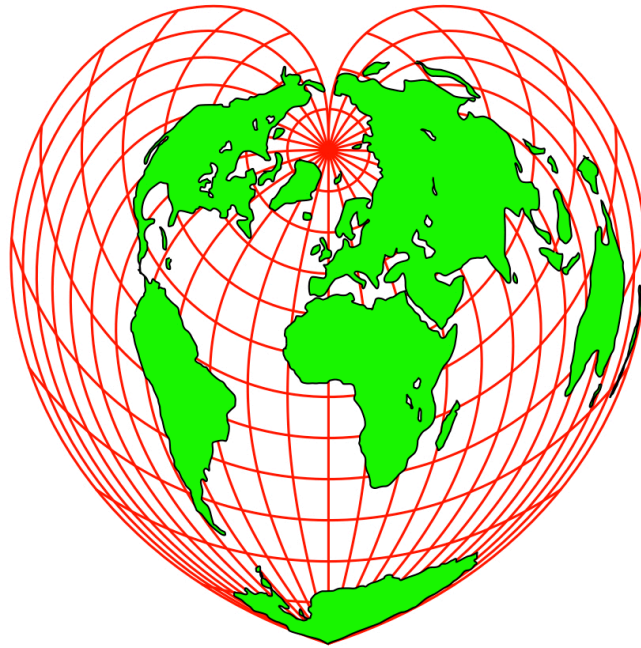


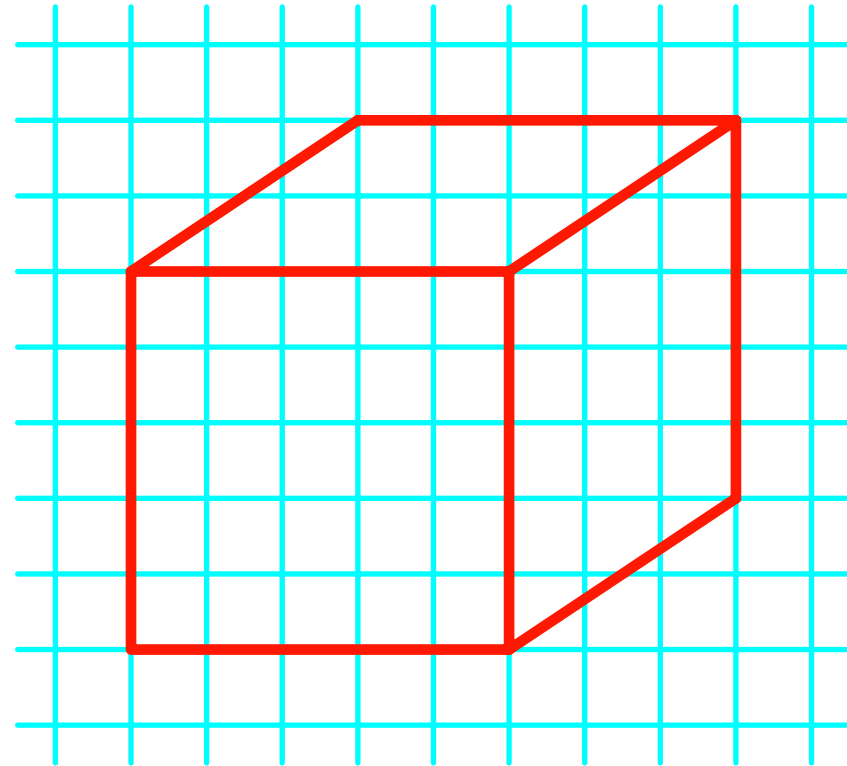
Hans Walser

Maßstab 1:1



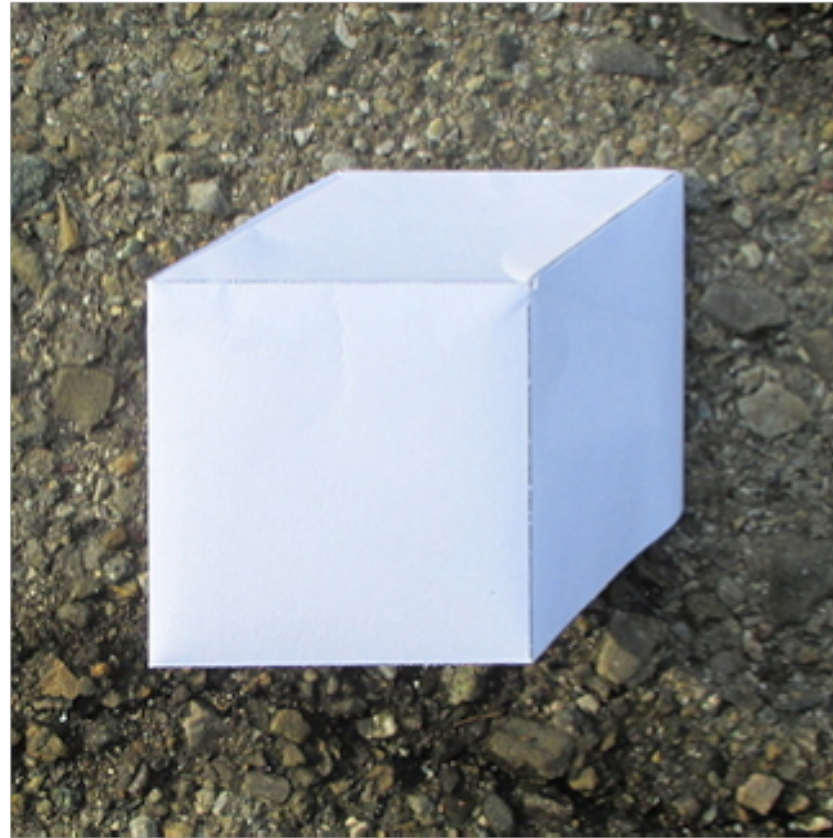
[www.walser-h-m.ch/hans](http://www.walser-h-m.ch/hans)

Was sehen wir?



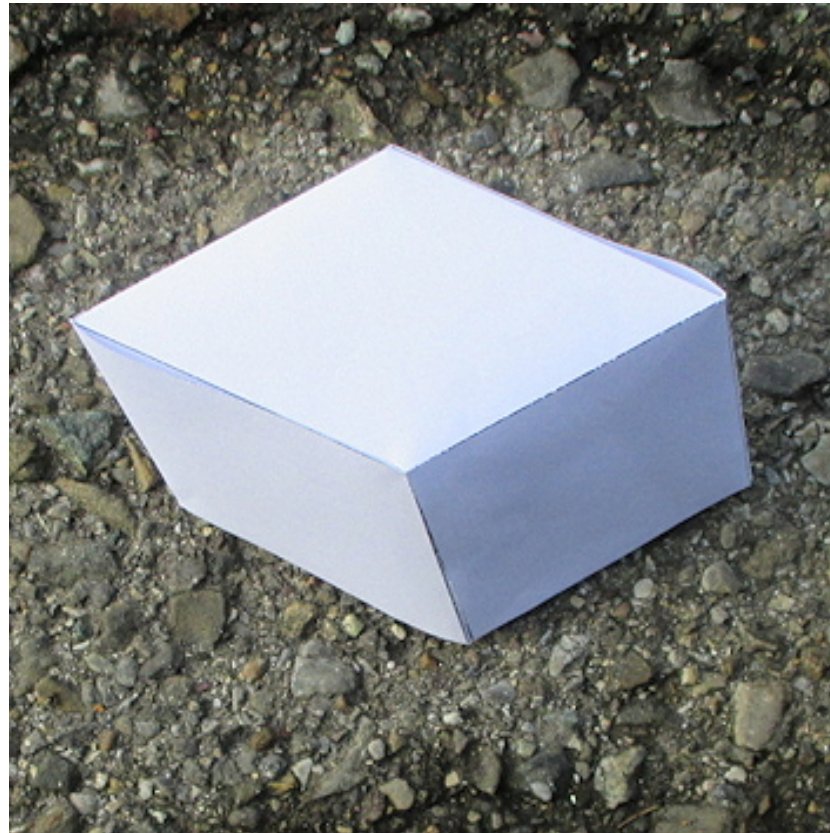
Was stellt es dar?  
Was bedeutet es?

Was sehen wir?



