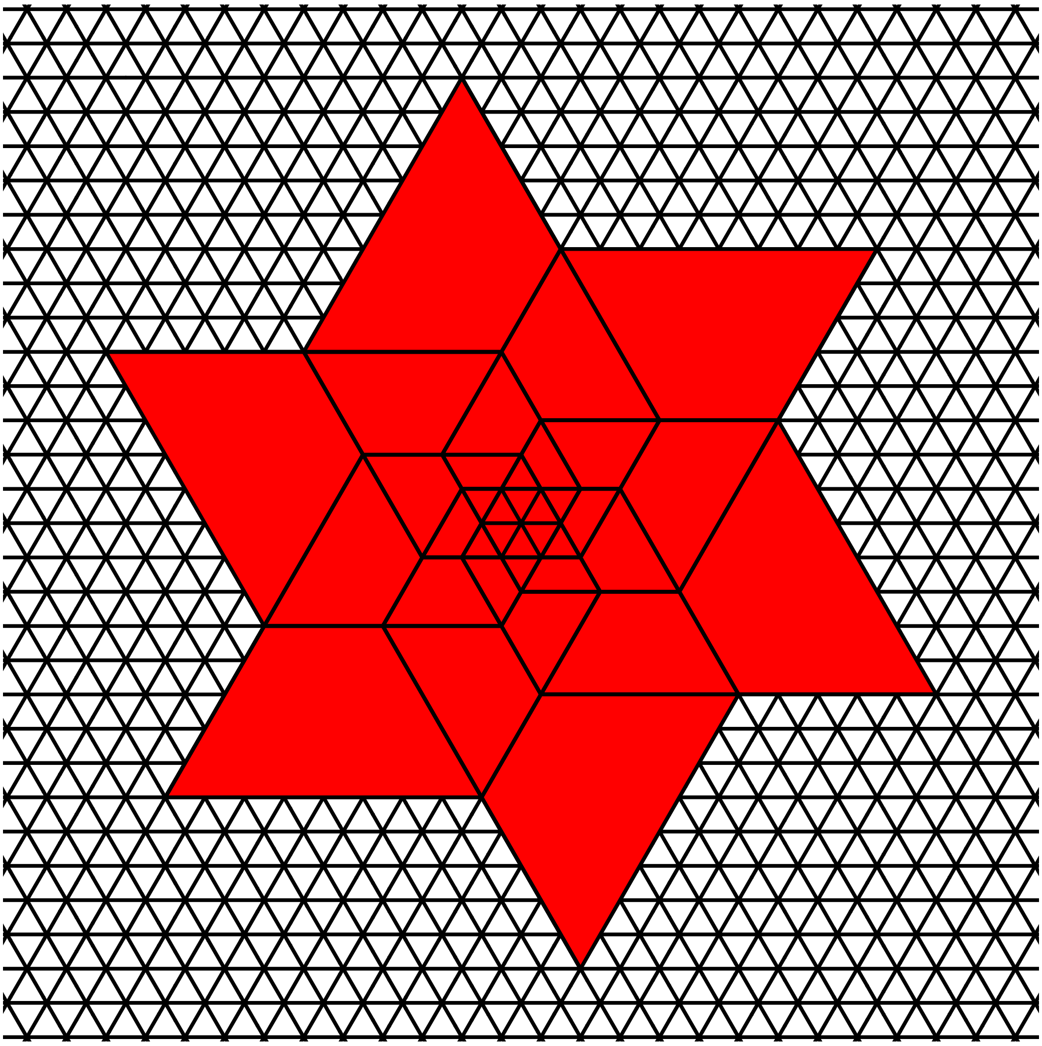
Just for fun: The Fibonacci Hexagon.



