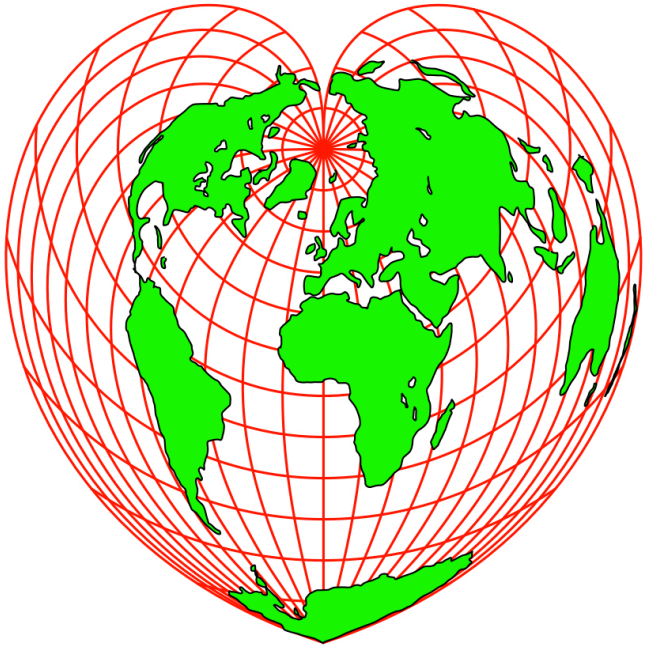
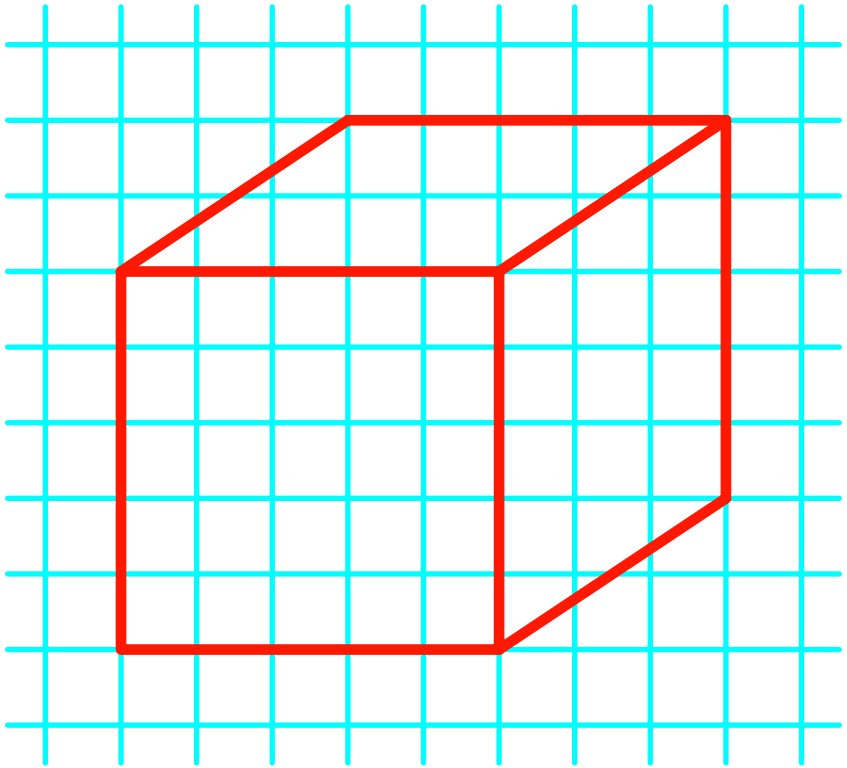
Hans Walser

Maßstab 1:1



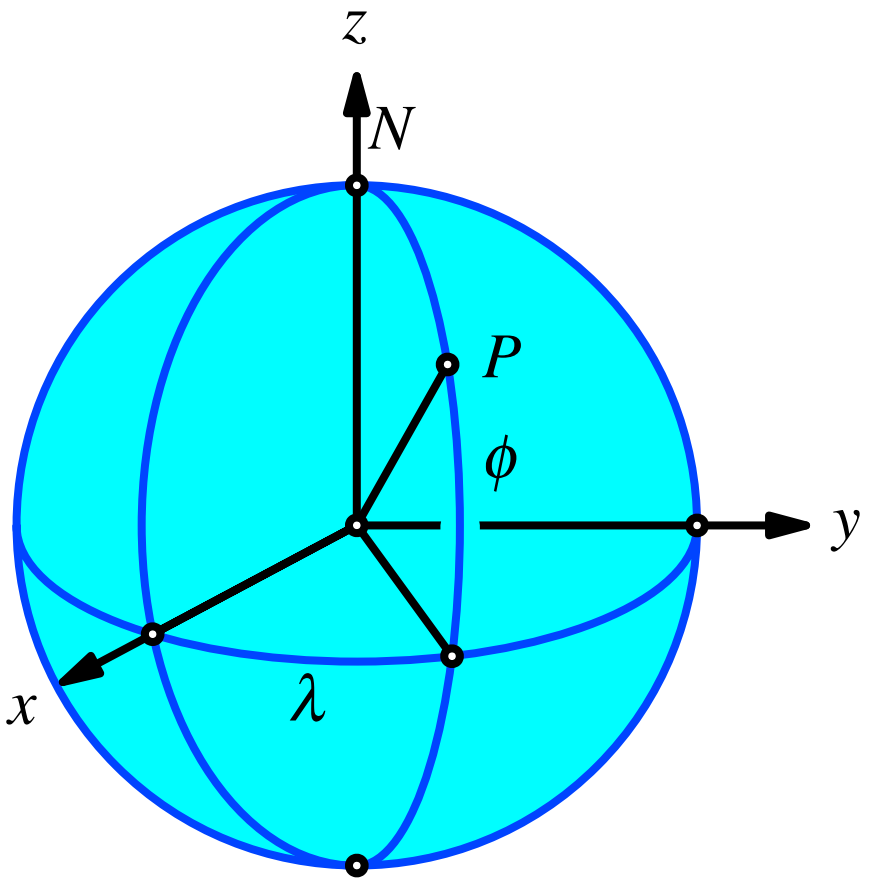
www.walser-h-m.ch/hans

Was sehen wir?



Was stellt es dar?
Was bedeutet es?

Was sehen wir?



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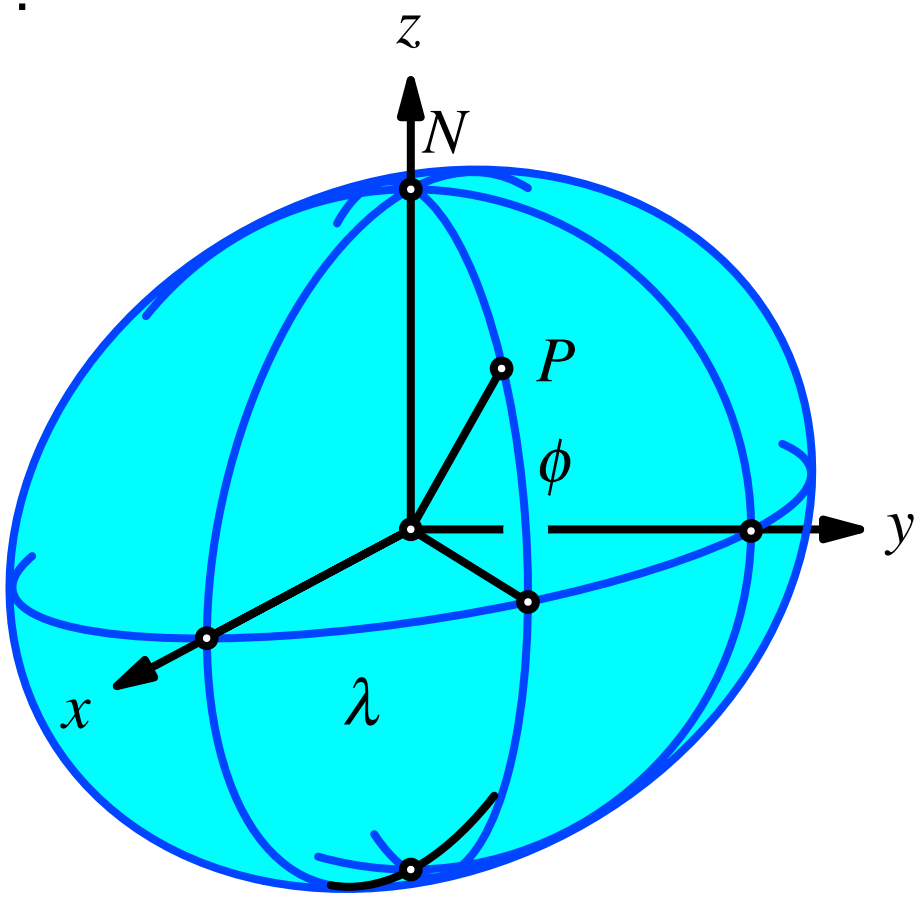
Schrägbild

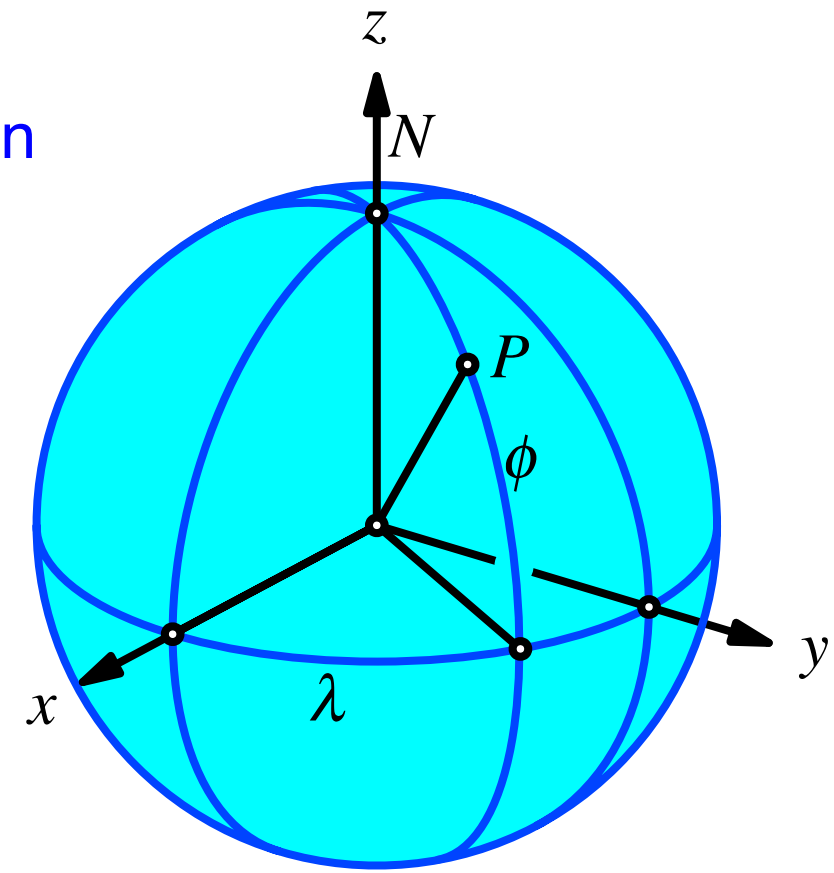


Was sehen wir?

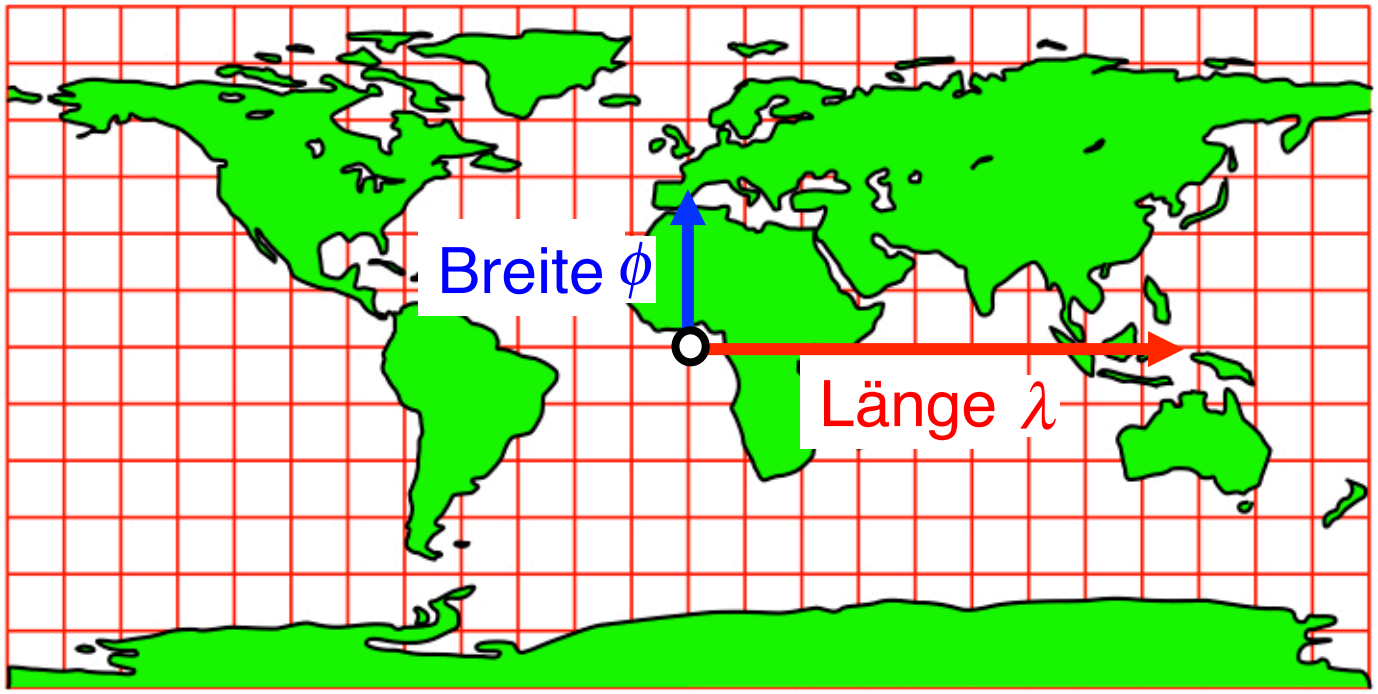
Schrägbild

Was sehen wir?

Normalprojektion

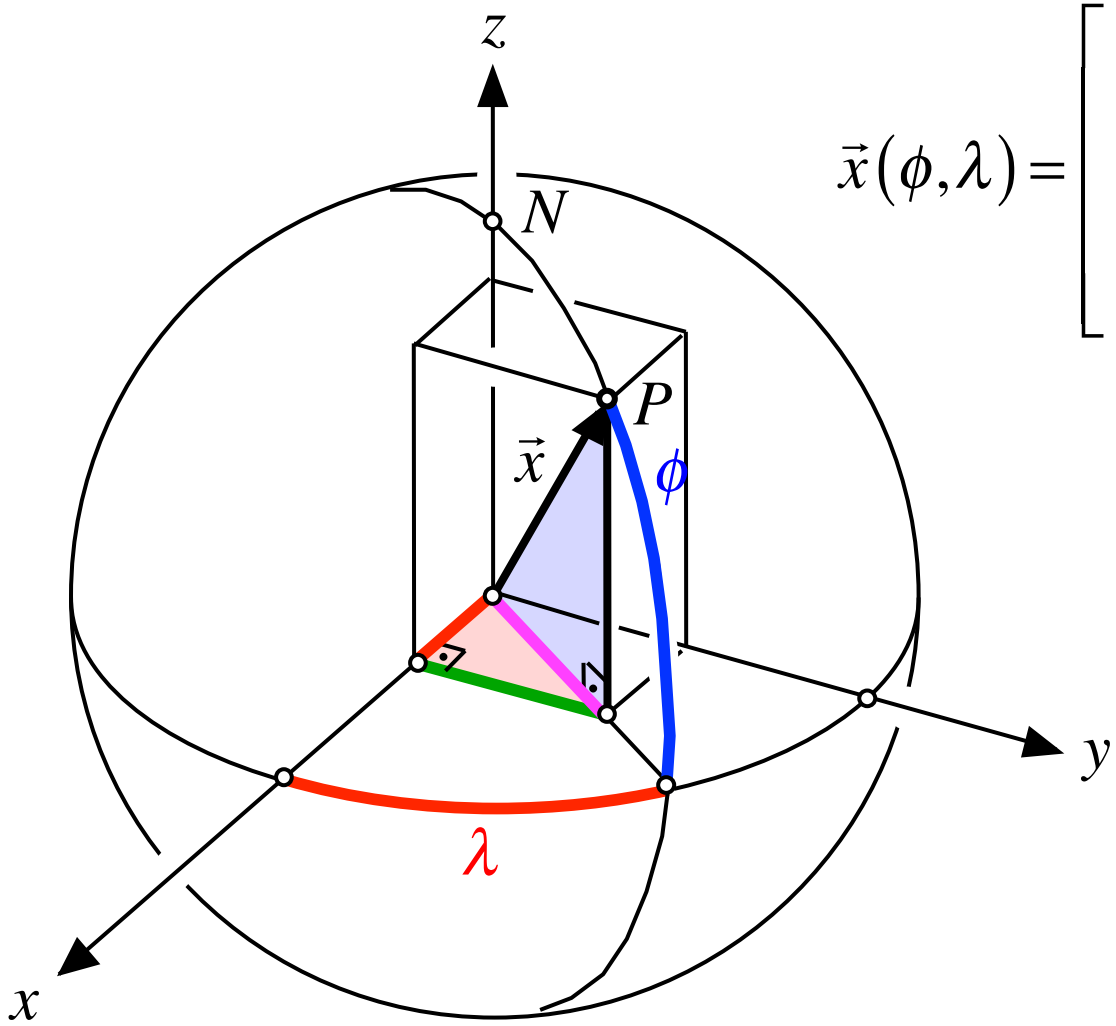


Längen oder Winkel?