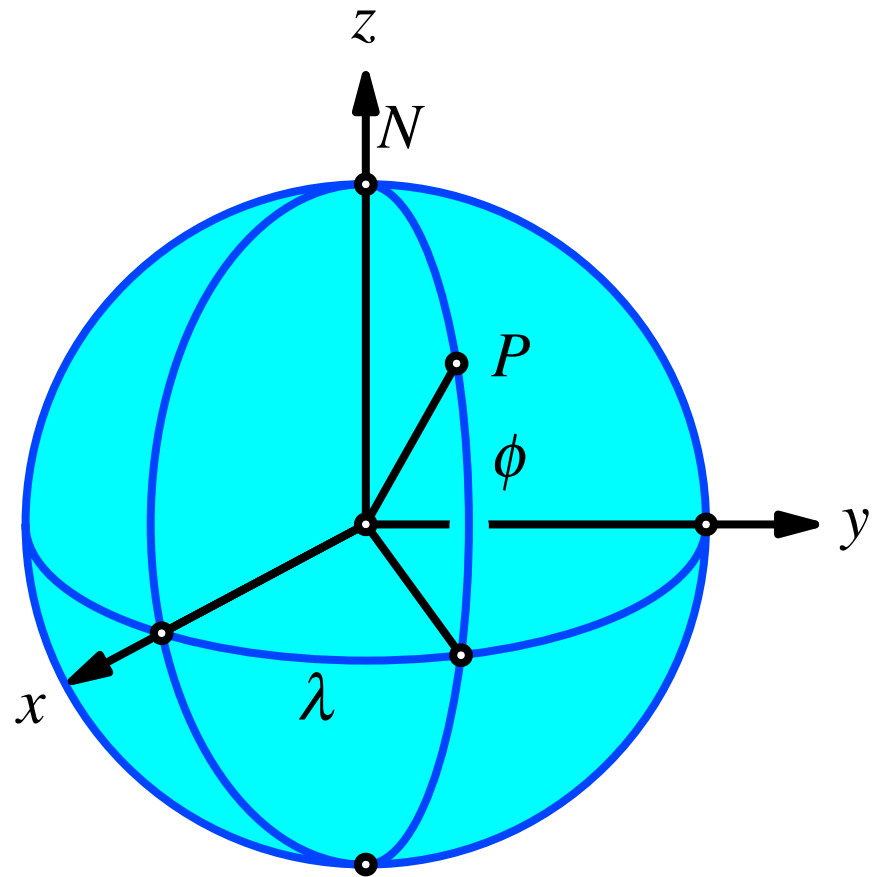
Was stellt es dar?  
Was bedeutet es?

Was sehen wir?



Was stellt es dar?  
Was bedeutet es?

Was sehen wir?

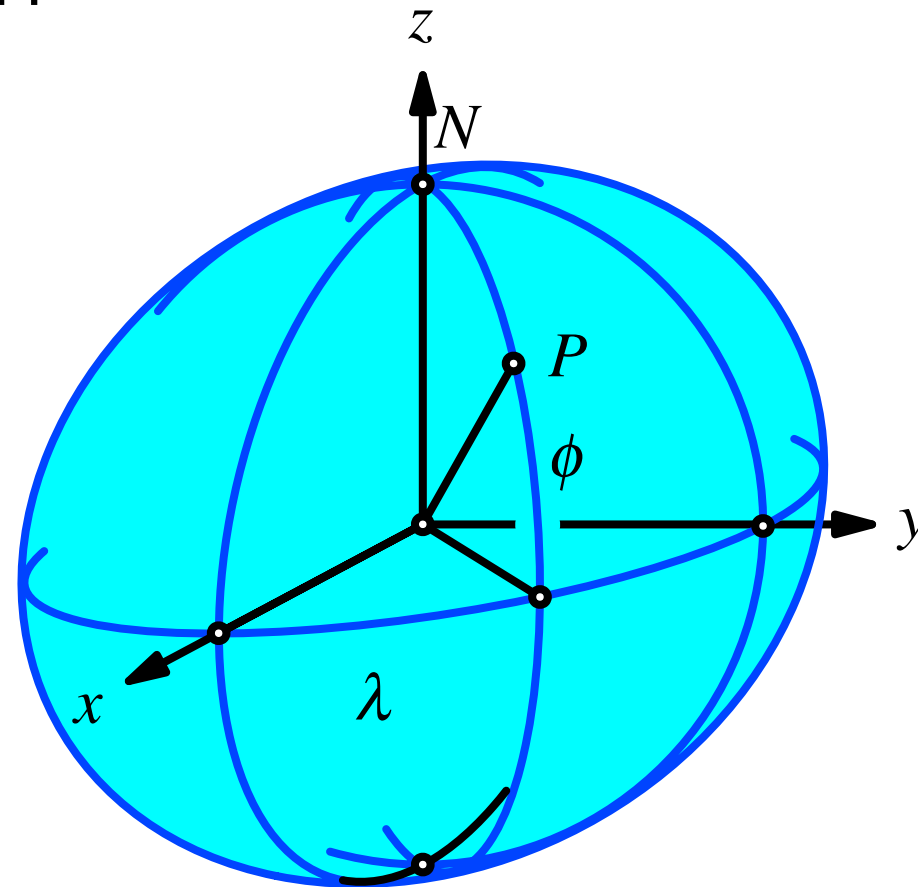


Was stellt es dar?  
Was bedeutet es?

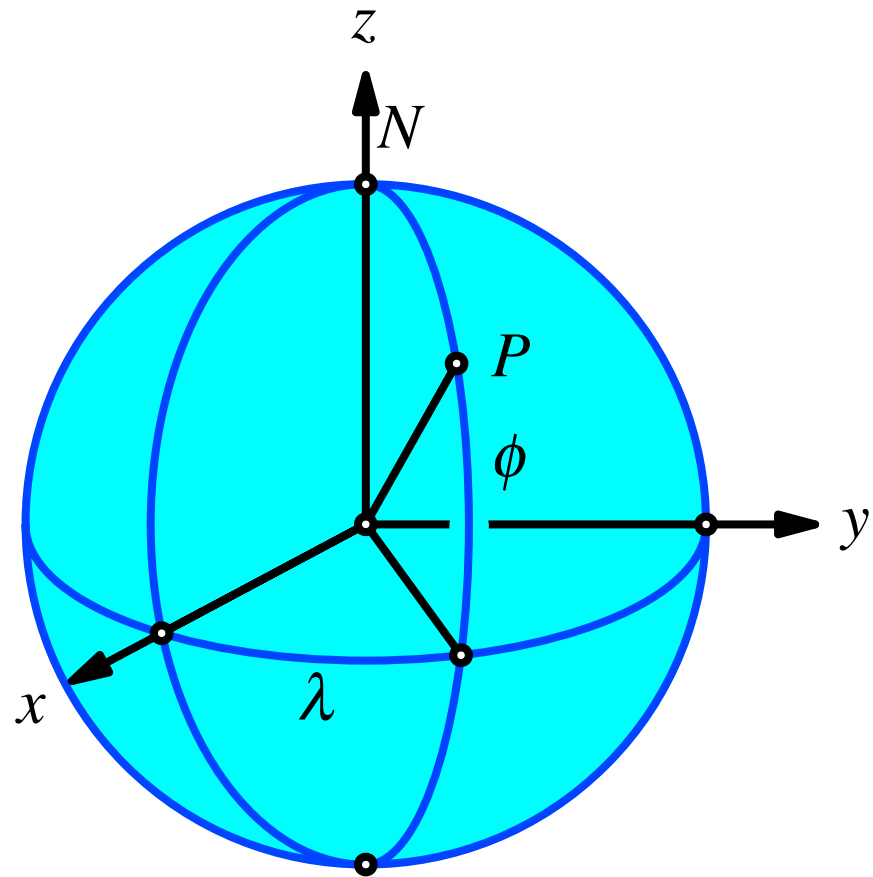


Was sehen wir?

Schrägbild

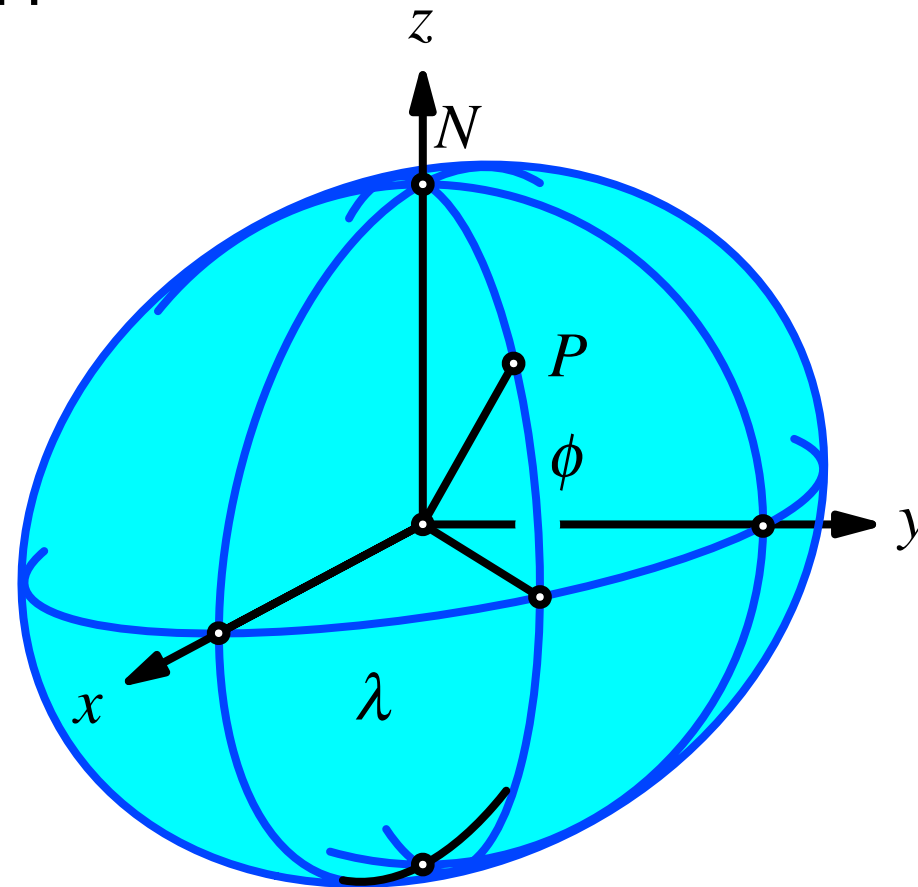


Was sehen wir?