Fibonacci Hexagon

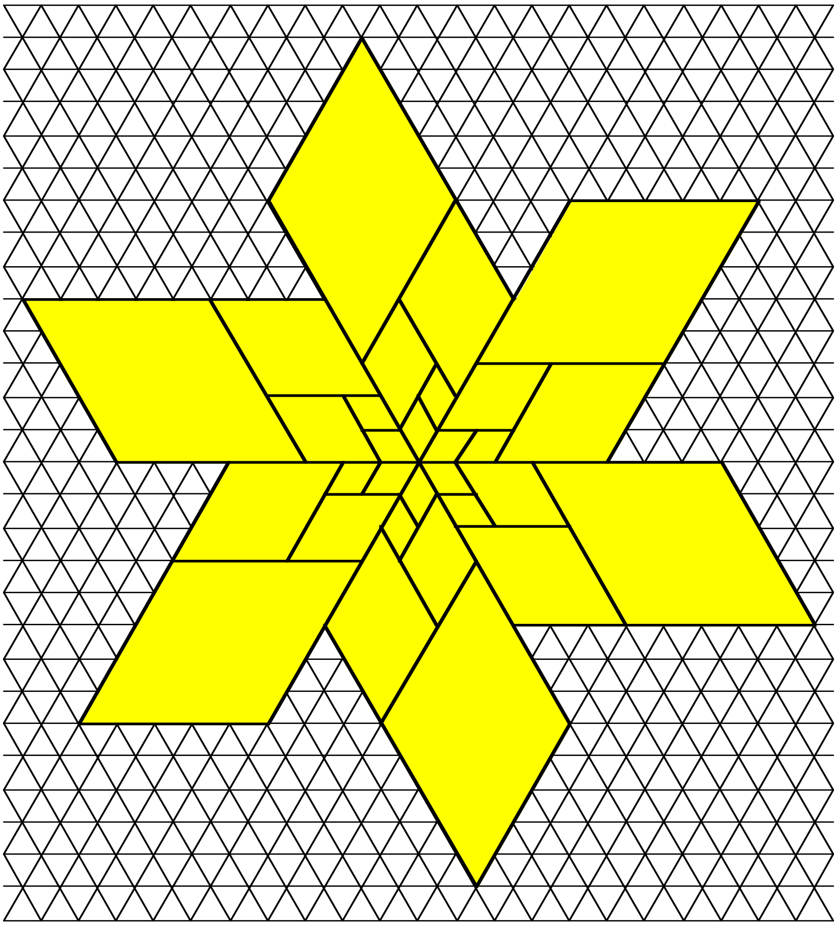
3 Stars

We can remove the rhombuses and reassemble the trapezoids to get a star.