Plattkarte

Längen oder Winkel?



$$\vec{x}(\phi,\lambda) = \begin{bmatrix} \cos(\phi)\cos(\lambda) \\ \cos(\phi)\sin(\lambda) \\ \sin(\phi) \end{bmatrix}$$

$$\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \lambda \in [-\pi, \pi]$$

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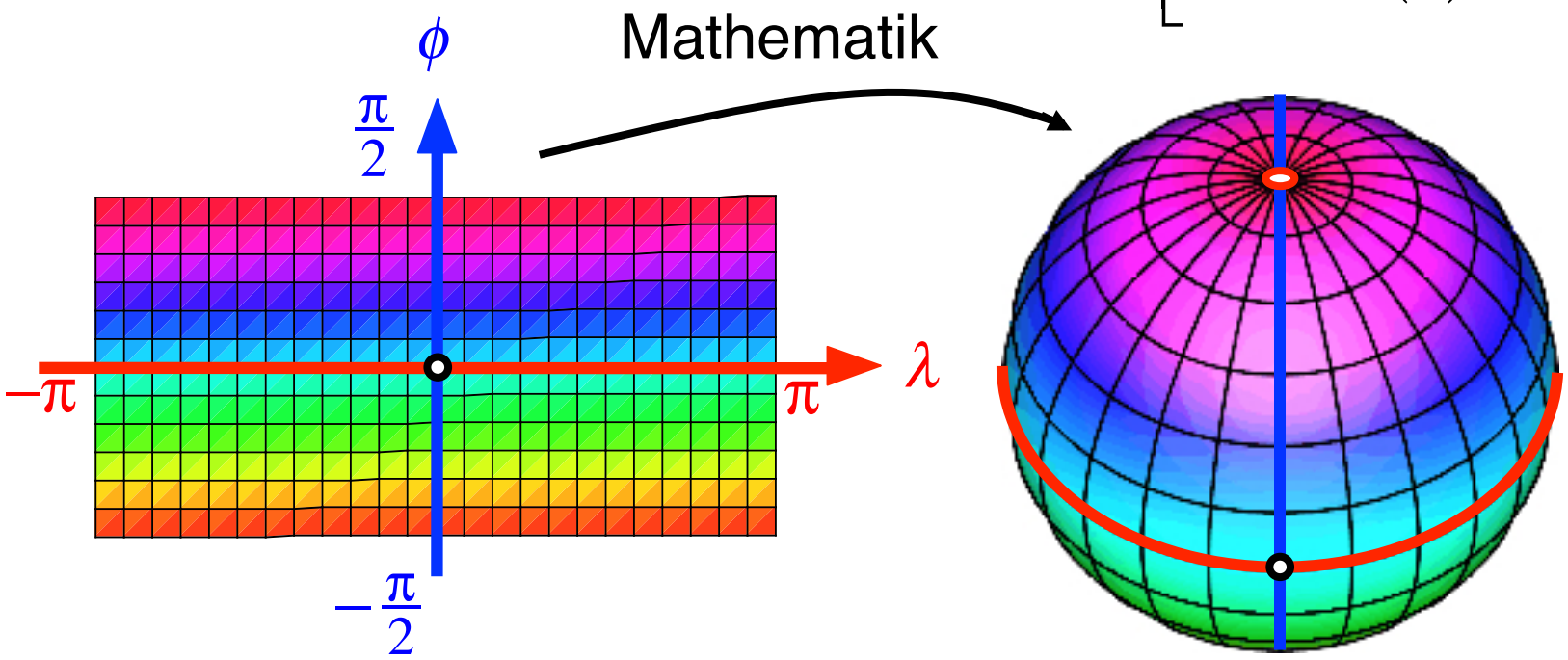

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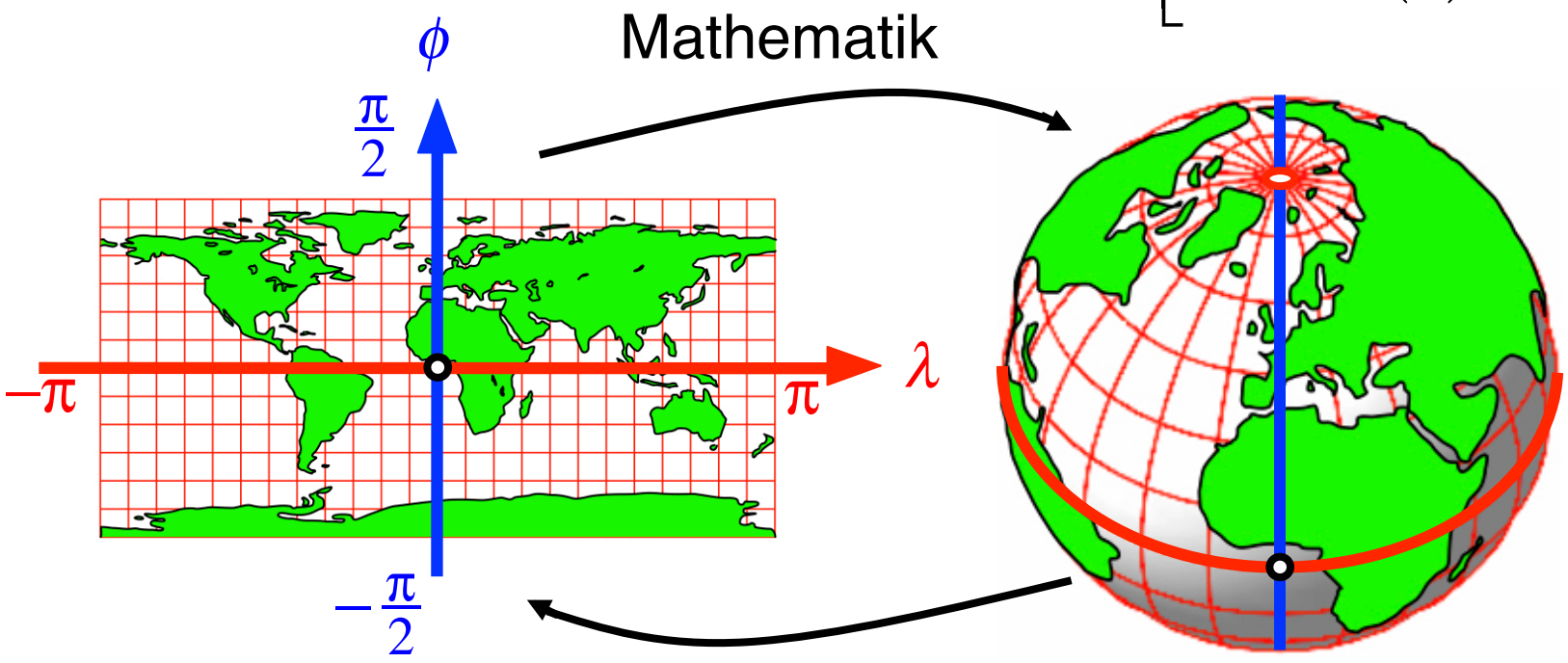

Idee eines Schülers:
Plattkarte Hochformat

$$\phi \in [-\pi,\pi], \lambda \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

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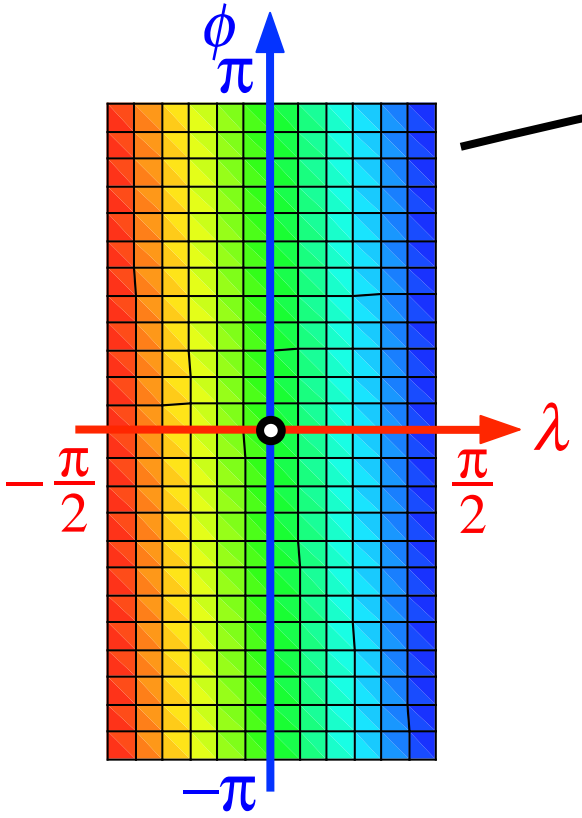

?

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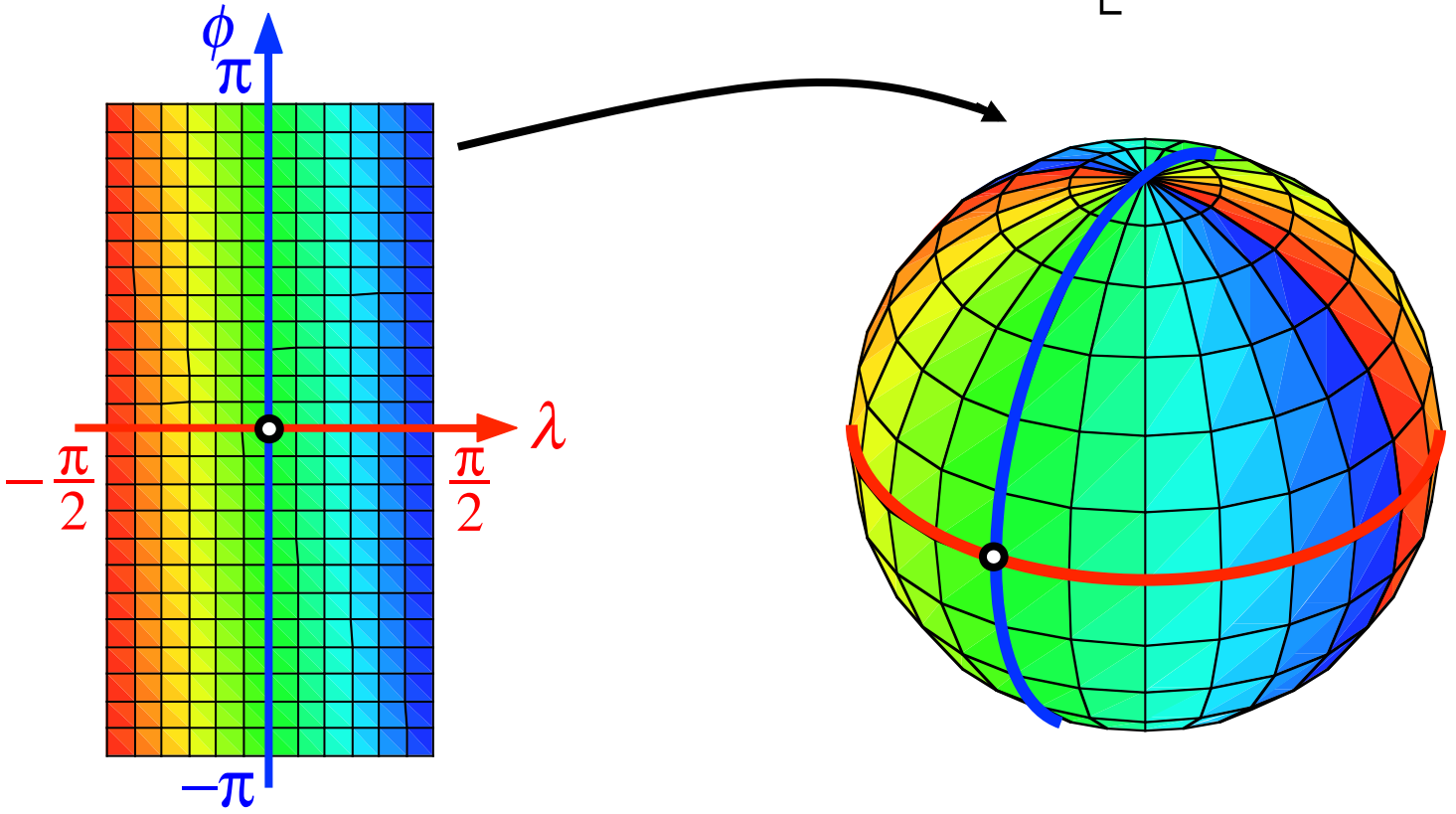
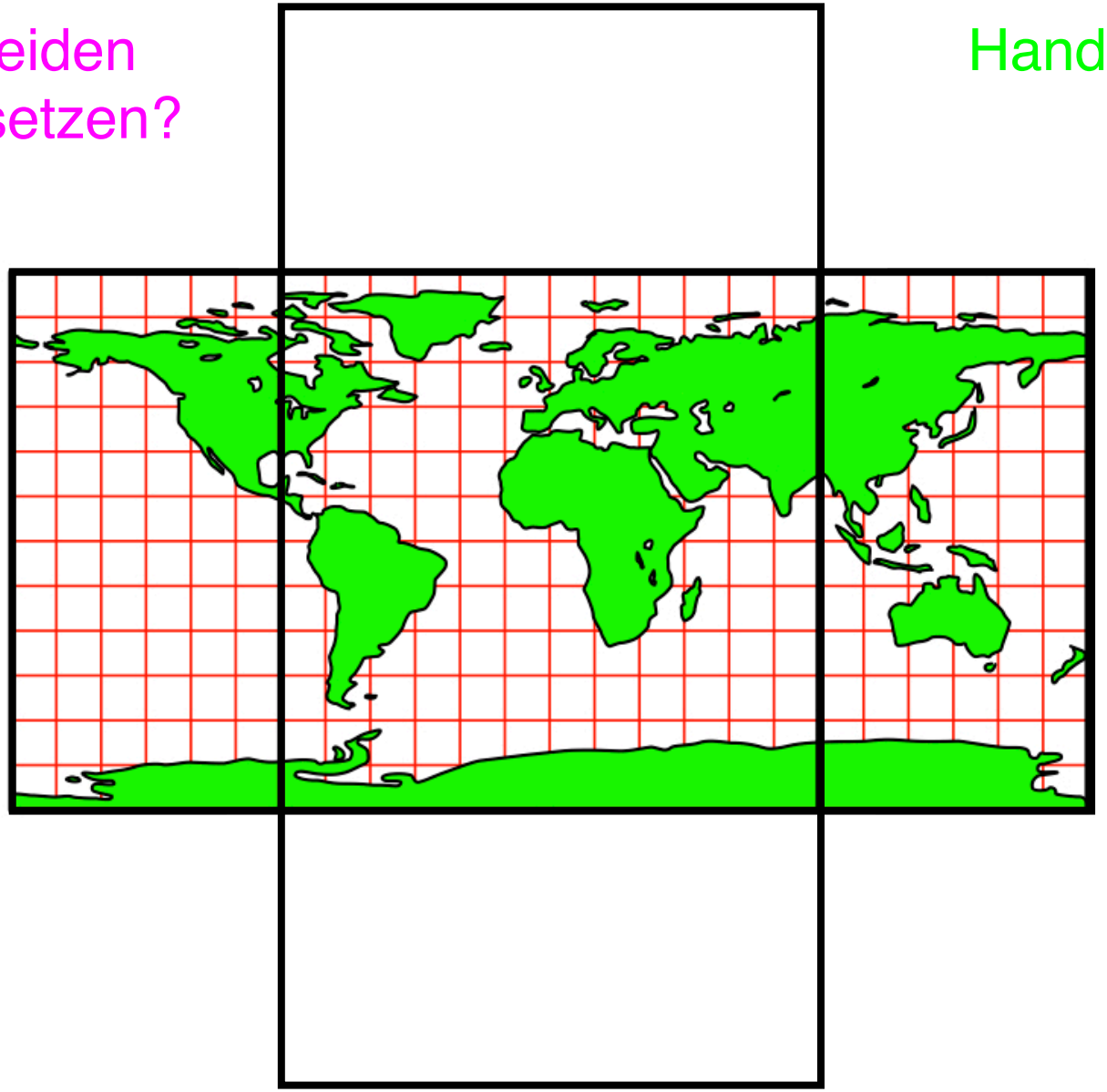