Was sehen wir?

Schrägbild





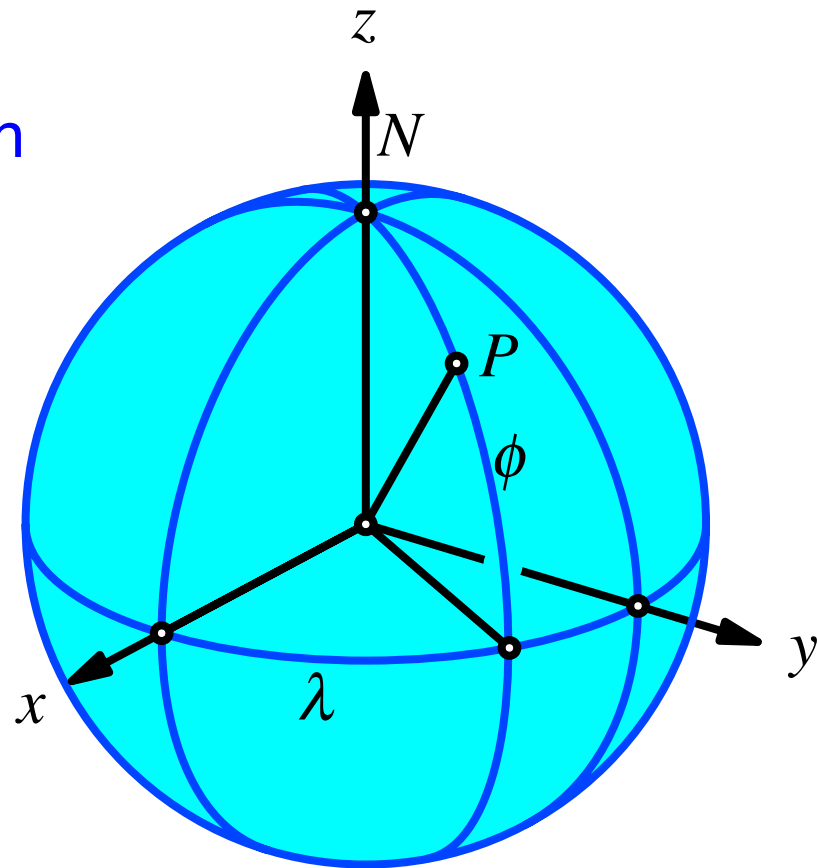
Was sehen wir?

Schrägbild

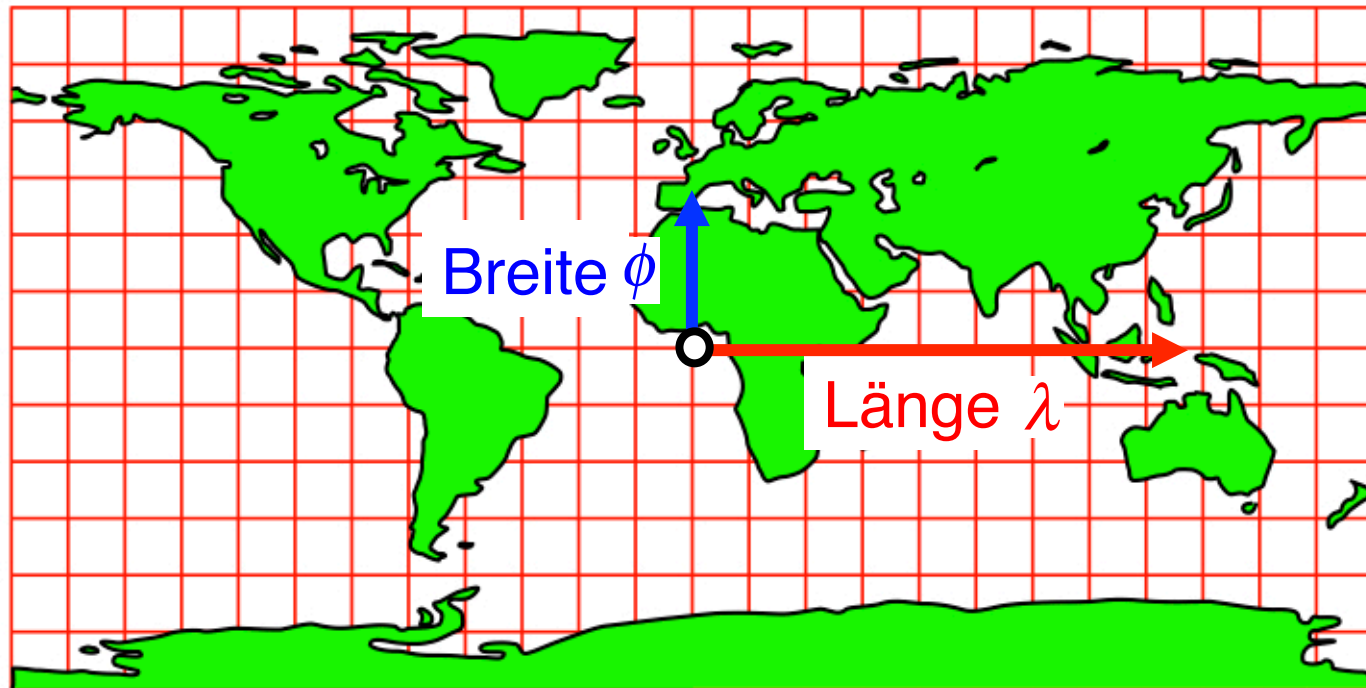


Was sehen wir?

Normalprojektion



Längen oder Winkel?



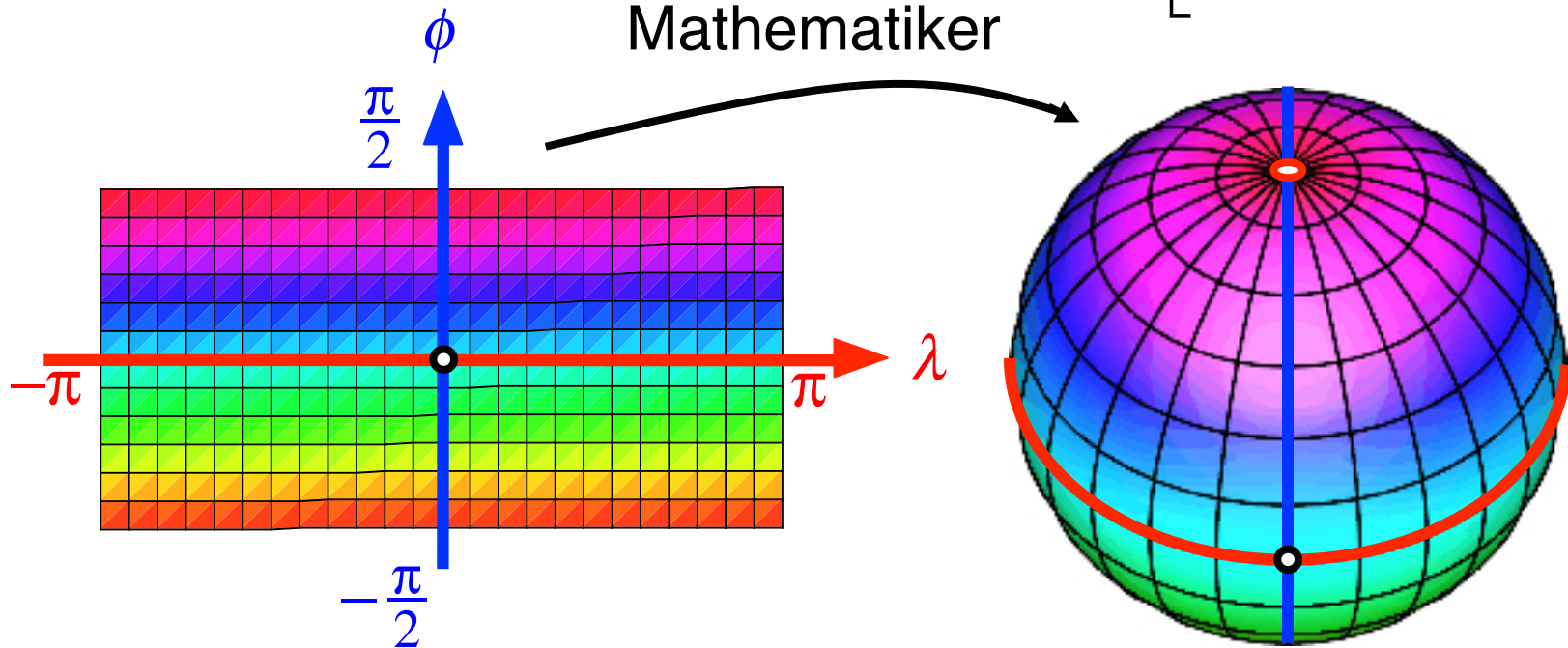
(Quadratische) Plattkarte



$$\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \lambda \in [-\pi, \pi]$$

