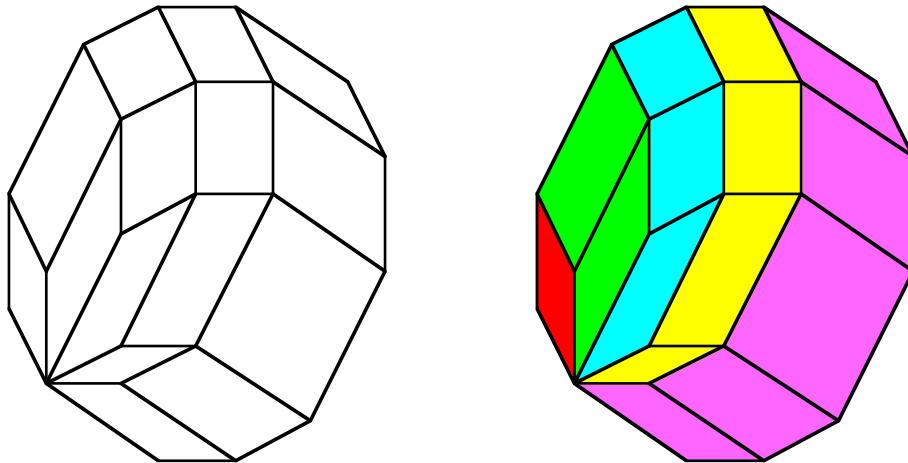Hans Walser, [20090720a]

Dissection of a convex polygon with central symmetry

1   General case

A polygon with central symmetry has an even number of vertices.
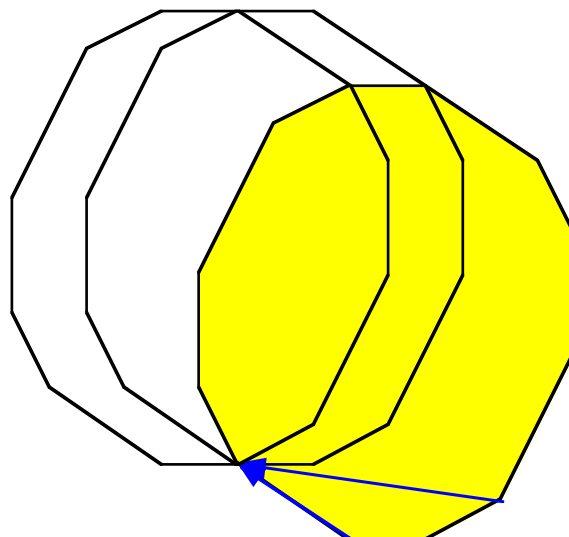
Theorem:

> A convex polygon with central symmetry and $2n$ vertices can be dissected into $\binom{n}{2}$ parallelograms.
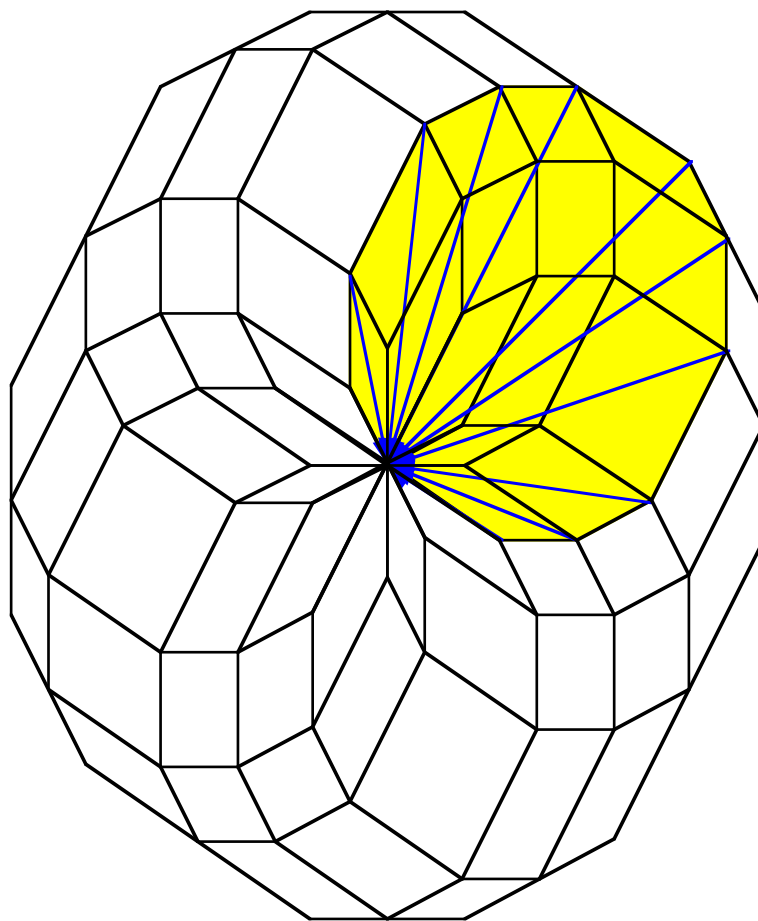