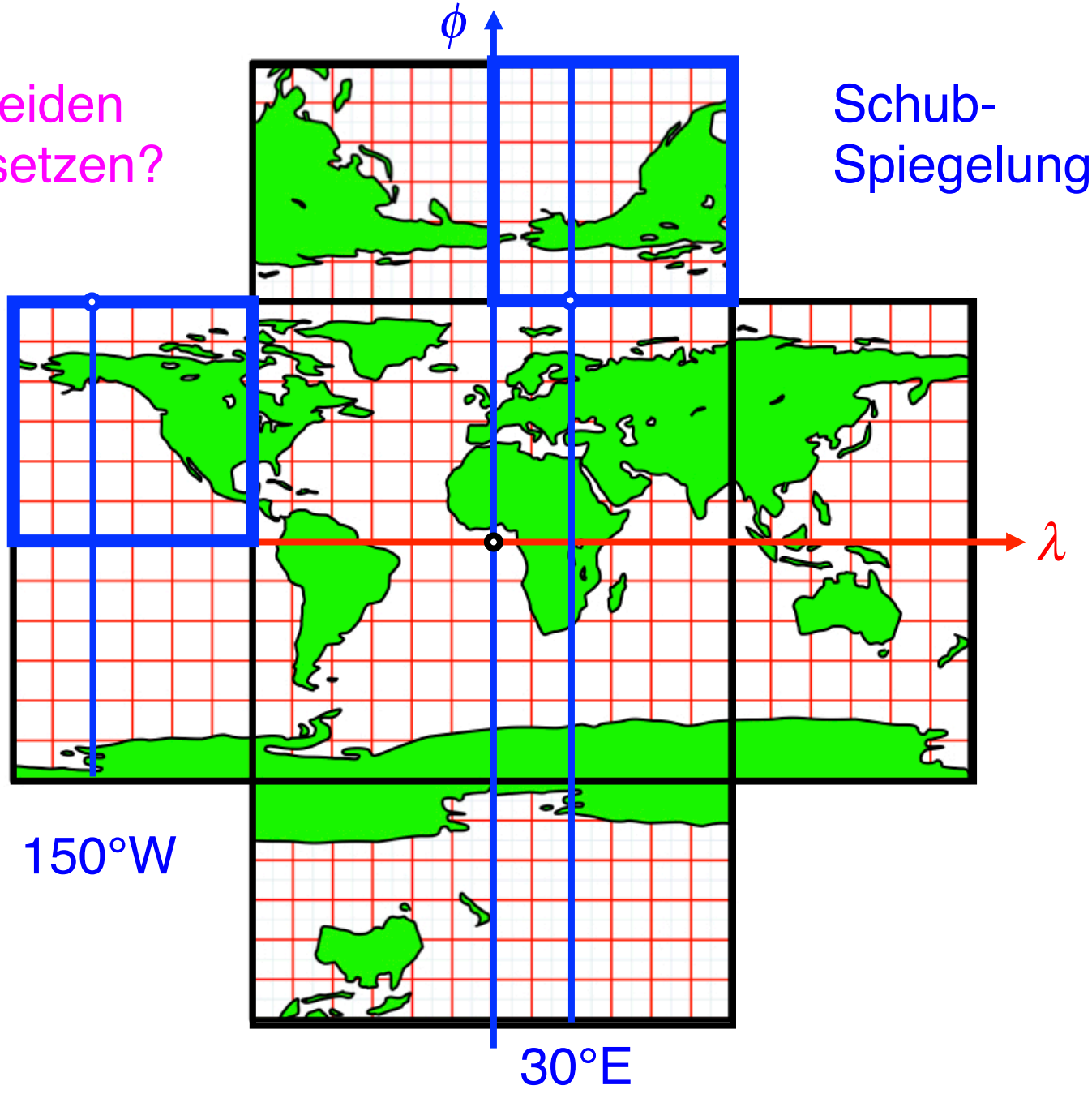
Abschneiden
und ansetzen?

Handout

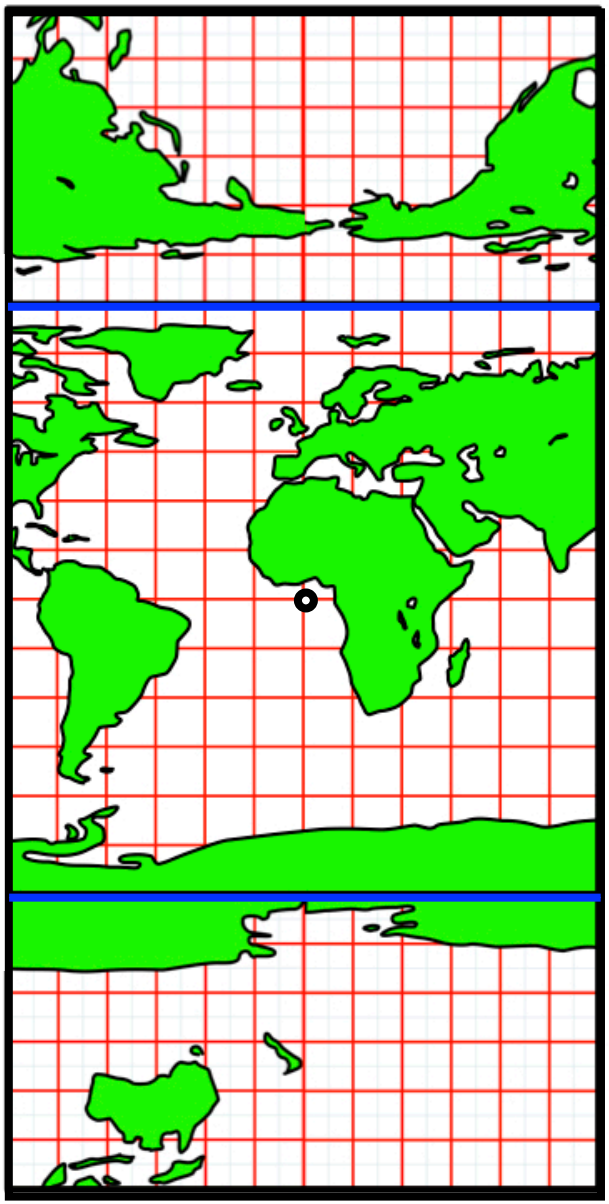


Abschneiden
und ansetzen?

Schub-
Spiegelung

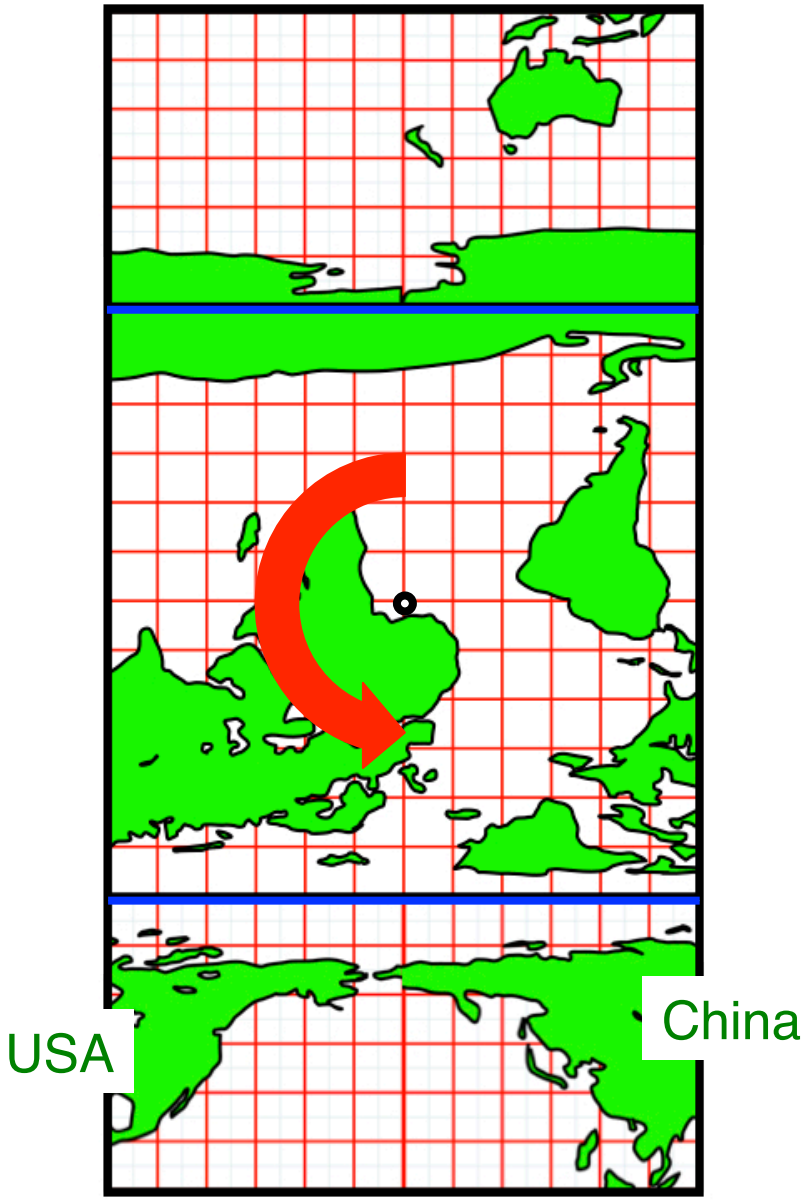


Die Welt im Hochformat



Verpackung Schokokugeln

Die Welt im Hochformat



Immer der Nase nach

... so geh hübsch sittsam und lauf nicht vom Wege ab!

La línia recta és creació de l'home; la línia corba, de Déu.

ANTONI GAUDÍ

Keine Seitenkrümmung

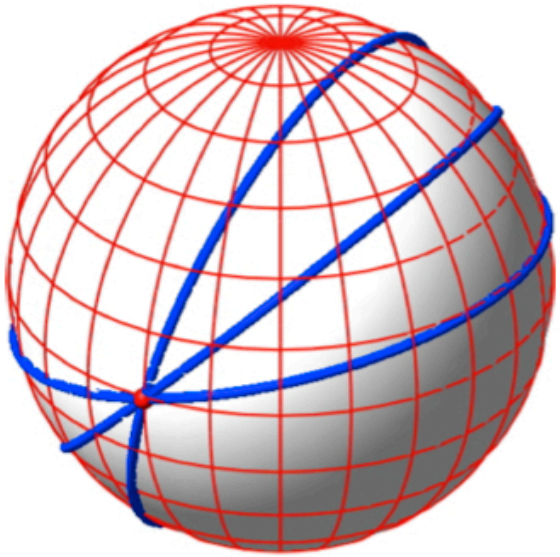
Geodätische Linie $\ddot{x}^m + \Gamma^m_{kl} \dot{x}^k \dot{x}^l = 0$

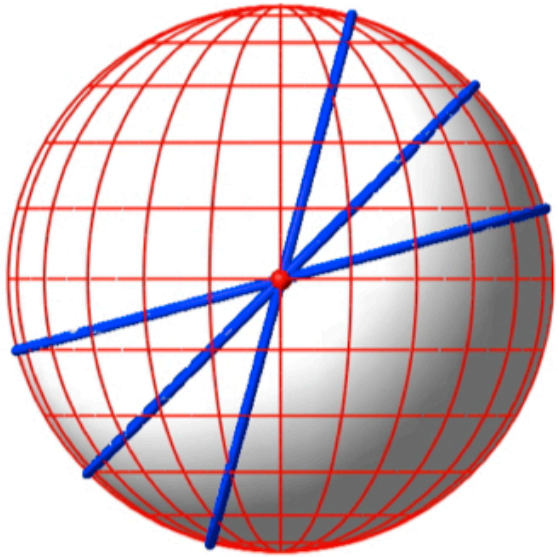
Durchgehendes Plastikband

Großkreis (Orthodrome)

Immer der Nase nach

Großkreise statt Geraden

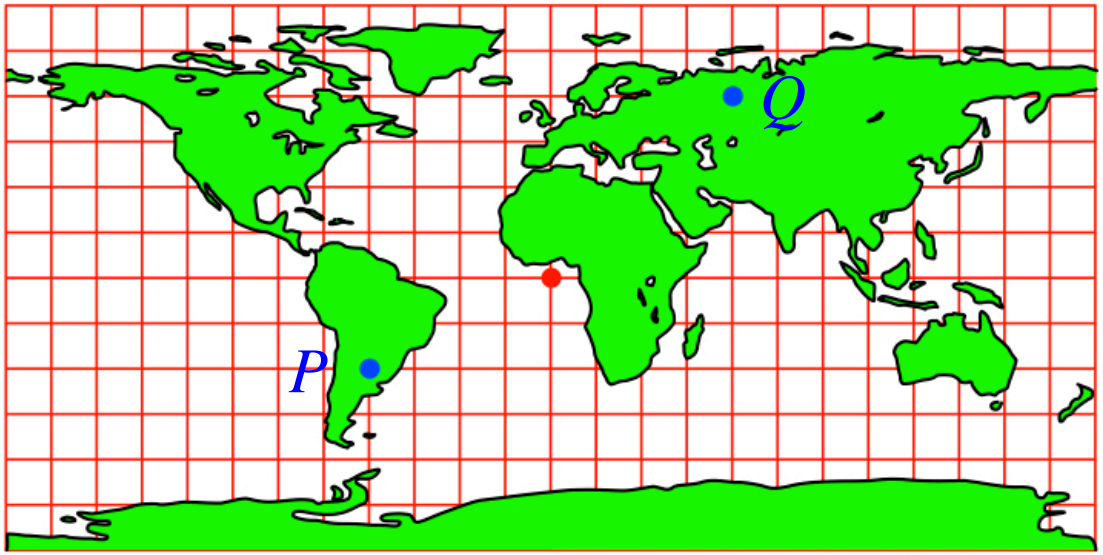


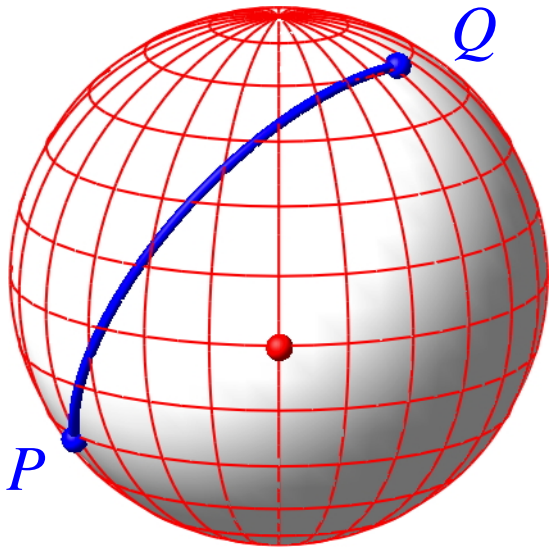


Blick von der Seite

Immer der Nase nach

Großkreisbogen von P nach Q?