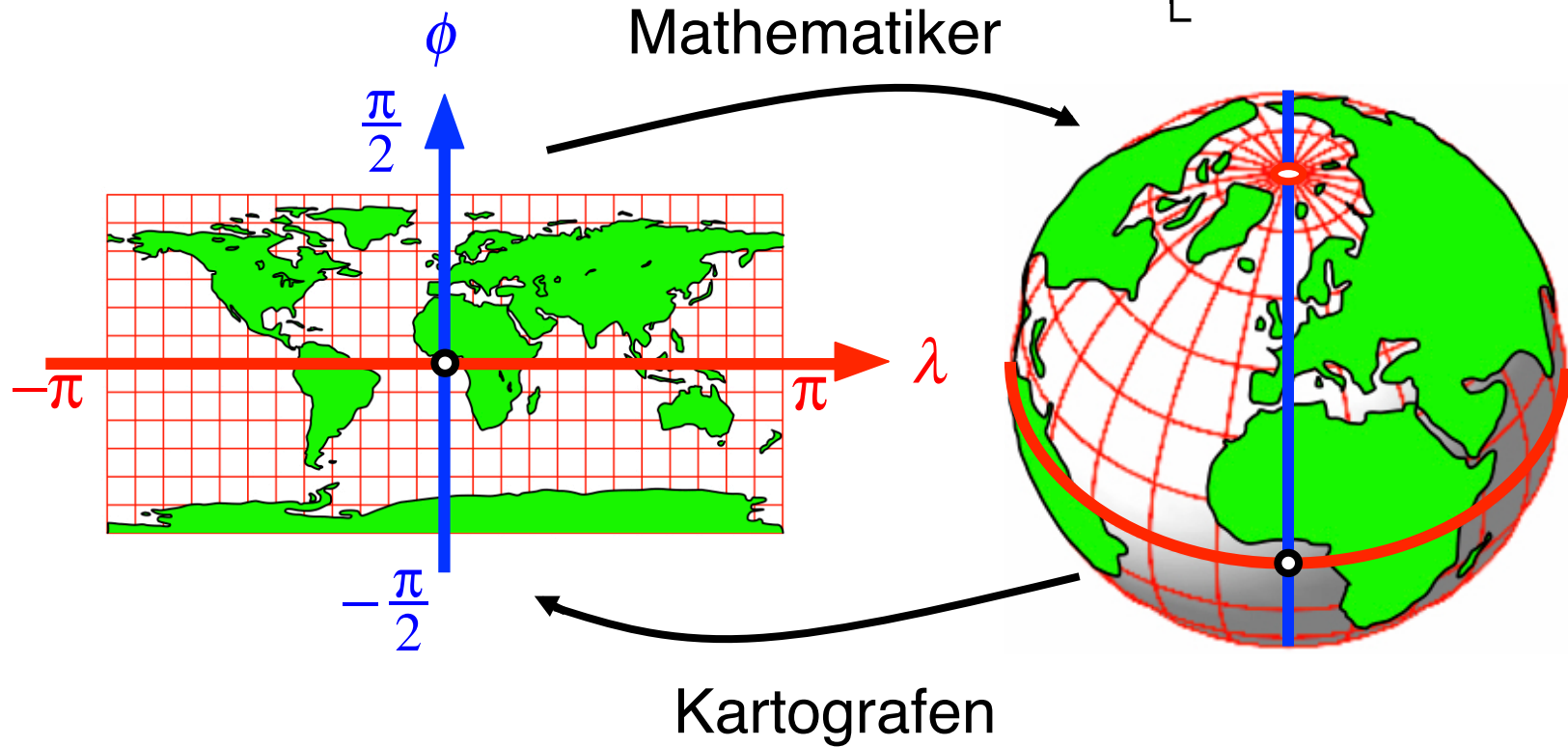
$$\vec{x}(\phi, \lambda) = \begin{bmatrix} \cos(\phi)\cos(\lambda) \\ \cos(\phi)\sin(\lambda) \\ \sin(\phi) \end{bmatrix}$$

Mathematiker



$$\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \lambda \in [-\pi, \pi]$$

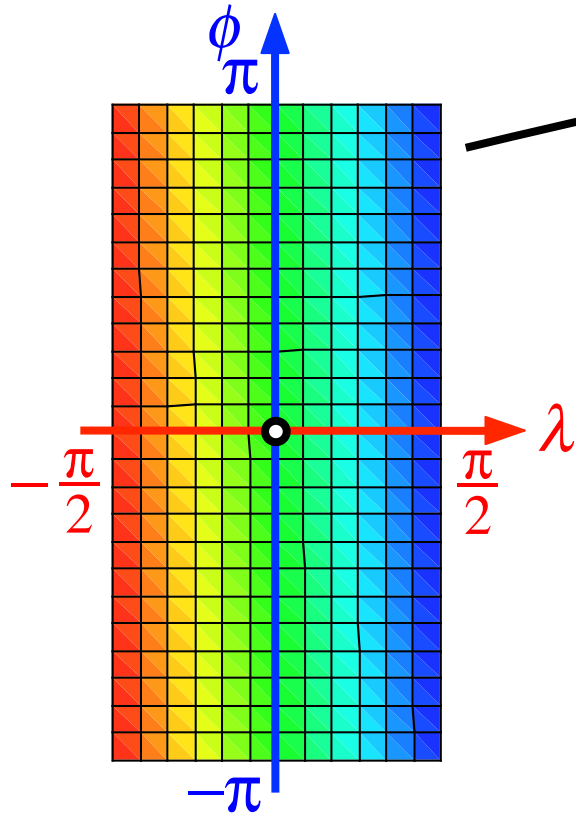
$$\vec{x}(\phi, \lambda) = \begin{bmatrix} \cos(\phi)\cos(\lambda) \\ \cos(\phi)\sin(\lambda) \\ \sin(\phi) \end{bmatrix}$$



Idee eines Schülers:  
Plattkarte Hochformat

$$\phi \in [-\pi, \pi], \lambda \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

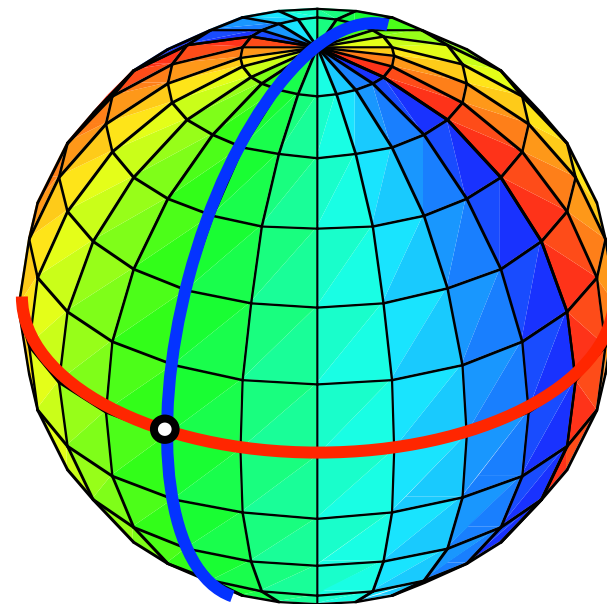
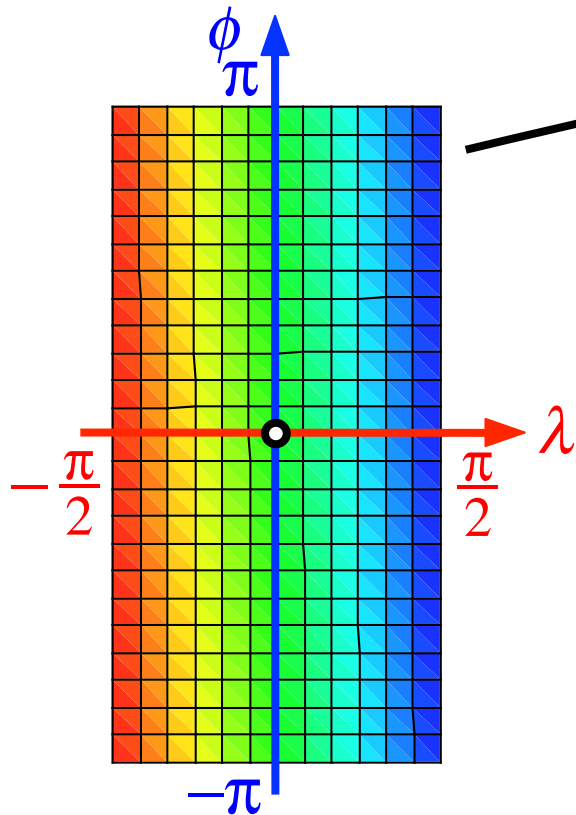
$$\vec{x}(\phi, \lambda) = \begin{bmatrix} \cos(\phi)\cos(\lambda) \\ \cos(\phi)\sin(\lambda) \\ \sin(\phi) \end{bmatrix}$$