Dissection of a 12-gon

Proof by induction according to the colors [Walser 1983].

We can easily find the dissection by moving copies of the polygon such that every vertex comes once to a particular vertex of the original polygon.
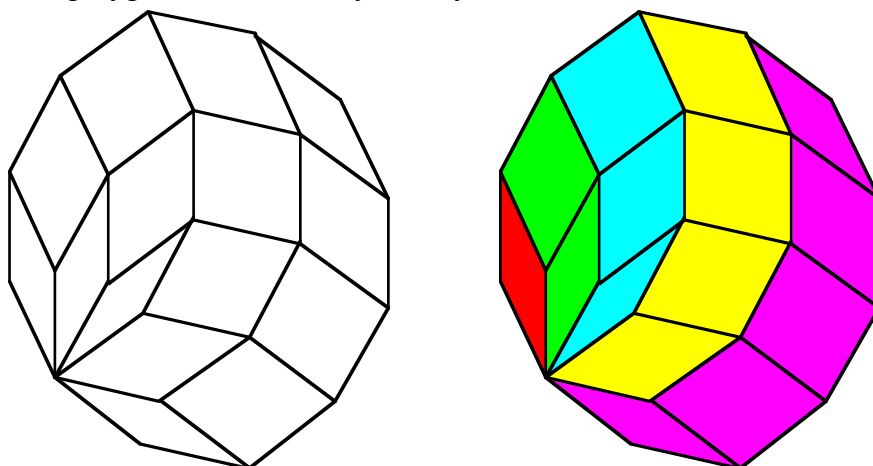

First two steps

Dissection

2   Special cases
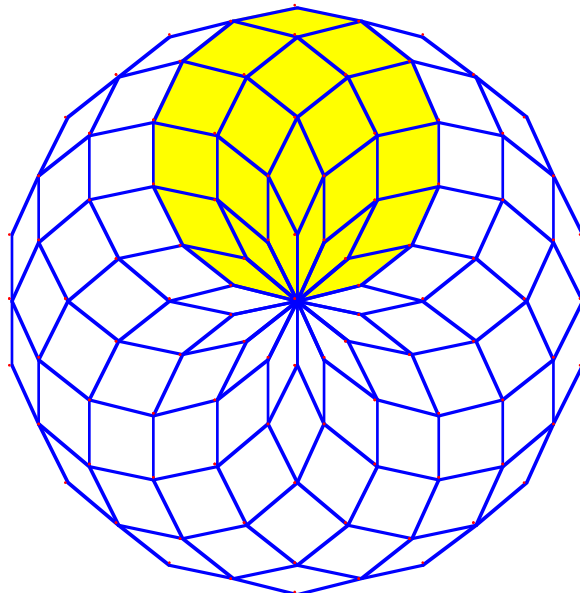
2.1   Equilateral polygons

An equilateral polygon with central symmetry can be dissected into rhombi.