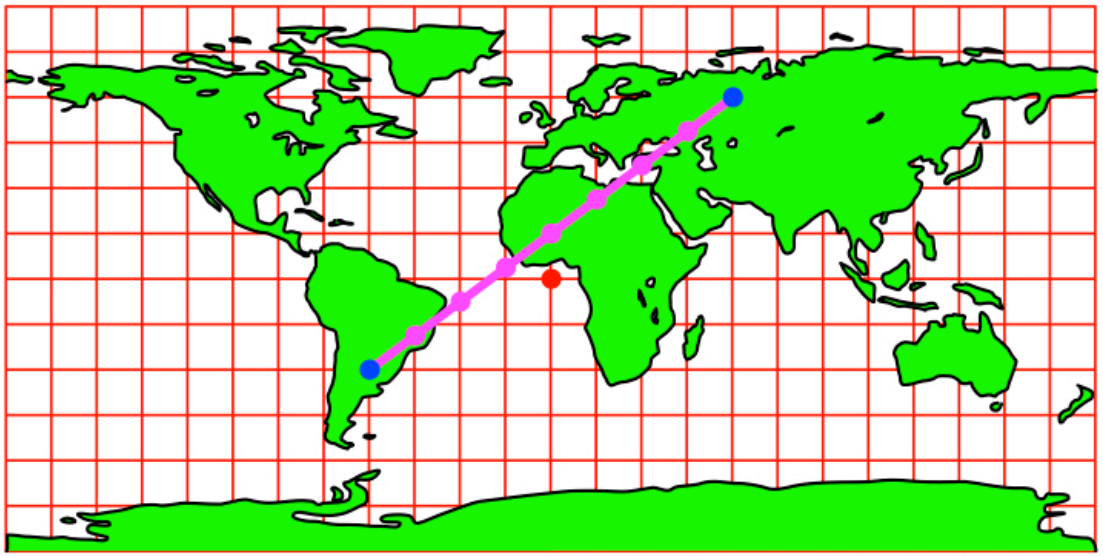


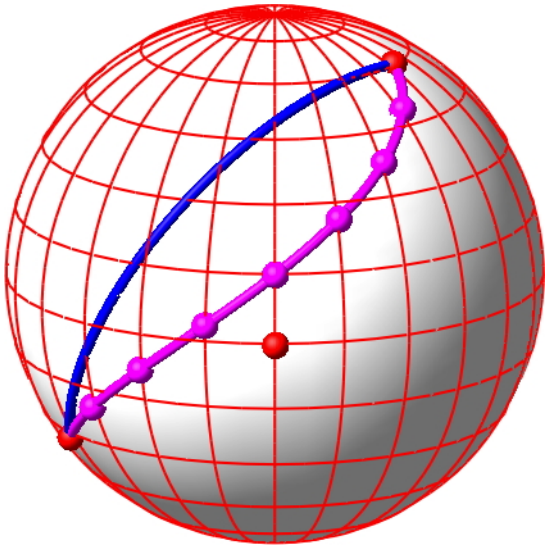


Immer der Nase nach

Großkreisbogen von P nach Q

Eine falsche Idee

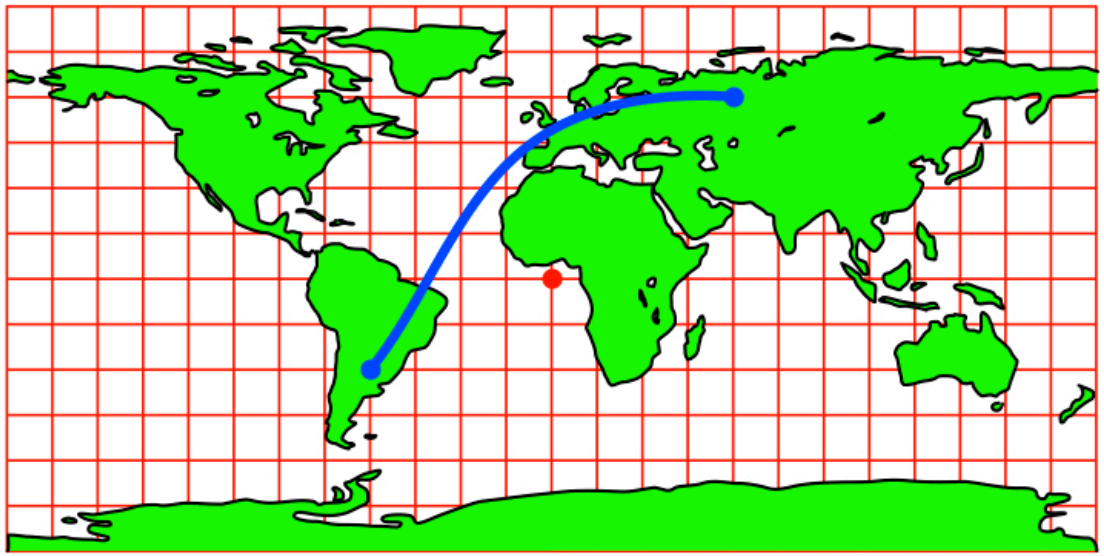


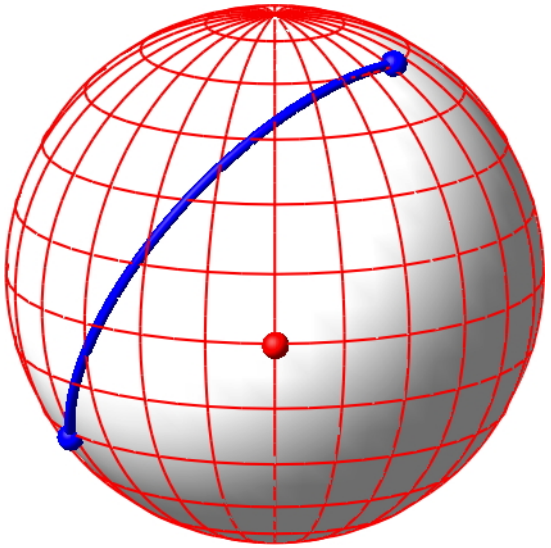


Immer der Nase nach

Großkreisbogen von P nach Q

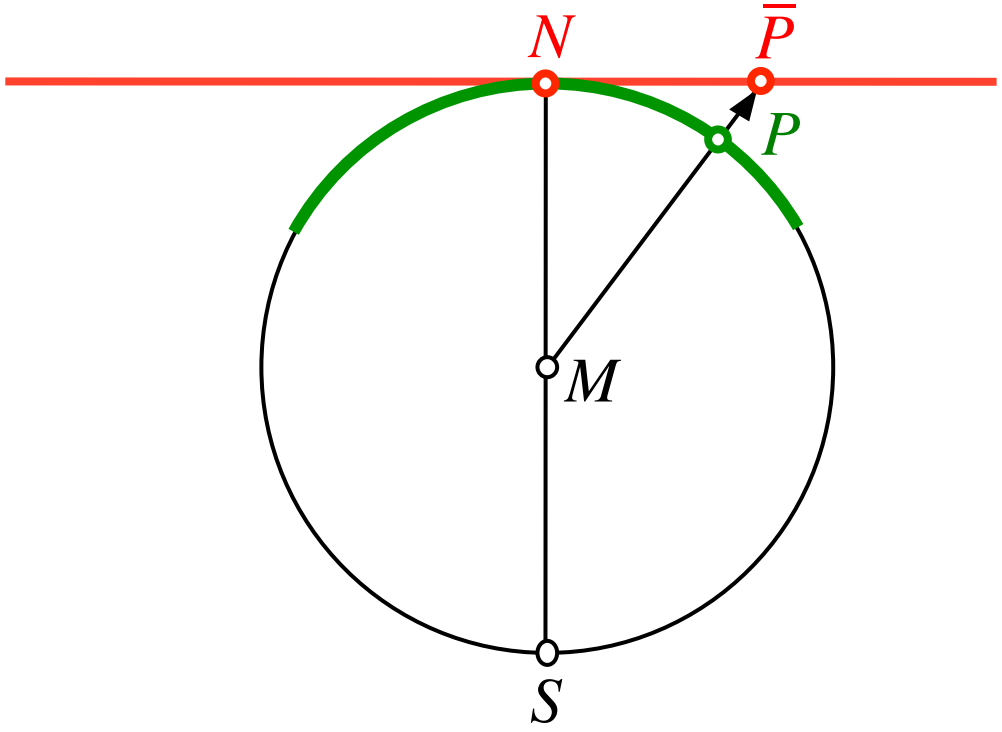
Die richtige Kurve

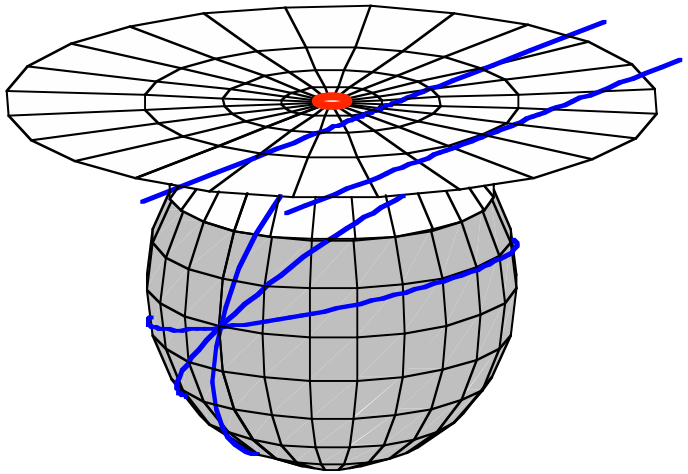




Großkreis als Gerade auf der Karte?

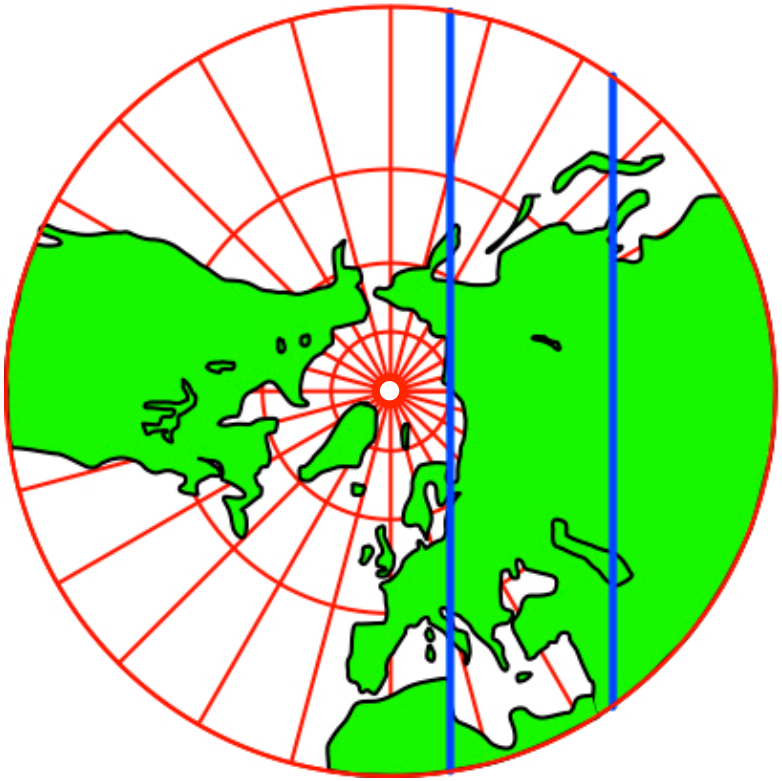
Gnomonische Projektion
Zentralprojektion von Kugelmitte aus

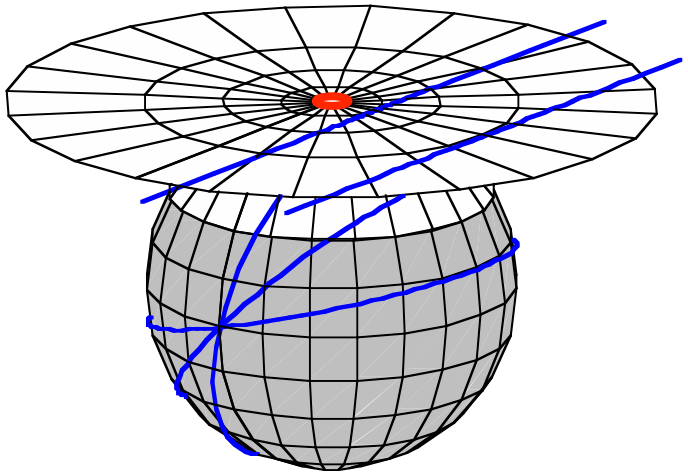




Großkreis als Gerade auf der Karte?

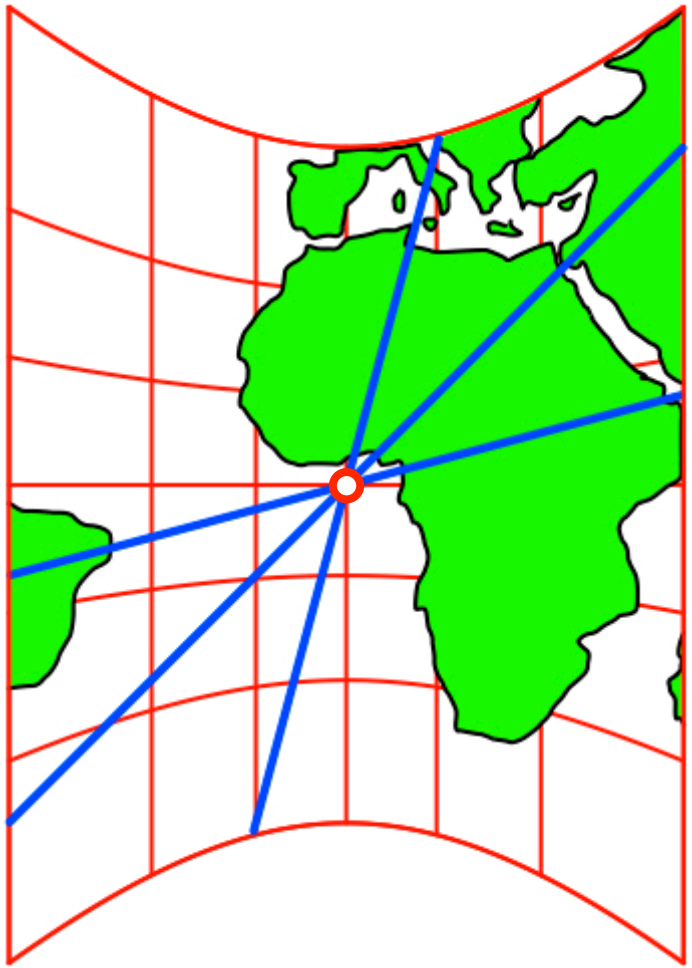
Gnomonische Projektion

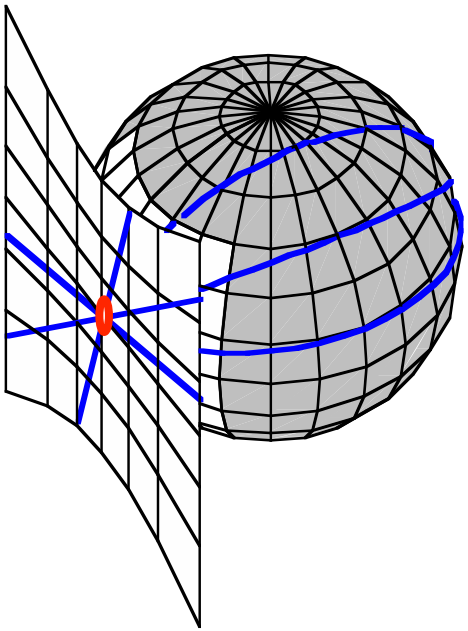




Großkreis als Gerade auf der Karte?

Gnomonische Projektion





Großkreis als Gerade auf der Karte?

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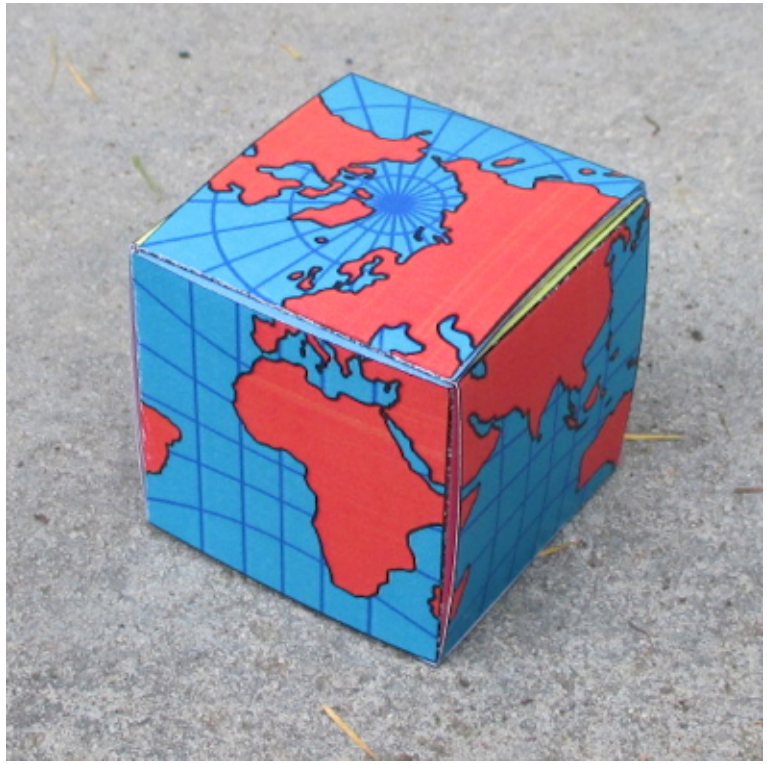
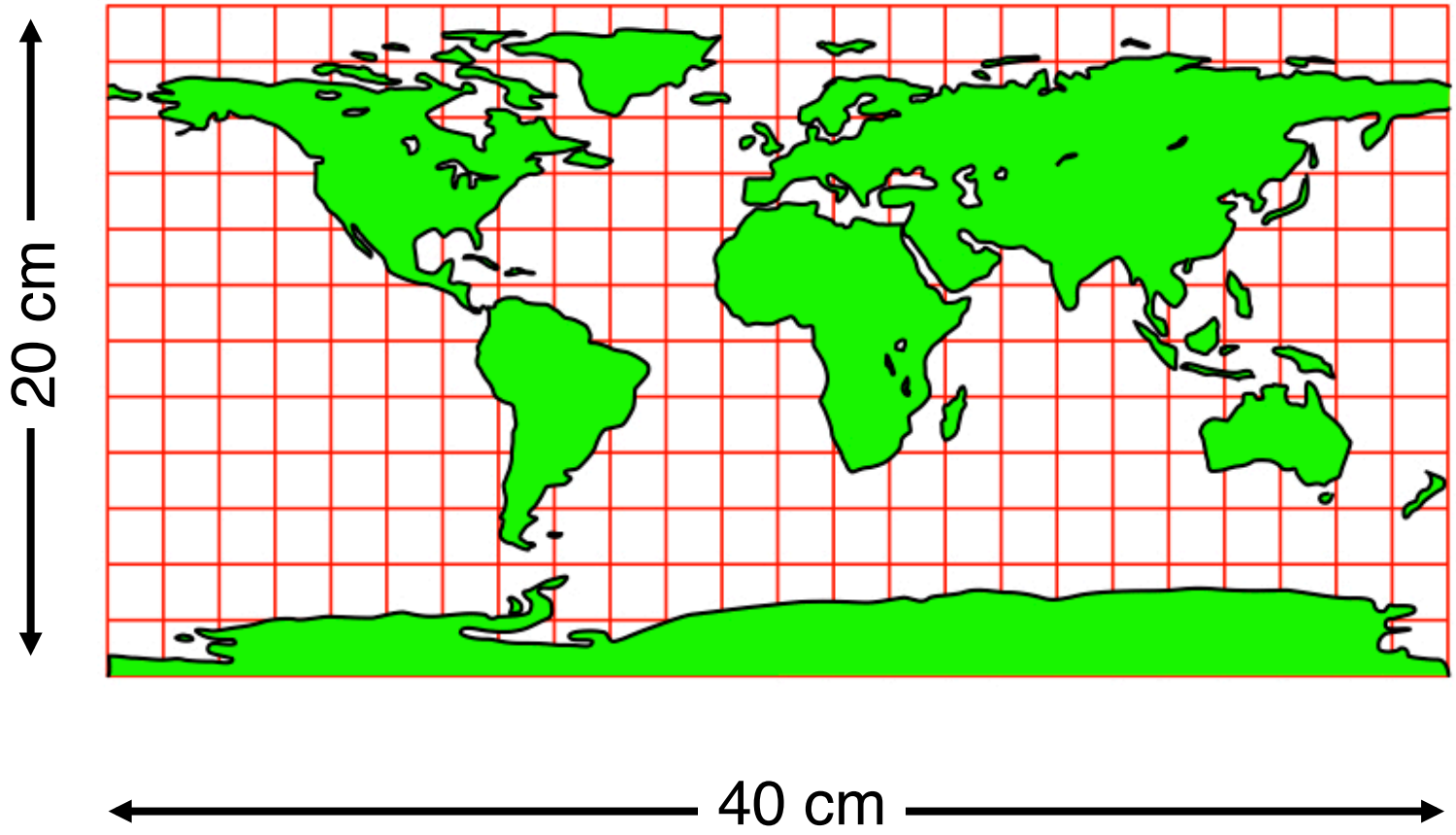


Handout Würfelwelt

Maßstab eins zu eins?