Idee eines Schülers:  
Plattkarte Hochformat

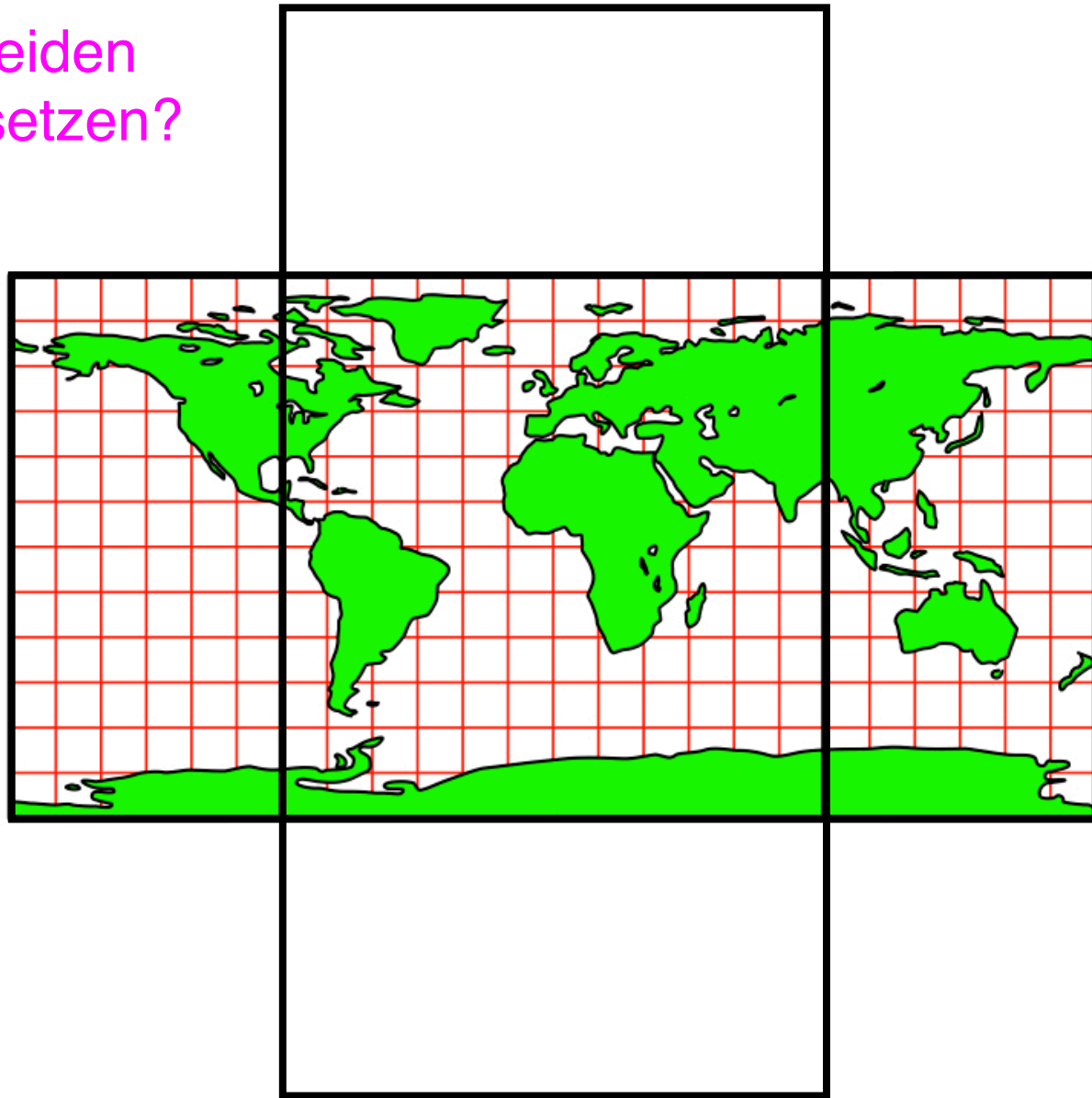
$$\phi \in [-\pi, \pi], \lambda \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\vec{x}(\phi, \lambda) = \begin{bmatrix} \cos(\phi)\cos(\lambda) \\ \cos(\phi)\sin(\lambda) \\ \sin(\phi) \end{bmatrix}$$



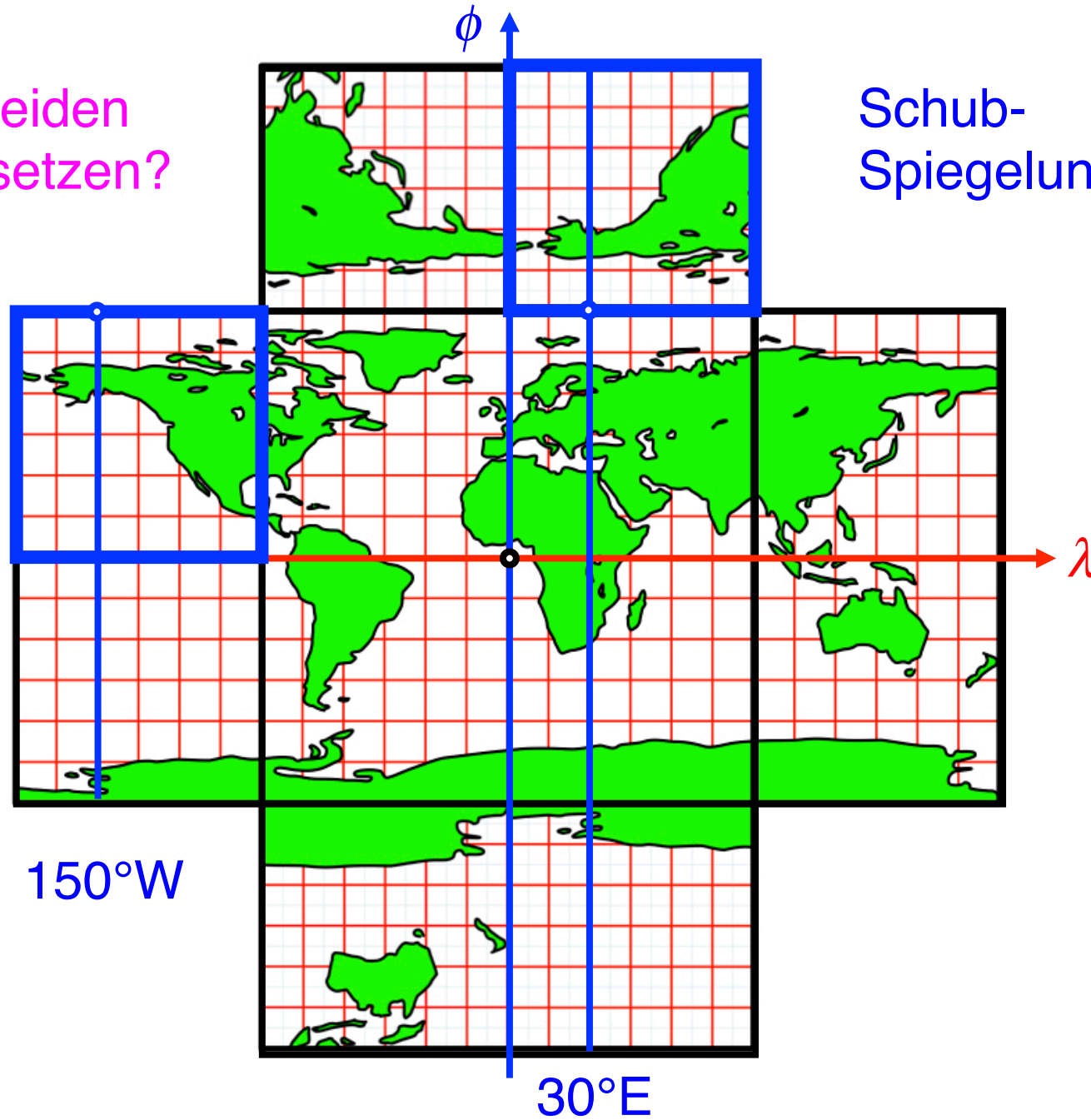


Abschneiden  
und ansetzen?

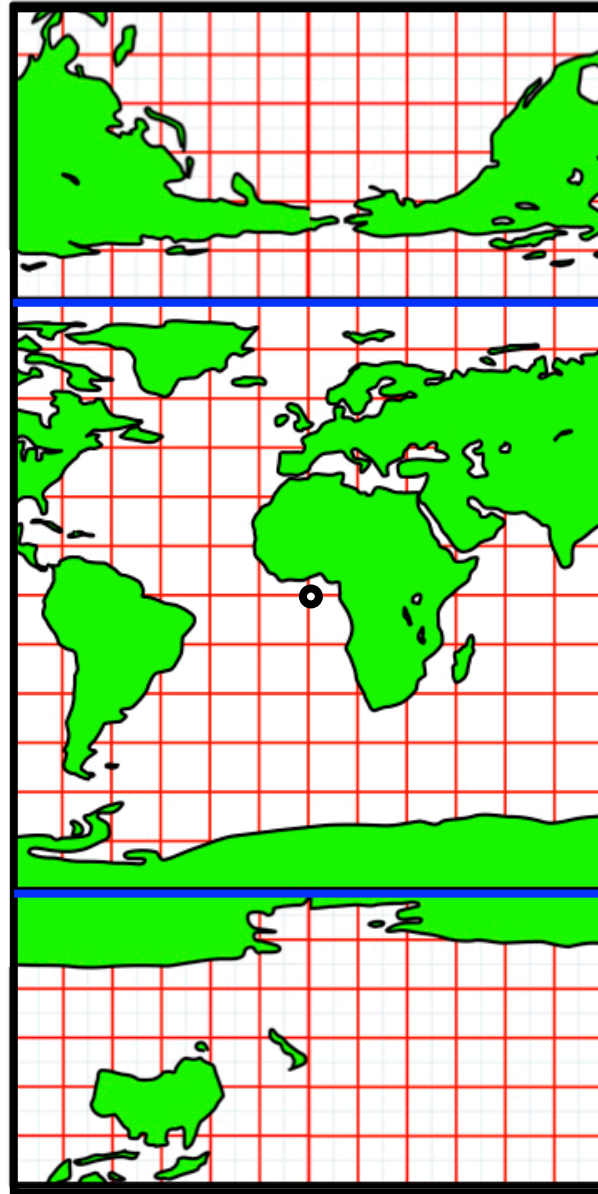


Abschneiden  
und ansetzen?