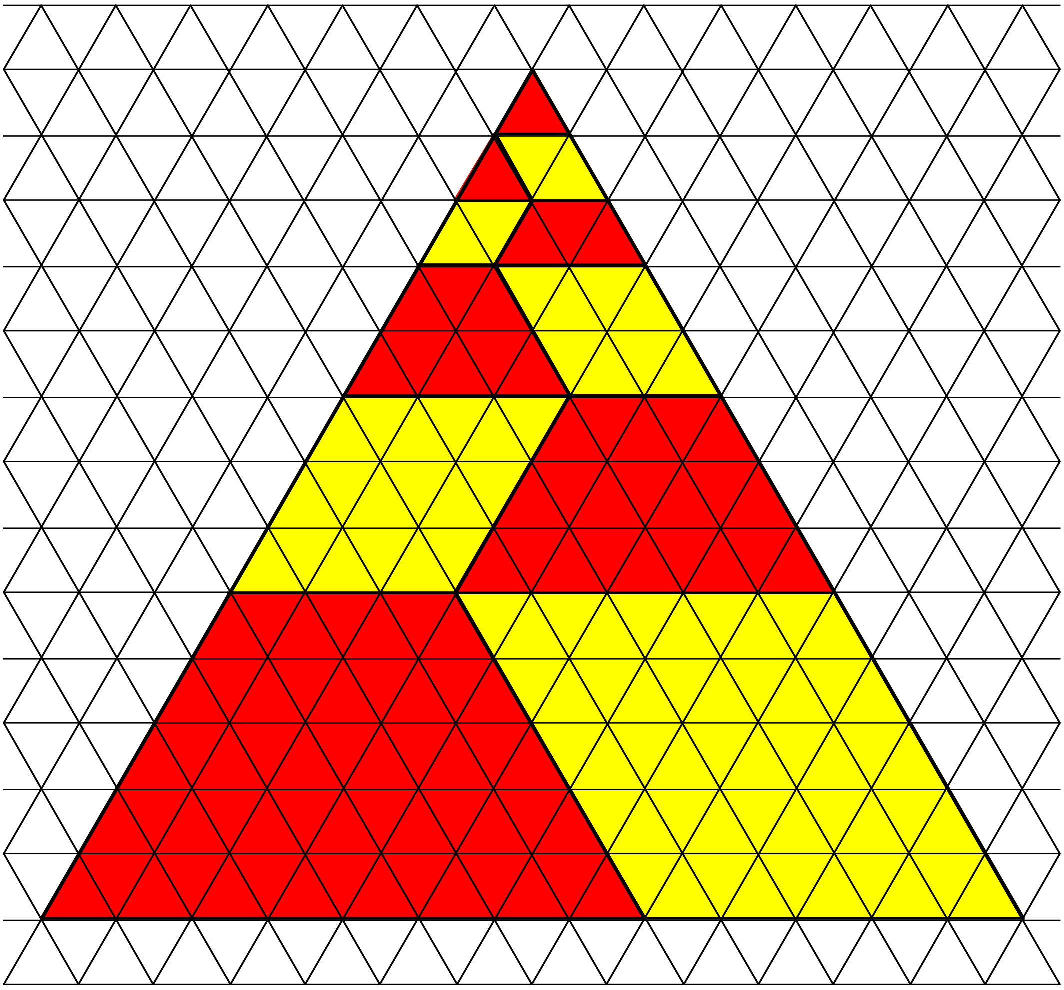
Fibonacci Star

And of course we can also reassemble the rhombuses to get another star.



Another Fibonacci Star

4 Proof without words



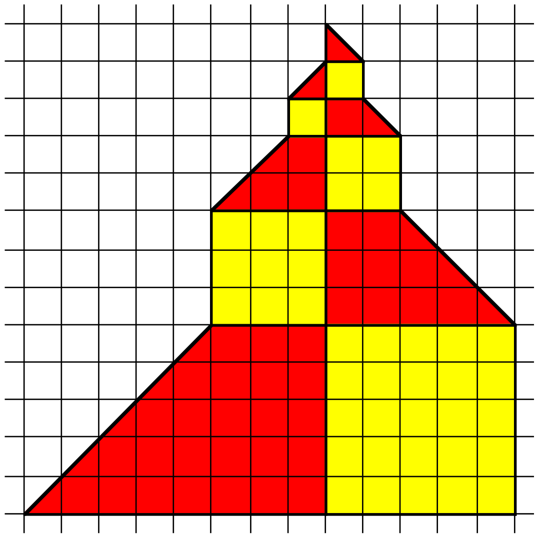
The Fibonacci Triangle again

Using the Fibonacci Triangle we can prove the identity:

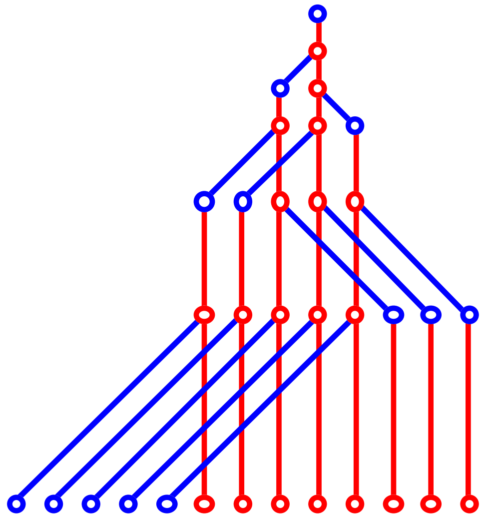
$$1+\sum_{k=1}^{n} f_k = f_{n+2}$$

5 Variations in the Square Lattice

We can transfer the Fibonacci triangle into a square lattice. Compare with the Matterhorn in the Swiss Alps.



In the Square Lattice. Matterhorn



Just Lines