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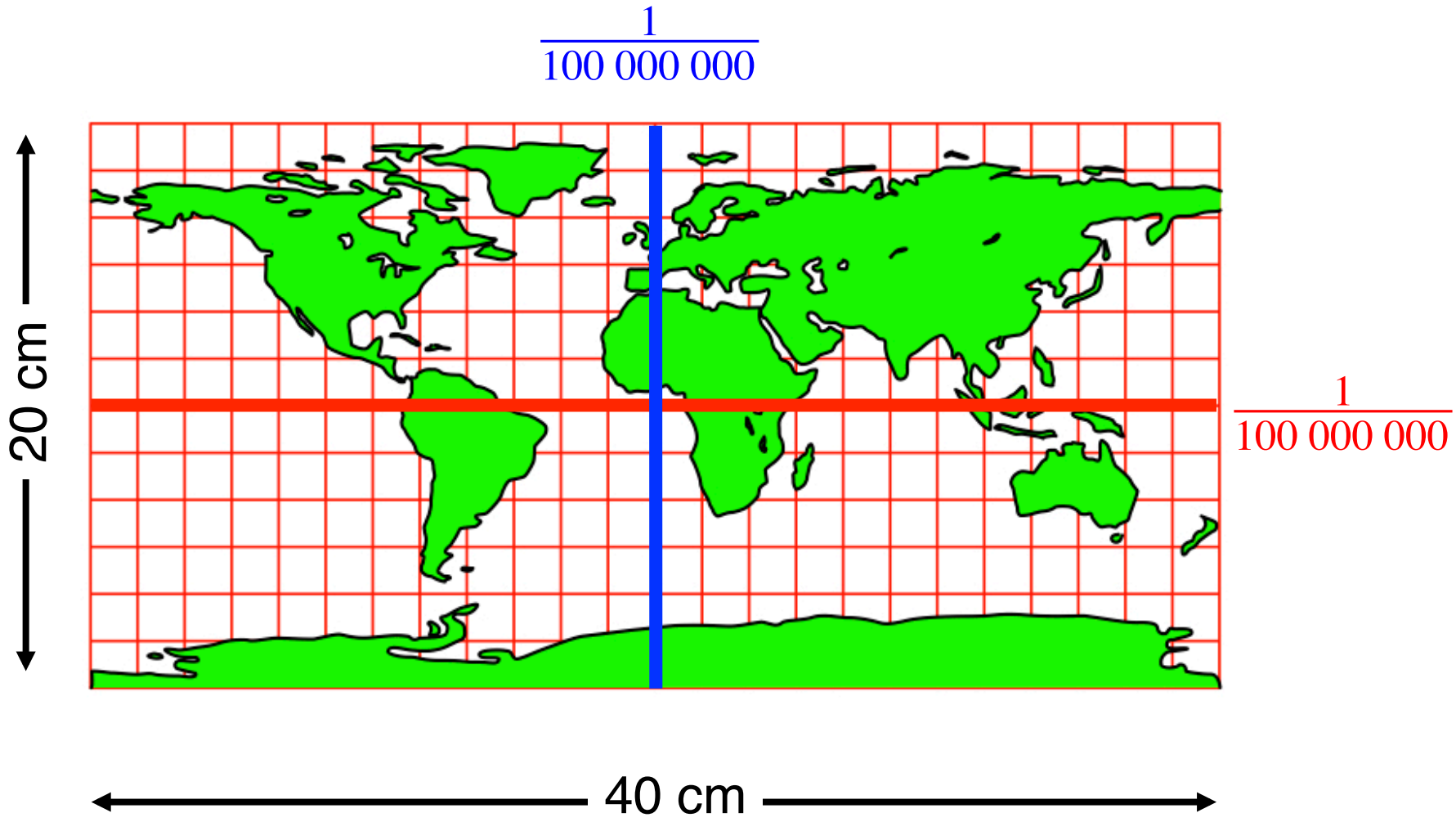

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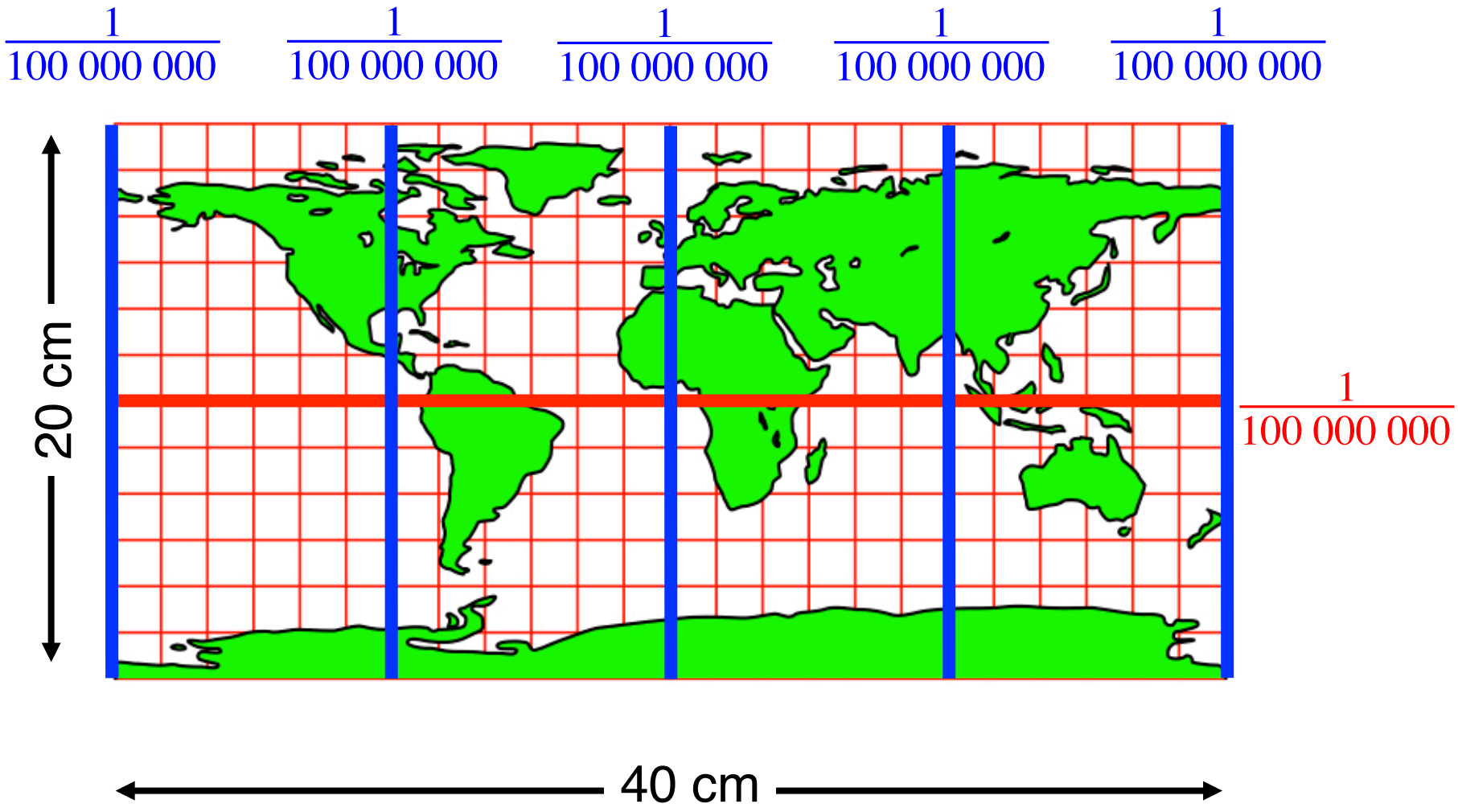

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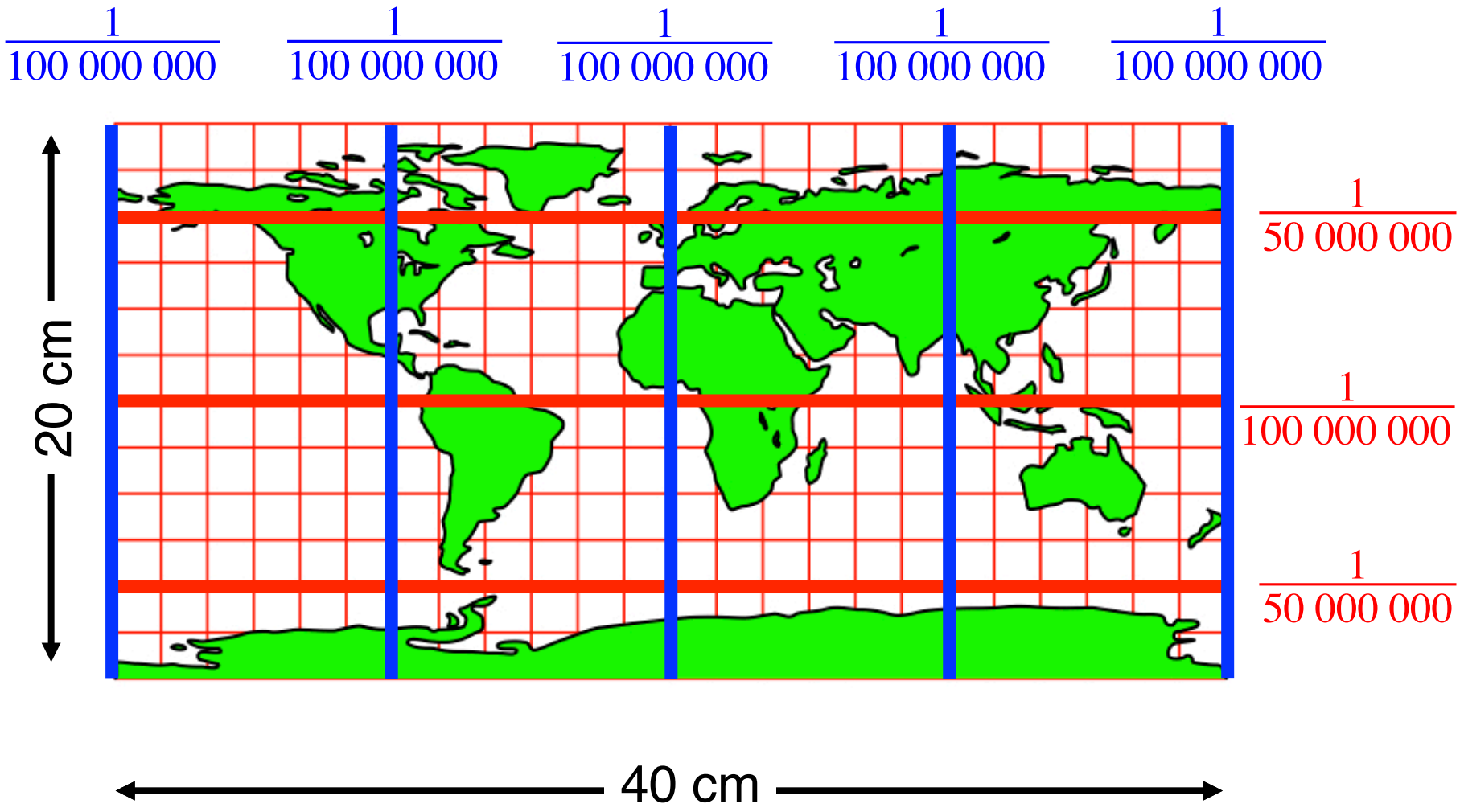

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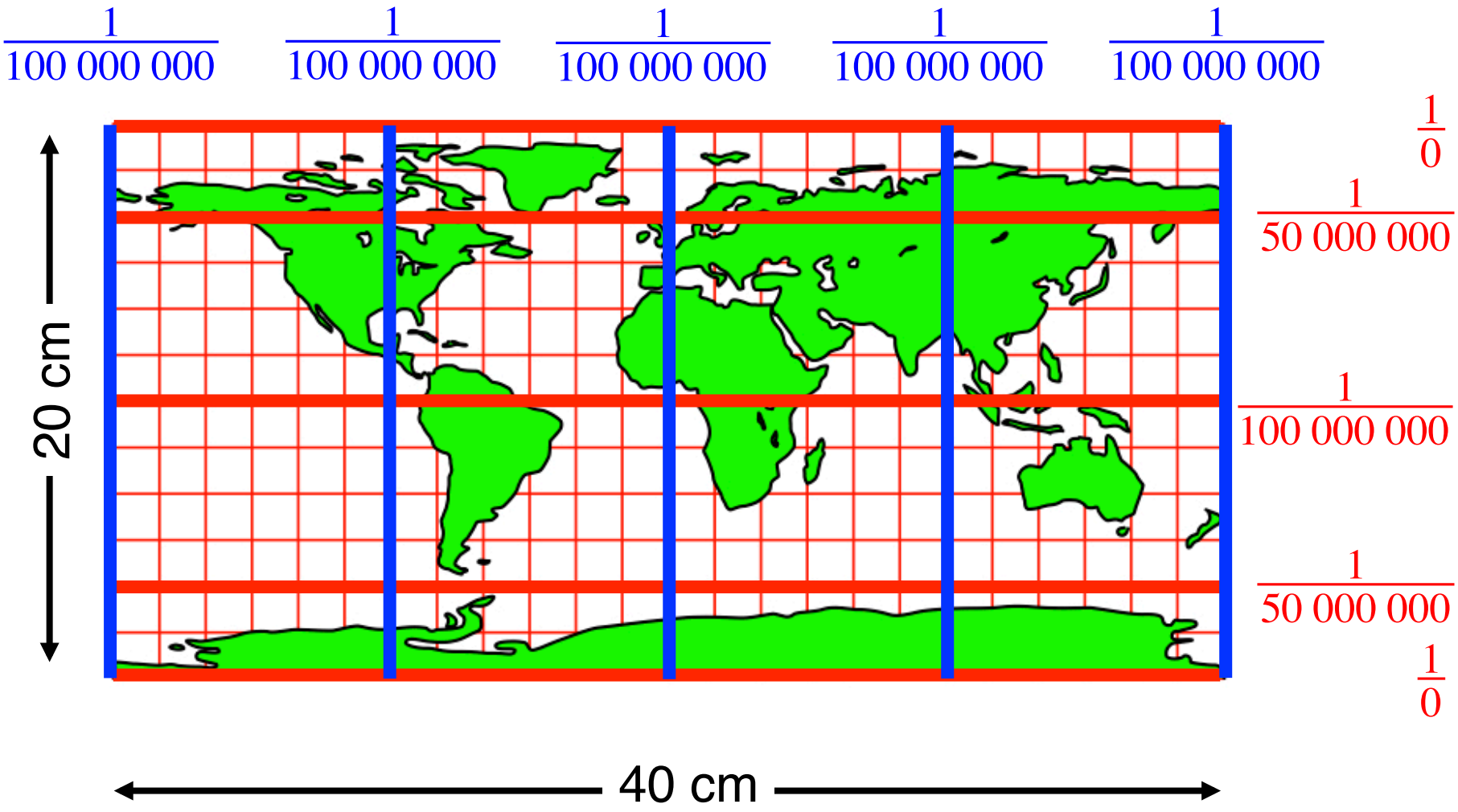
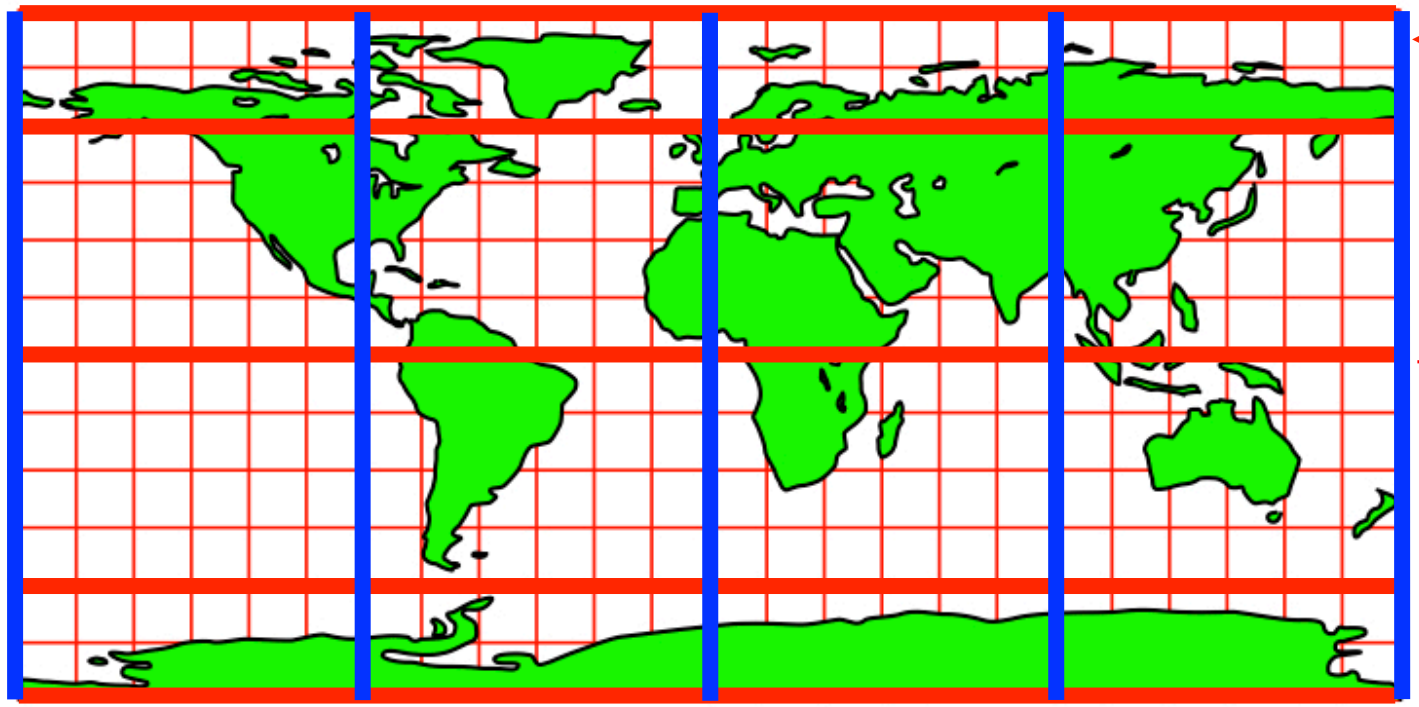


Maßstab eins zu eins?