Rhombi

2.2   Regular polygons

We discuss now regular polygons with $2n$ vertices. The dissection we can obtain by moving like above or by rotating the $2n$-gon by angles of multiples of $\frac{\pi}{n}$ around a particular vertex ([Lindgren 1972], p. 109, and cover).
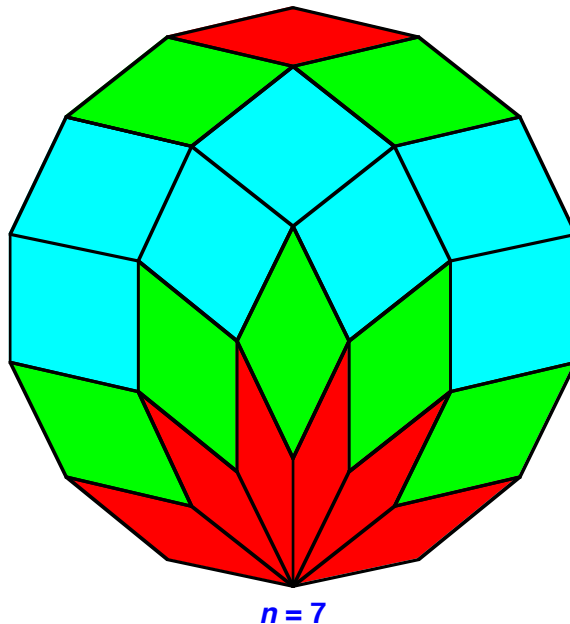

Dissection of a regular 14-gon

If we are just interested in the vertices of the rhombi, we can work with circles.
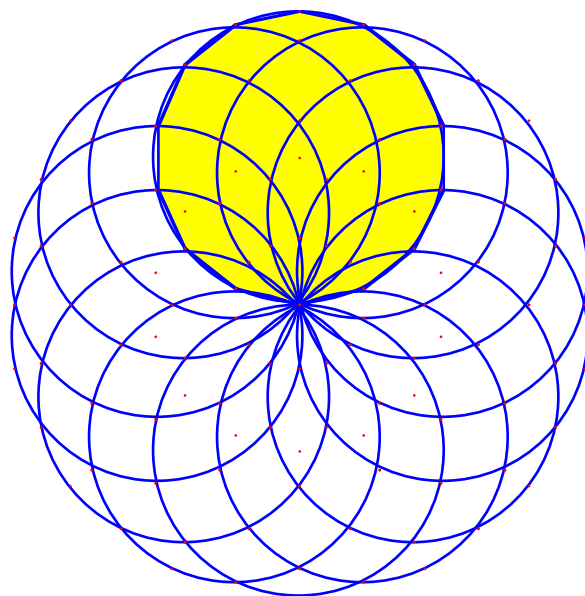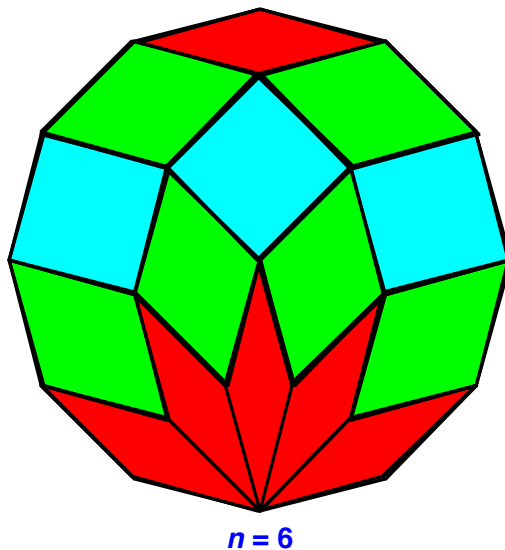

Circles will work

2.2.1 *n* odd

In this case we get $\frac{n-1}{2}$ sets, each consisting of *n* congruent rhombi. The acute angles of the rhombi are $\frac{\pi}{n}, 2\frac{\pi}{n}, \ldots, \frac{n-1}{2}\frac{\pi}{n}$.

n = 7

2.2.2 *n* even

Solution of M. N. Deshpande's Problem 1650 in Mathematics Magazine (2002), p. 227 ([Fischer 2003], with beautiful examples).

We get $\frac{n-2}{2}$ sets, each consisting of *n* congruent rhombi, and a set of $\frac{n}{2}$ squares.

n = 6

References

[Lindgren 1972]   Lindgren, H.: Geometric Dissections. Revised and enlarged by Greg Frederickson. New York: Dover 1972.

[Walser 1983]   Walser, Hans: Ein Zerlegungssatz für punktsymmetrische konvexe Vielecke. Elemente der Mathematik (38), 1983, p. 159-160.

Links (2009 / 7 / 20)

[Fischer 2003]

http://www.calpoly.edu/~glfisher/MAA_Dahlia_Paper3.pdf