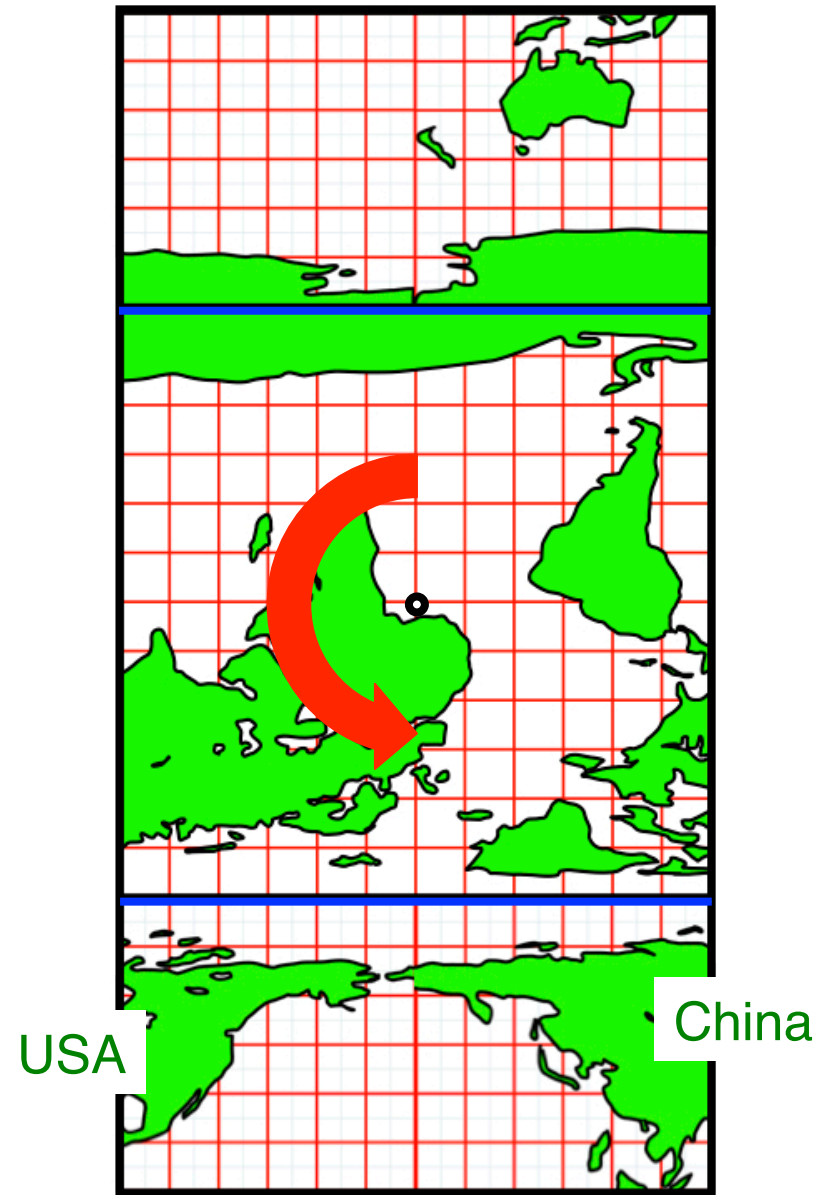
Schub-  
Spiegelung



# Die Welt im Hochformat



# Die Welt im Hochformat



Immer der Nase nach



... so geh hübsch sittsam und lauf nicht vom Wege ab!

Keine Seitenkrümmung

Geodätische Linie  $\ddot{x}^m + \Gamma_{kl}^m \dot{x}^k \dot{x}^l = 0$

Durchgehendes Plastikband

Großkreis (Orthodrome)

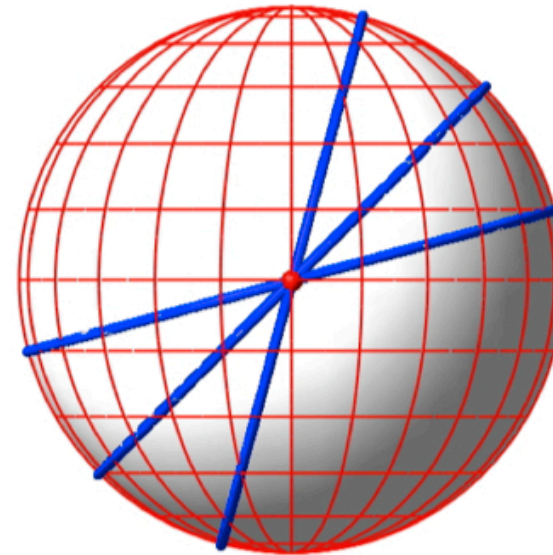
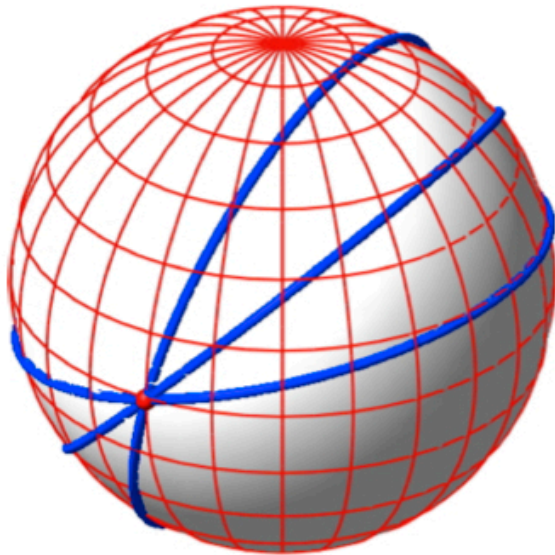
Immer der Nase nach



La línia recta és creació de l'home; la línia corba, de Déu.

ANTONI GAUDÍ

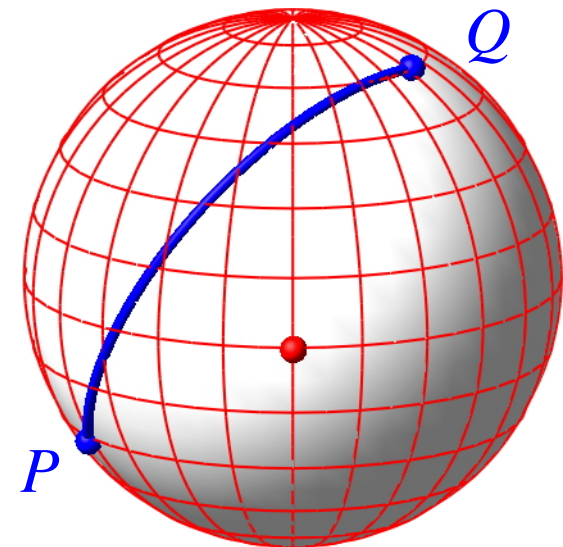
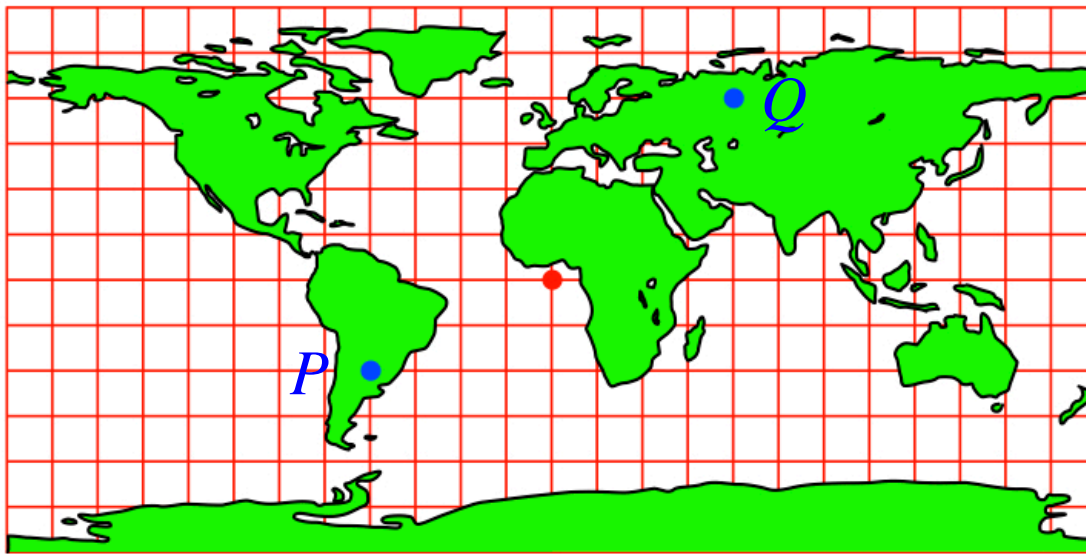
Immer der Nase nach  
Großkreise statt Geraden



Blick von der Seite

Immer der Nase nach

Großkreisbogen von  $P$  nach  $Q$ ?