6.37 cm vom Nordpol entfernt

$\frac{1}{100\,000\,000}$ $\frac{1}{100\,000\,000}$ $\frac{1}{100\,000\,000}$ $\frac{1}{100\,000\,000}$ $\frac{1}{100\,000\,000}$



$\frac{1}{0}$

$\frac{1}{50\,000\,000}$

$\frac{1}{100\,000\,000}$

$\frac{1}{50\,000\,000}$

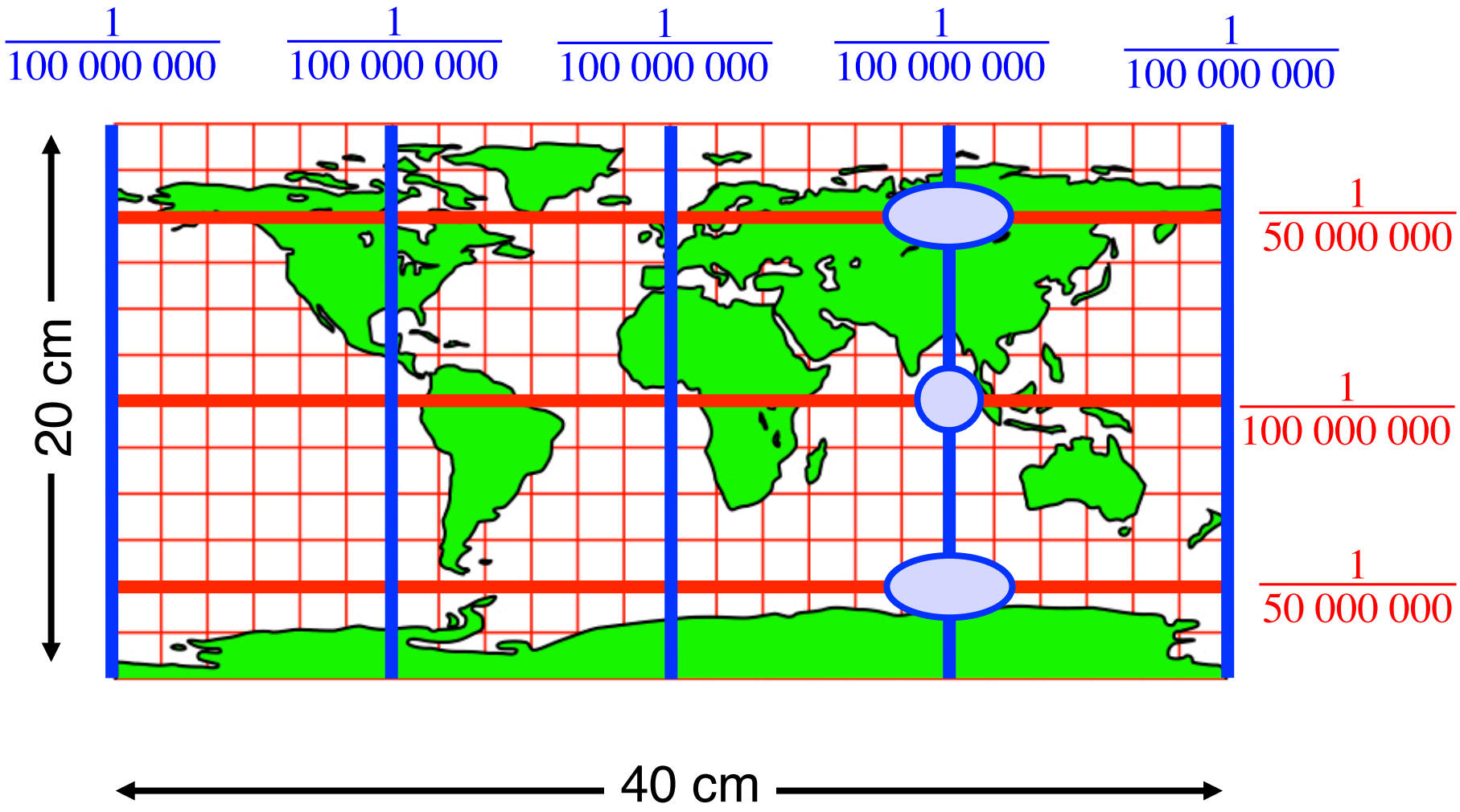
$\frac{1}{0}$

20 cm

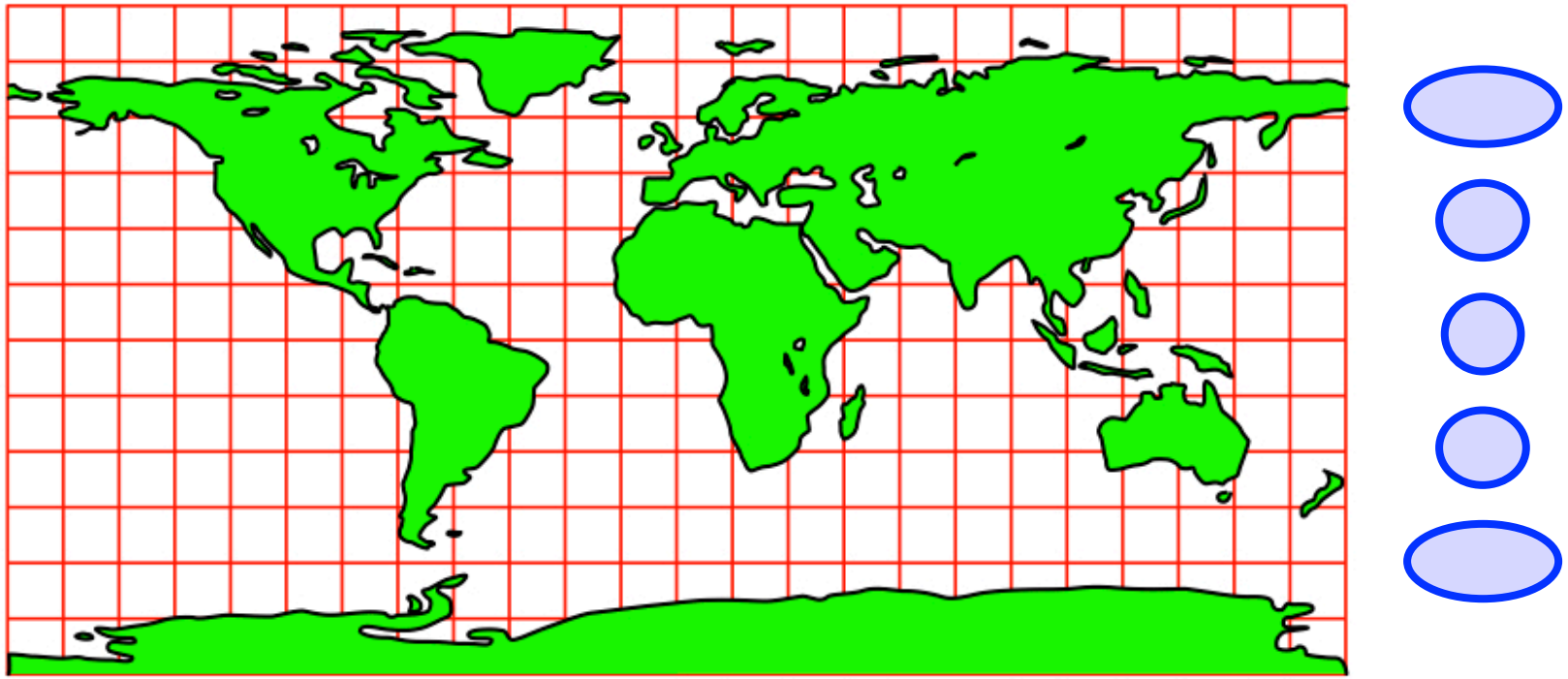
40 cm

$\phi = \pm 89.9999994° = \pm 89°\ 59'\ 59.998"$

Verzerrungsellipse, Tissotsche Indikatrix



Verzerrungsellipse, Tissotsche Indikatrix
Bild des Swimming Pools



Carl Friedrich Gauß
1777 - 1855

verzerrungsfreie, maßstäbliche

Theorema egregium:

Es gibt keine isometrische Abbildung von der Kugel auf die Ebene.

Carl Friedrich Gauß
1777 - 1855

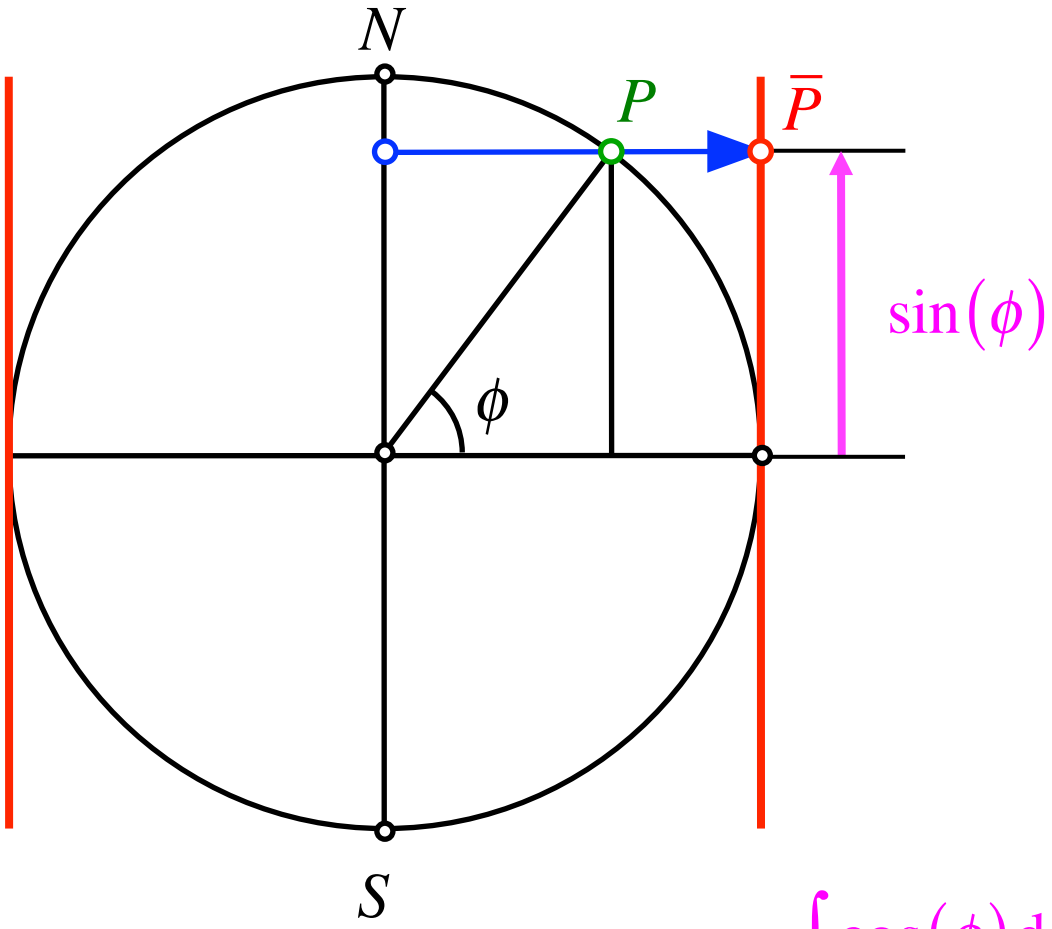
Theorema egregium:

Es gibt keine isometrische Abbildung von der Kugel auf die Ebene.

Hingegen gibt es:

- flächentreue Karten (equivalent)
- winkeltreue Karten (conformal)
- und viele andere Karten

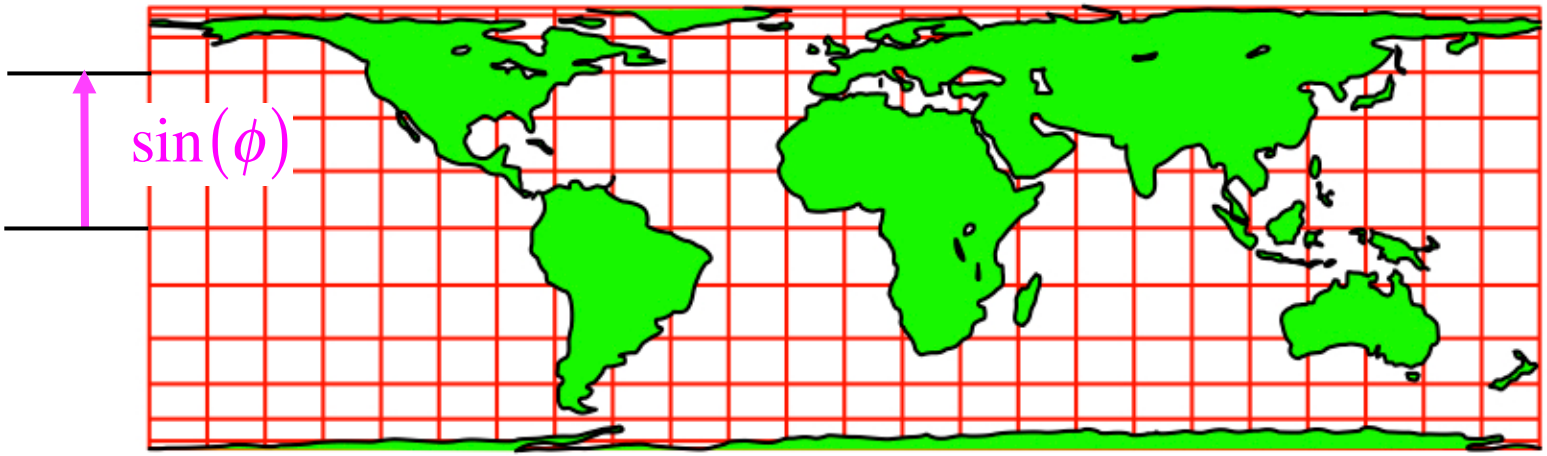
Flächentreu (equivalent), Archimedes / Lambert



$$\int \cos(\phi)\,\mathrm{d}\phi = \sin(\phi)$$

Flächentreu (equivalent), Archimedes / Lambert

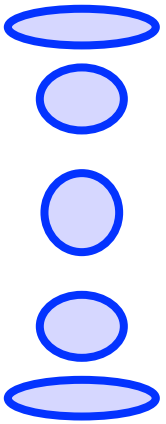
Bild des Swimming Pools



$$\int \cos(\phi)\,\mathrm{d}\phi = \sin(\phi)$$

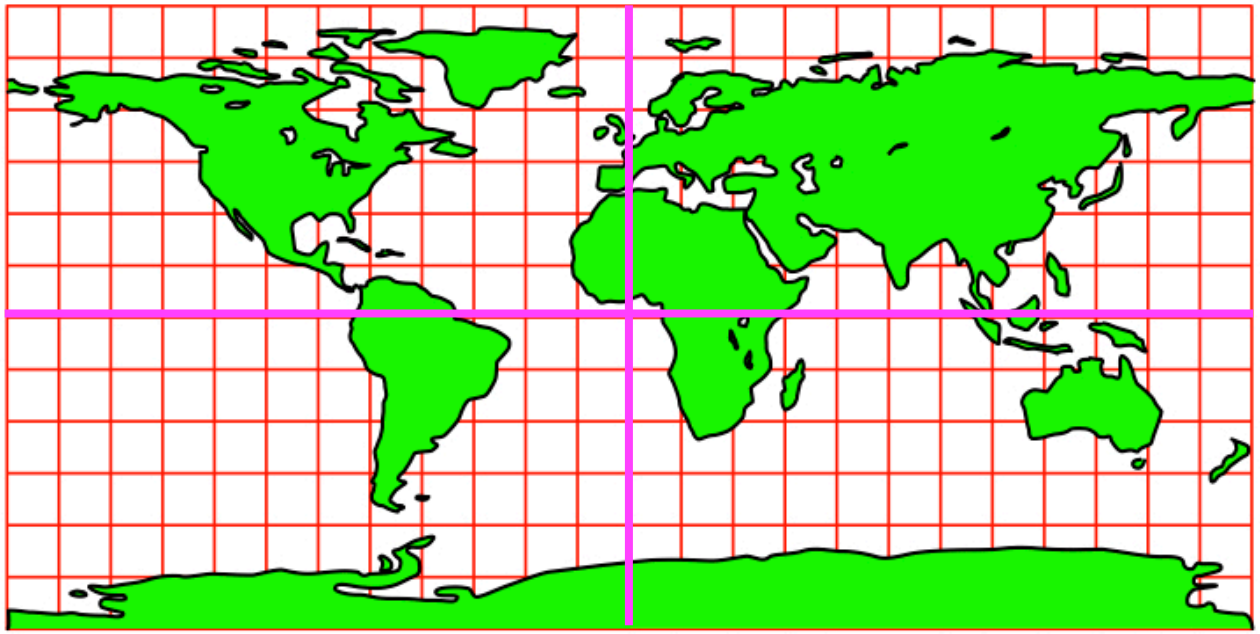
Flächentreu (equivalent), Archimedes / Lambert

Johann Heinrich Lambert
1728-1777



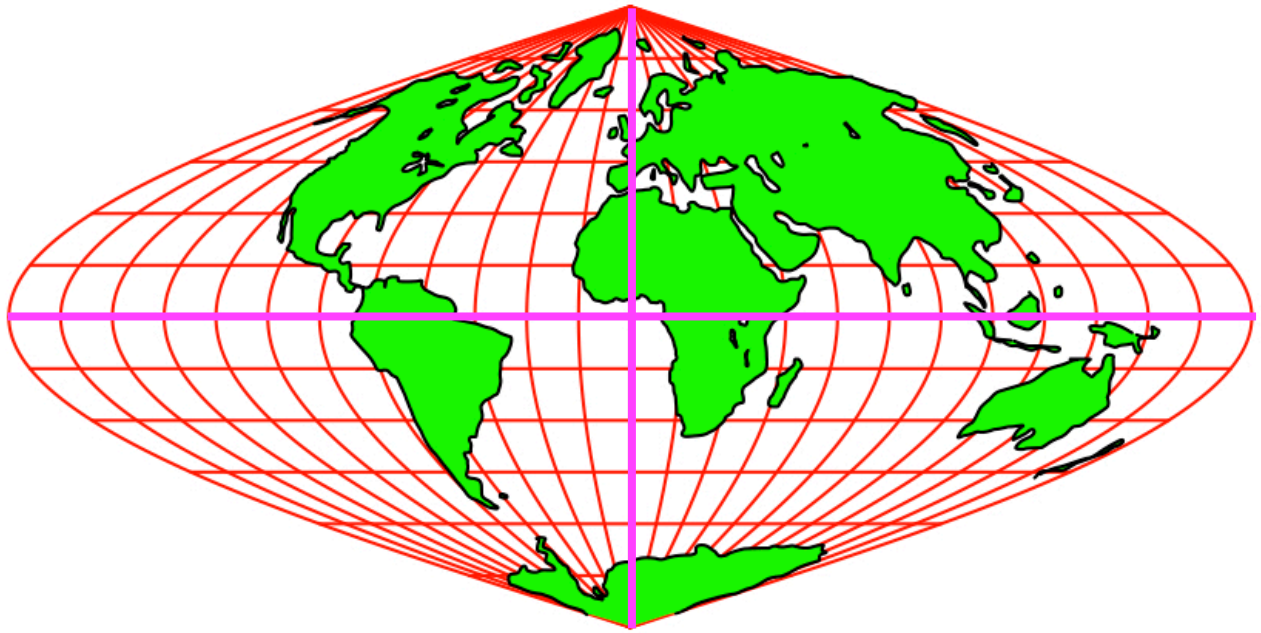
Flächentreu (equivalent), Mercator / Sanson

Flächentreu (equivalent), Mercator / Sanson



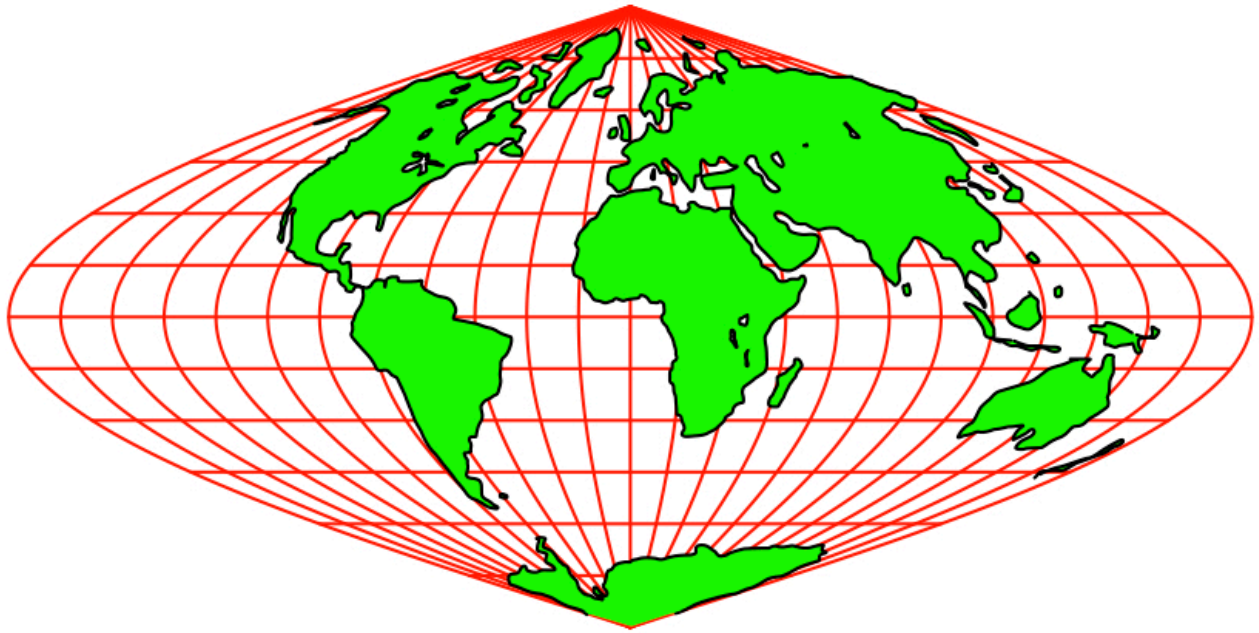
Idee: an den Polen zu Punkt einbrutzeln

Flächentreu (equivalent), Mercator / Sanson



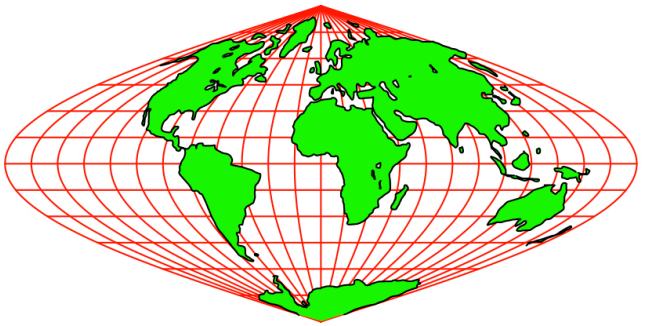
Idee: an den Polen zu Punkt einbrutzeln

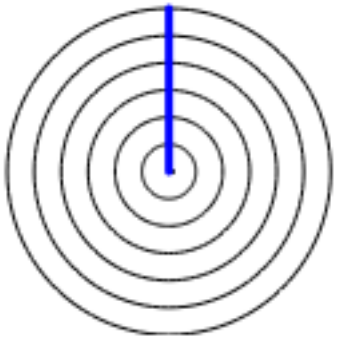
Flächentreu (equivalent), Mercator / Sanson



Erinnerung an die Schule

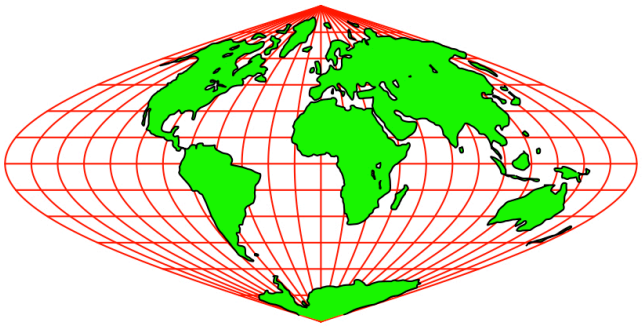
Kreis und Dreieck

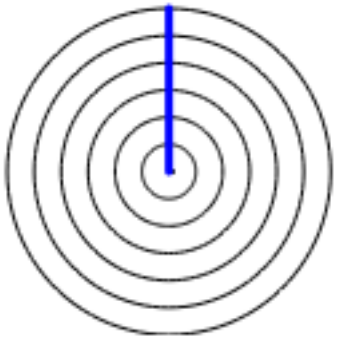




Erinnerung an die Schule

Kreis und Dreieck





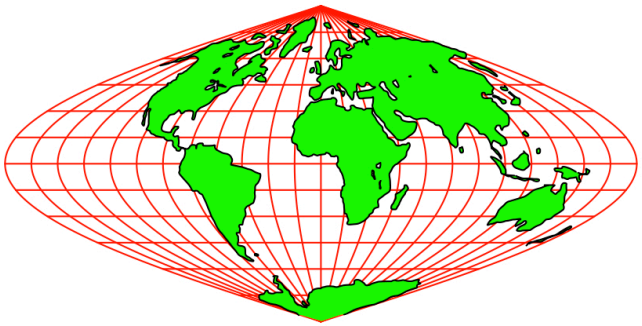
$$\text{Grundlinie} = 2r\pi$$

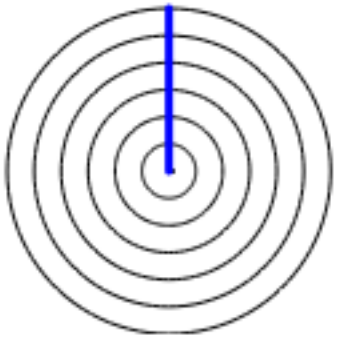
$$\text{Höhe} = r$$

$$\text{Flächeninhalt} = \frac{2r\pi \cdot r}{2} = r^2\pi$$

Erinnerung an die Schule

Kreis und Dreieck





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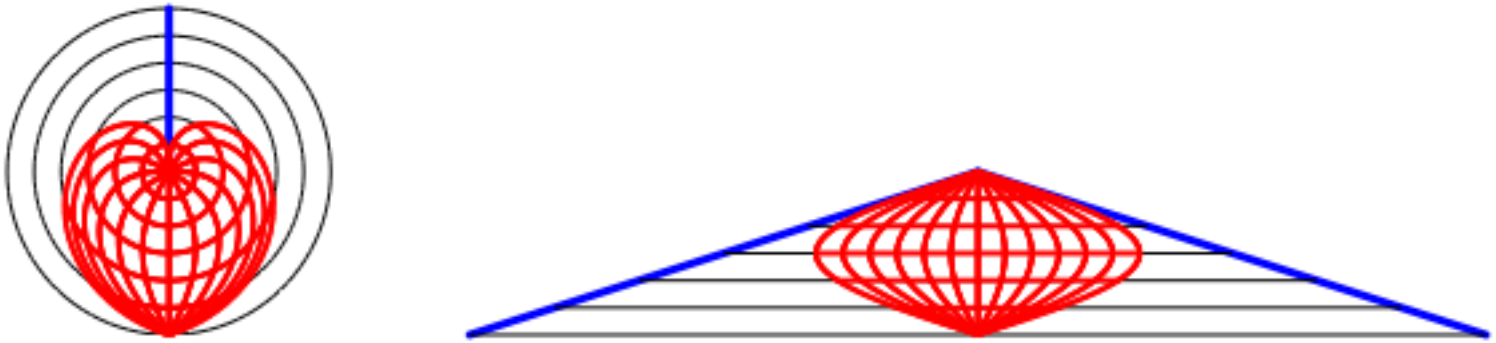
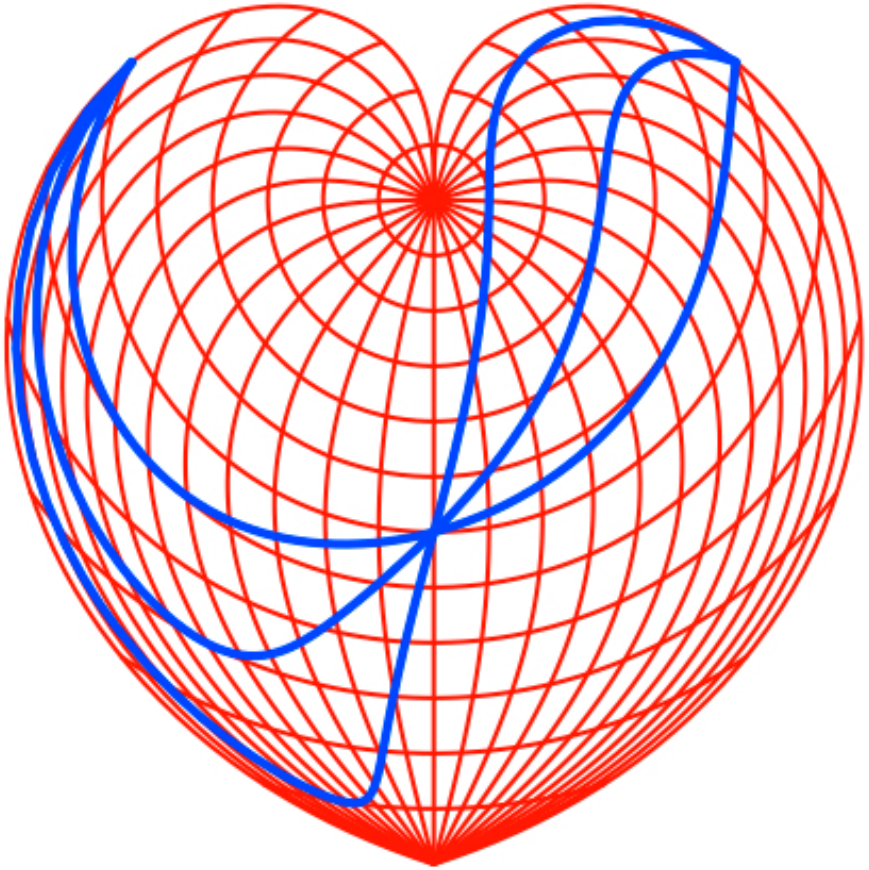
Erinnerung an die Schule

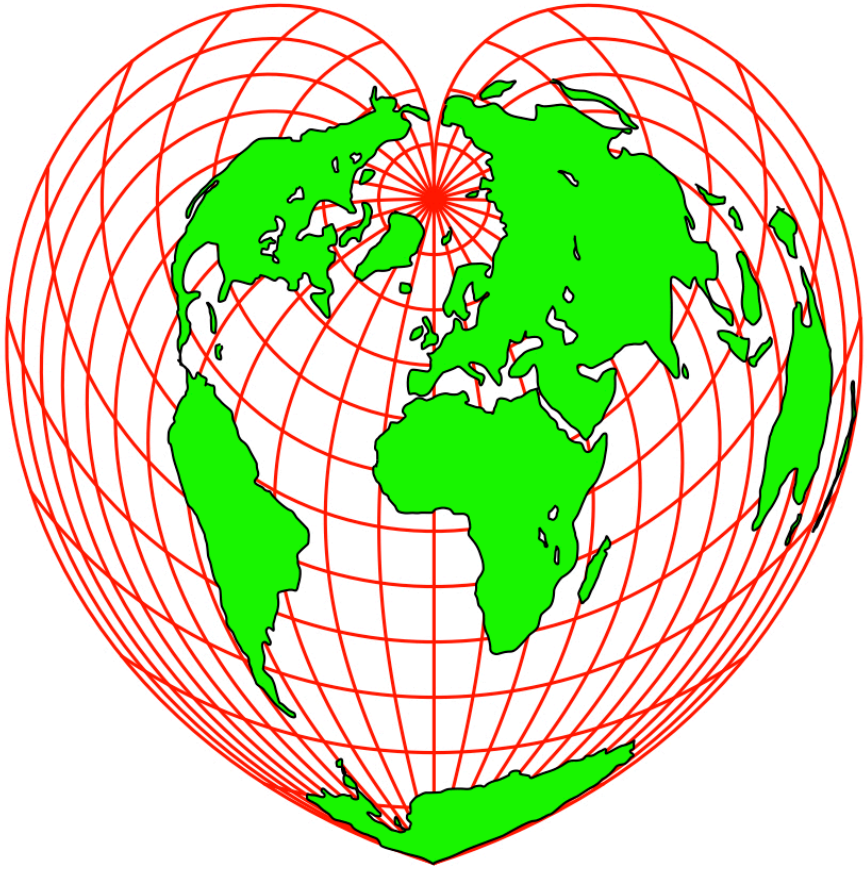
Kreis und Dreieck



Flächentreu (equivalent), Herzkarte von Stab / Werner 1514



Flächentreu (equivalent), Herzkarte von Stab / Werner 1514

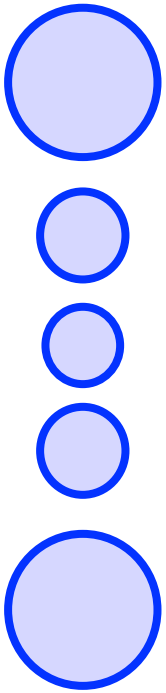


Flächentreu (equivalent), Herzkarte von Stab / Werner 1514

501 Jahre

Johannes Stabius
vor 1468 (bei Steyr) – 1522 (Graz)