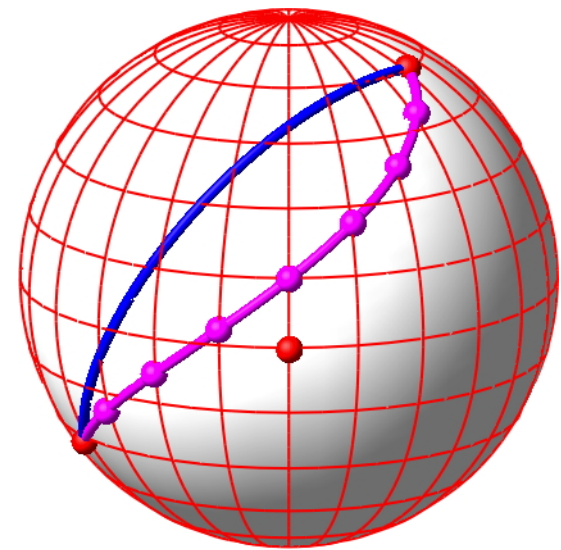
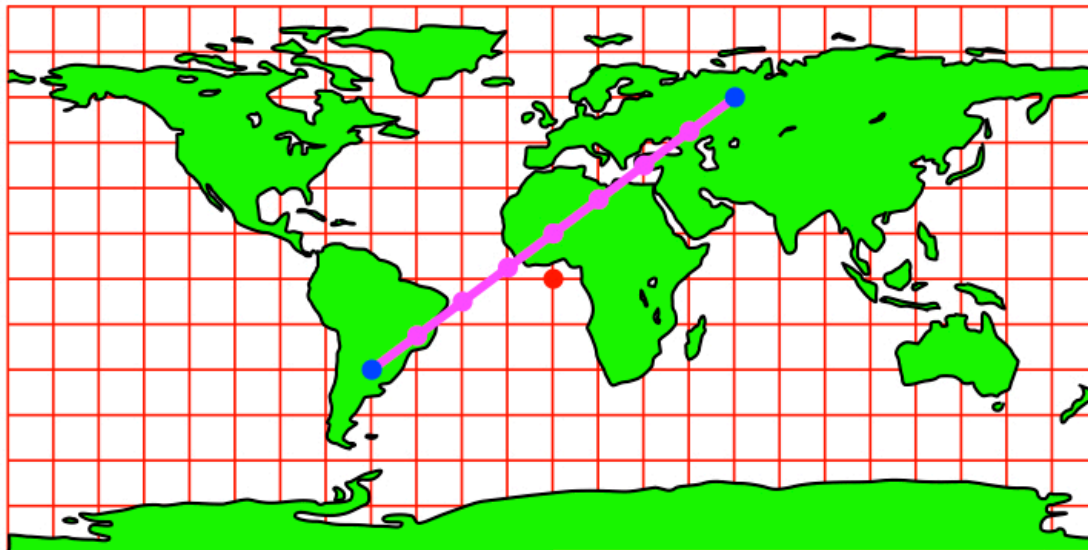




Immer der Nase nach

Großkreisbogen von  $P$  nach  $Q$

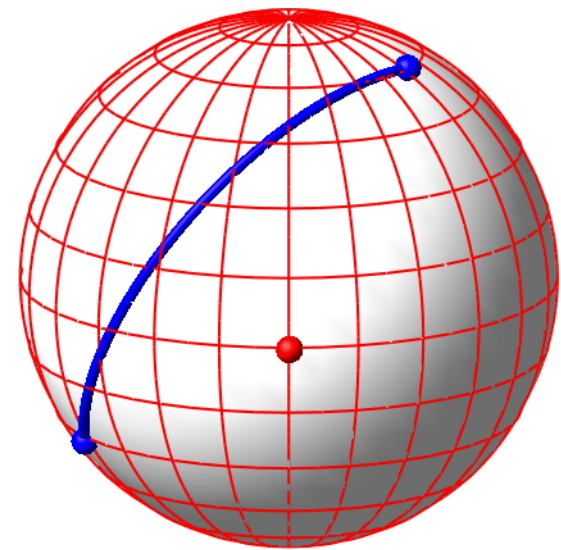
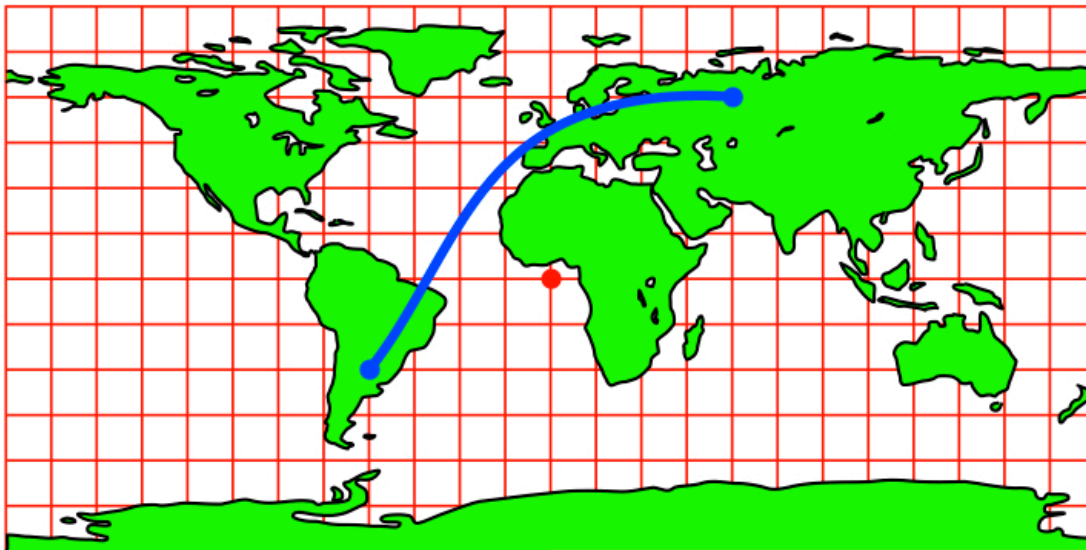
Eine falsche Idee



Immer der Nase nach

Großkreisbogen von  $P$  nach  $Q$

Die richtige Kurve

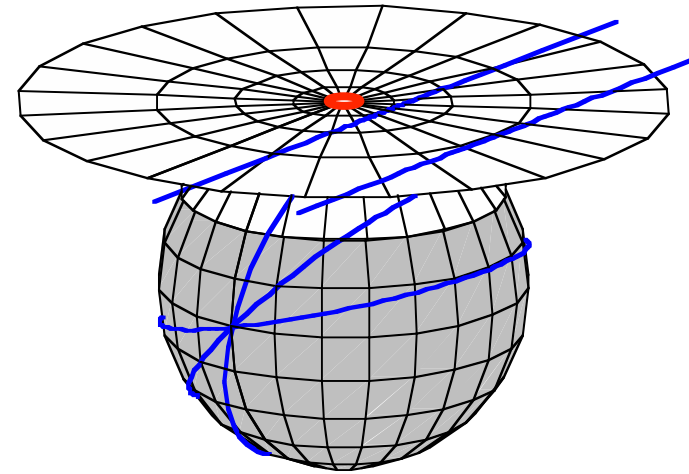
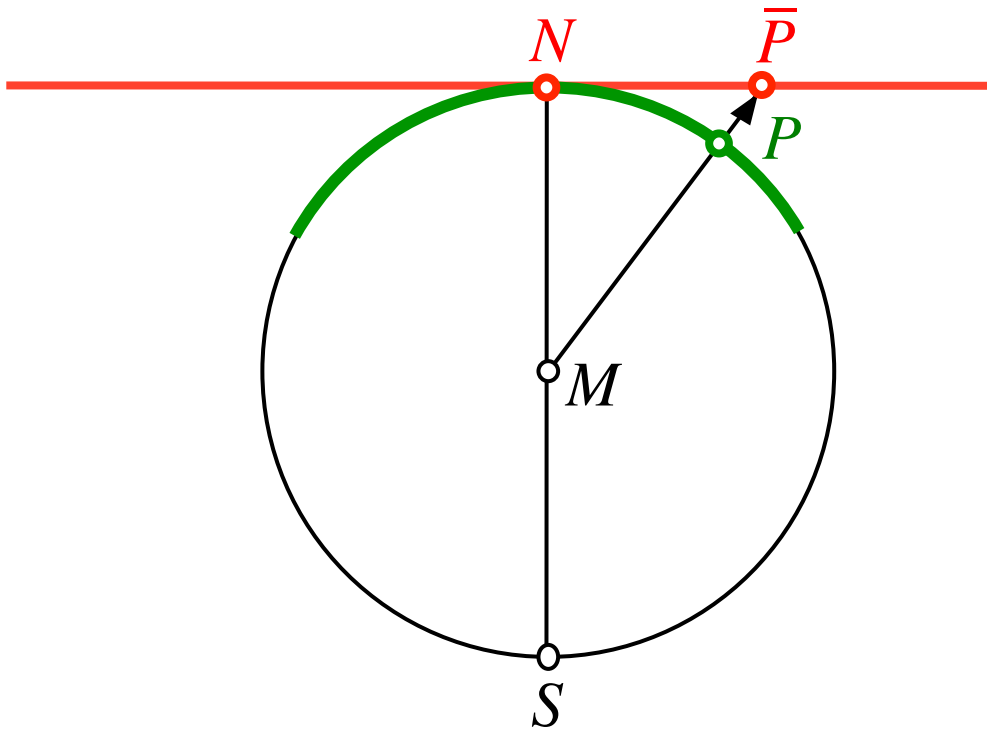


Großkreis als Gerade auf der Karte?