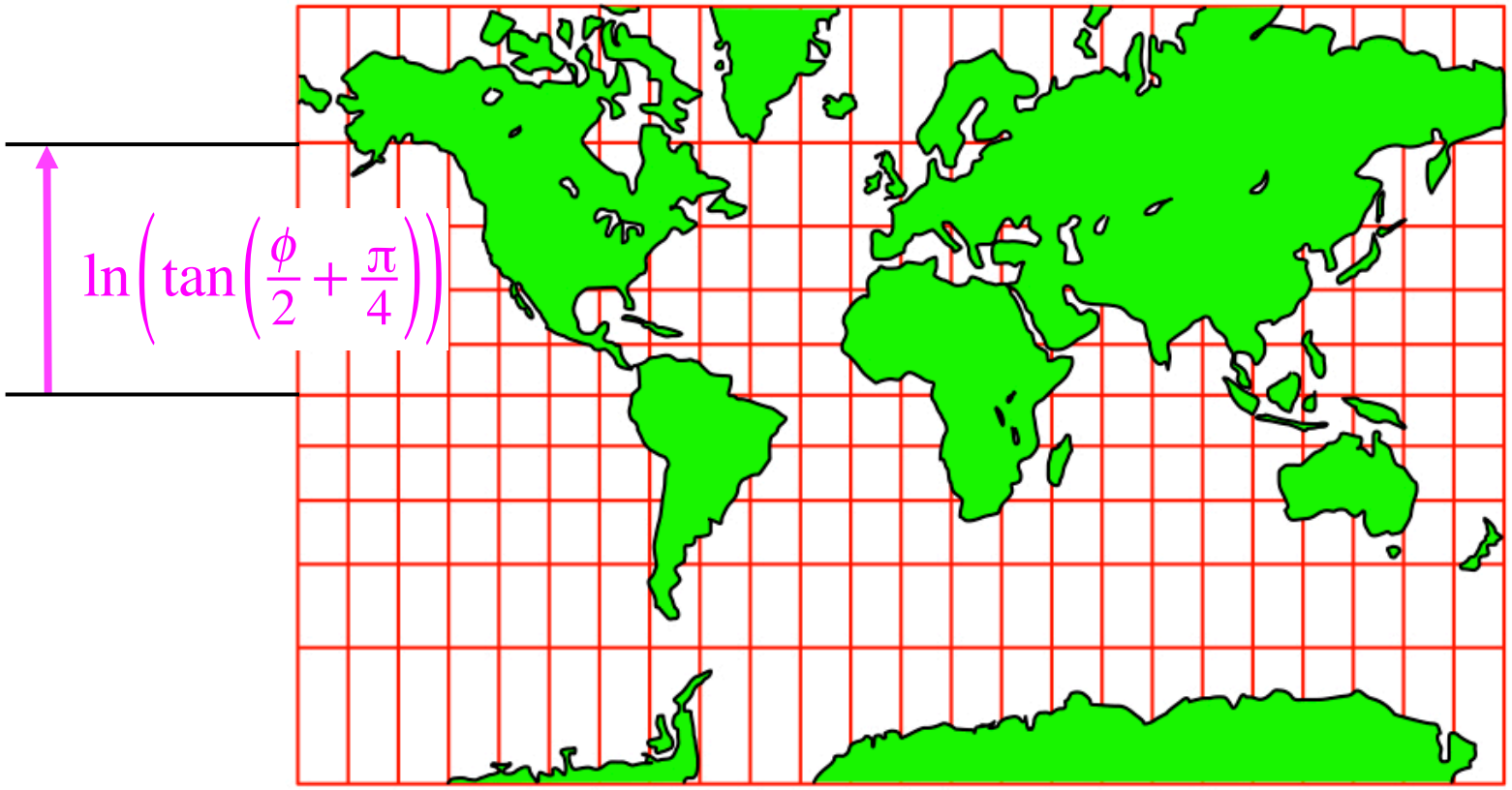
Winkeltreu (conformal), Mercator, 1569



Winkeltreu (conformal), Mercator, 1569

Gerhard Mercator
1512 - 1594

Winkeltreu (conformal), Mercator, 1569

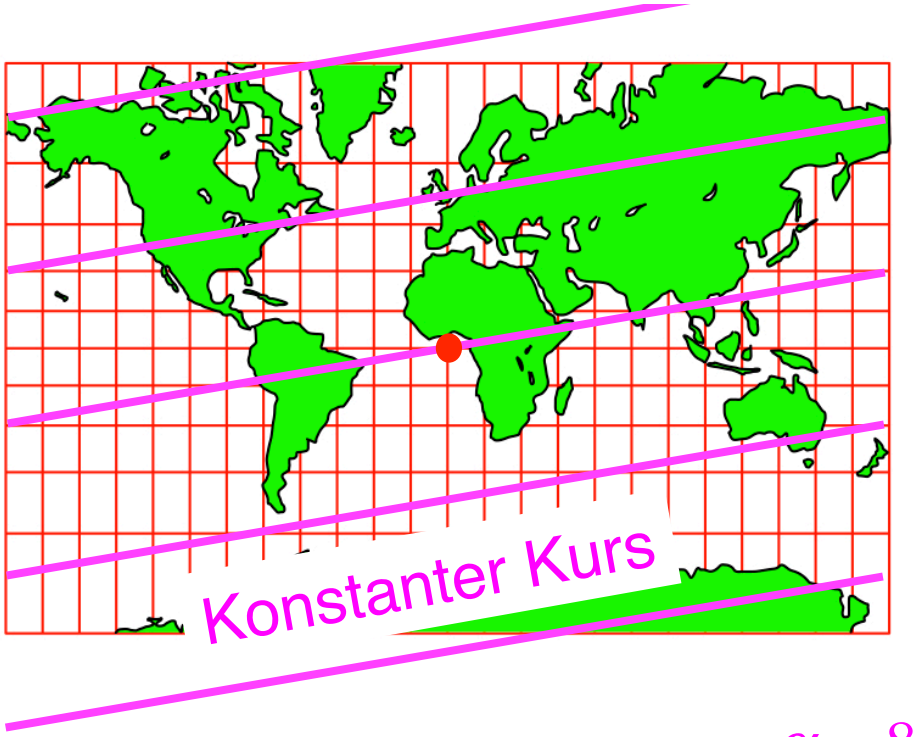


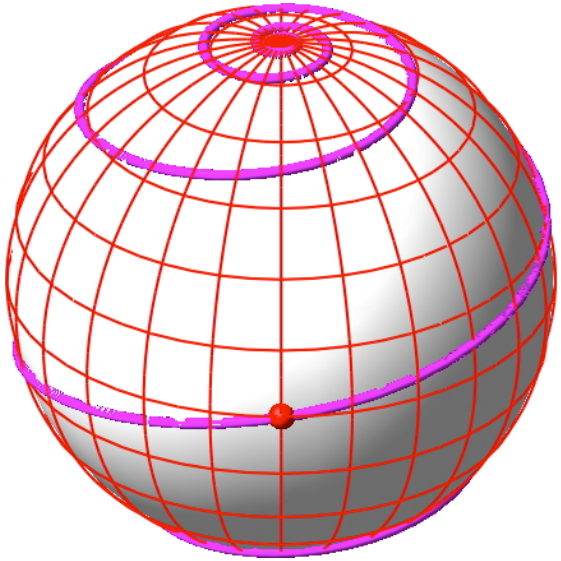
$$\int \frac{1}{\cos(\phi)}\,\mathrm{d}\phi = \ln\left(\tan\left(\frac{\phi}{2}+\frac{\pi}{4}\right)\right)$$

Winkeltreu (conformal), Mercator, 1569

Loxodrome:

Kurve mit konstantem Winkel α gegenüber Meridianen

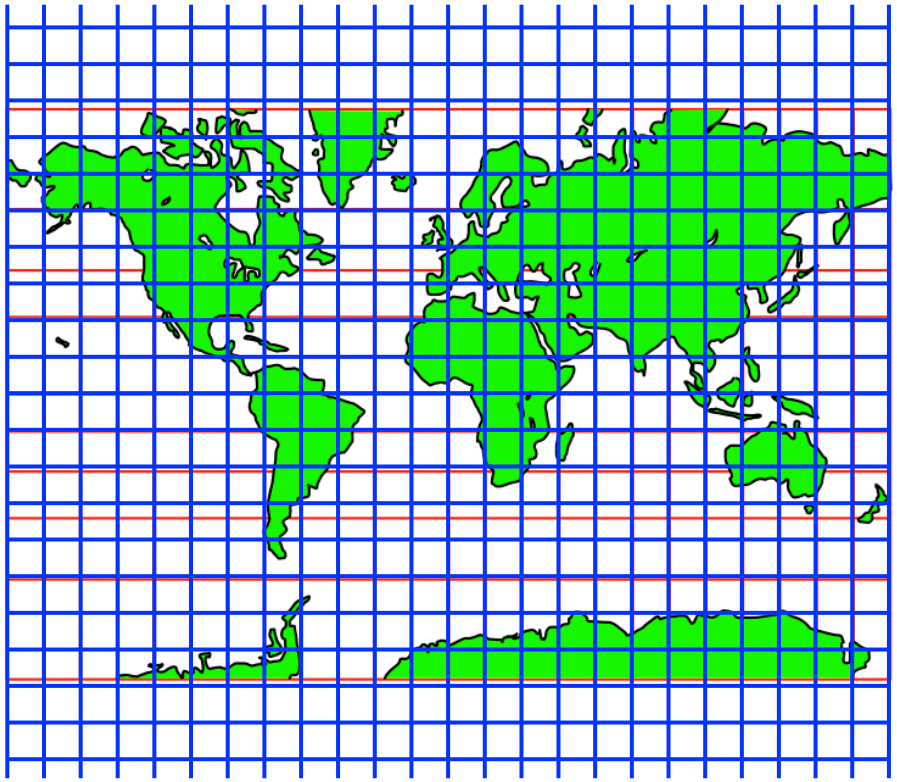


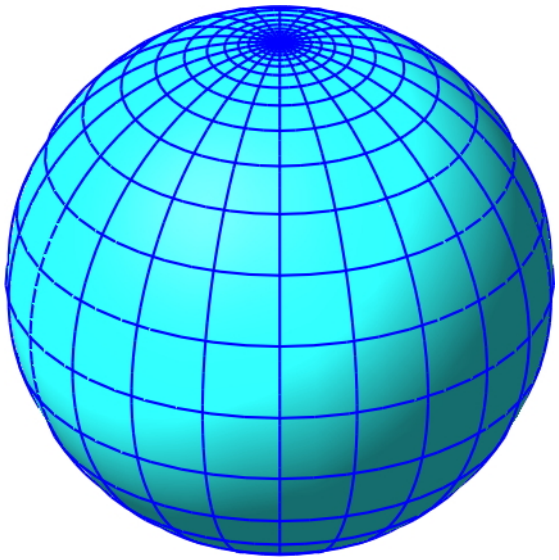


$\alpha = 80°$

Winkeltreu (conformal), Mercator, 1569

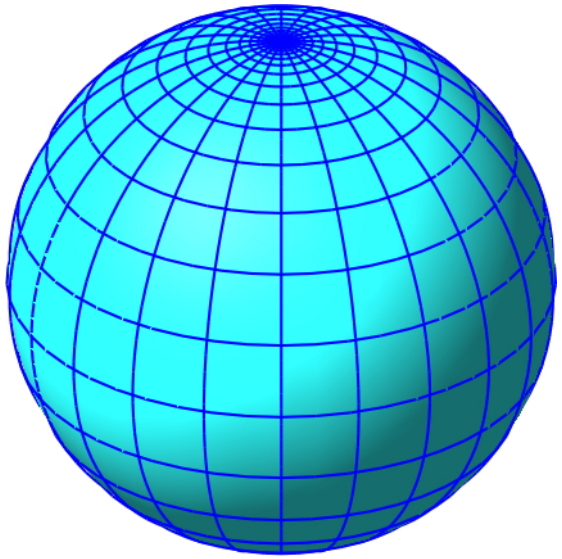
Schöne Kugel: Quadratraster Seekarte —> Kugel





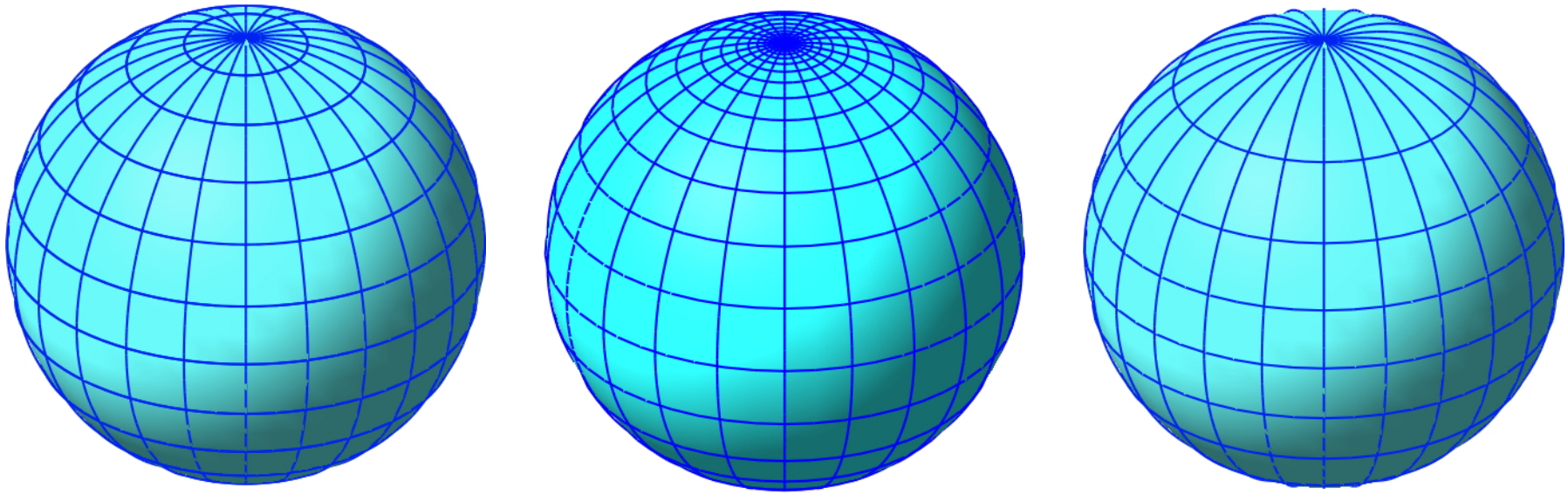
Winkeltreu (conformal), Mercator, 1569

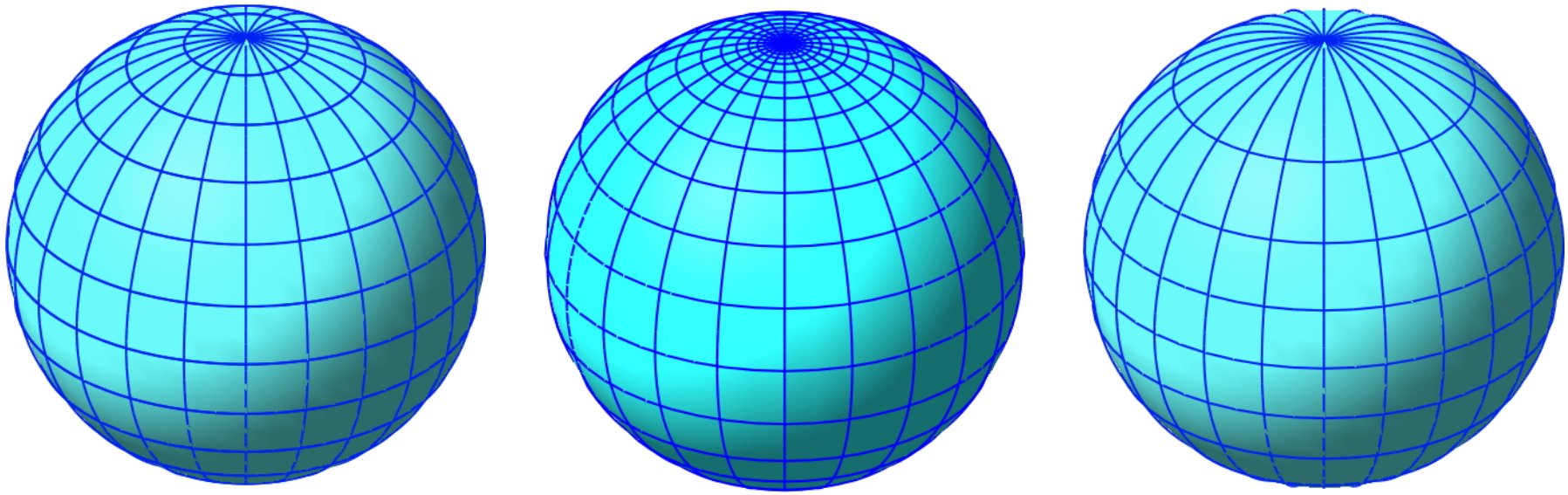
Schöne Kugel: Quadratraster Seekarte —> Kugel



(Handout)

Welches ist die schönste Kugel?





Danke