# Großkreis als Gerade auf der Karte?

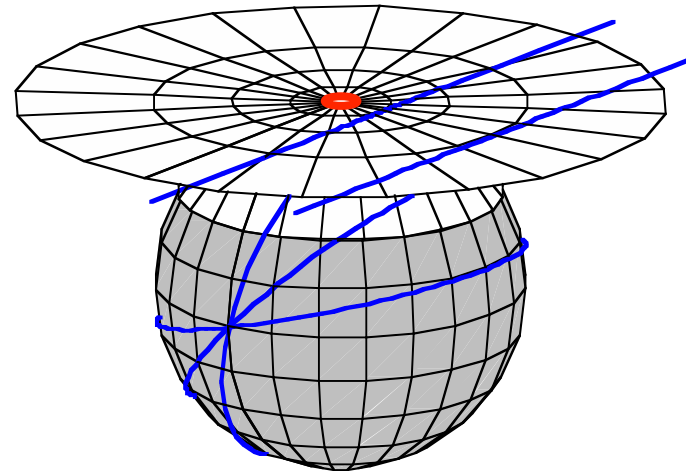
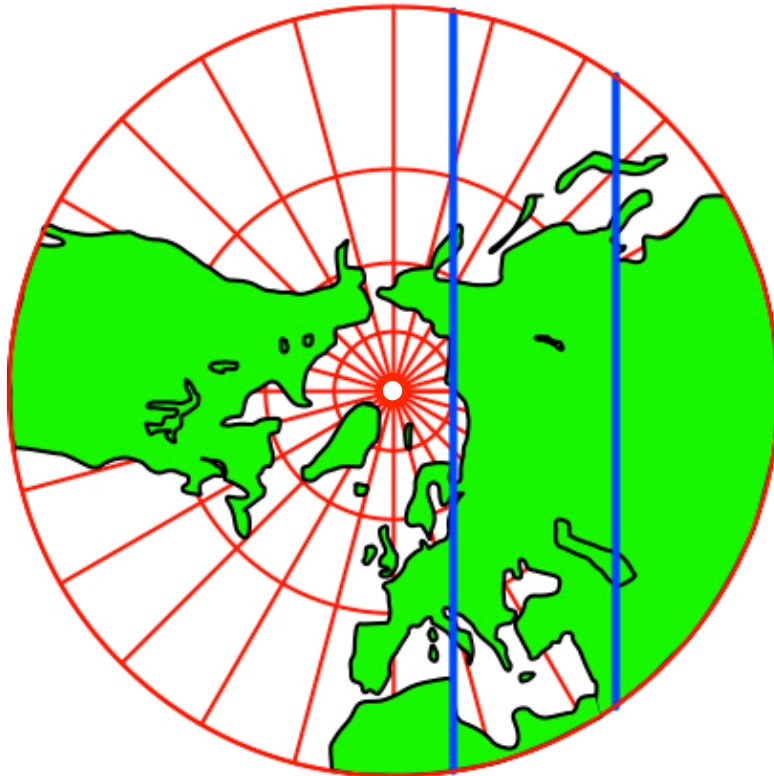
Gnomonische Projektion

Zentralprojektion von Kugelmitte aus



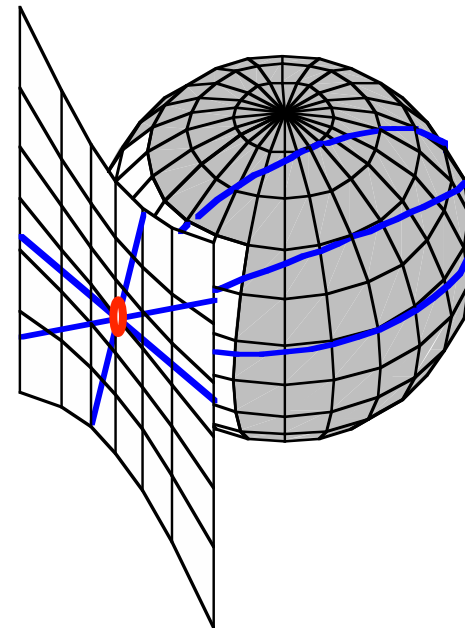
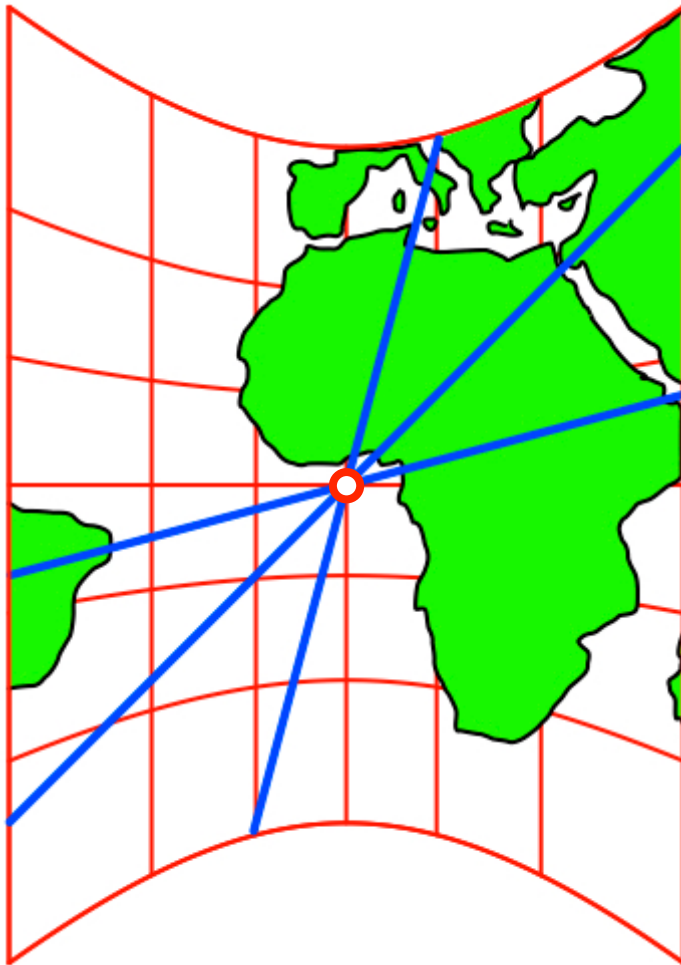
# Großkreis als Gerade auf der Karte?

## Gnomonische Projektion

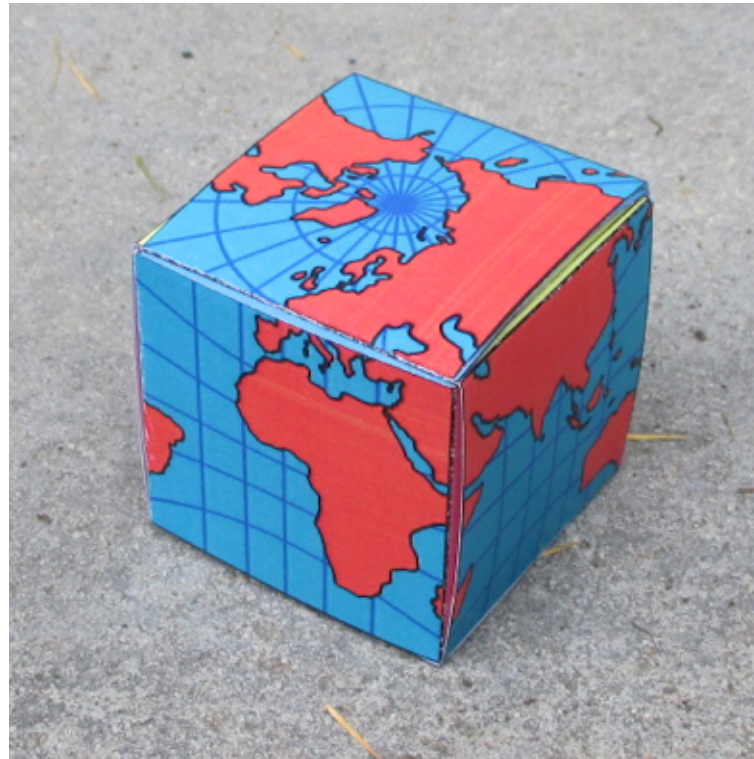


# Großkreis als Gerade auf der Karte?

## Gnomonische Projektion

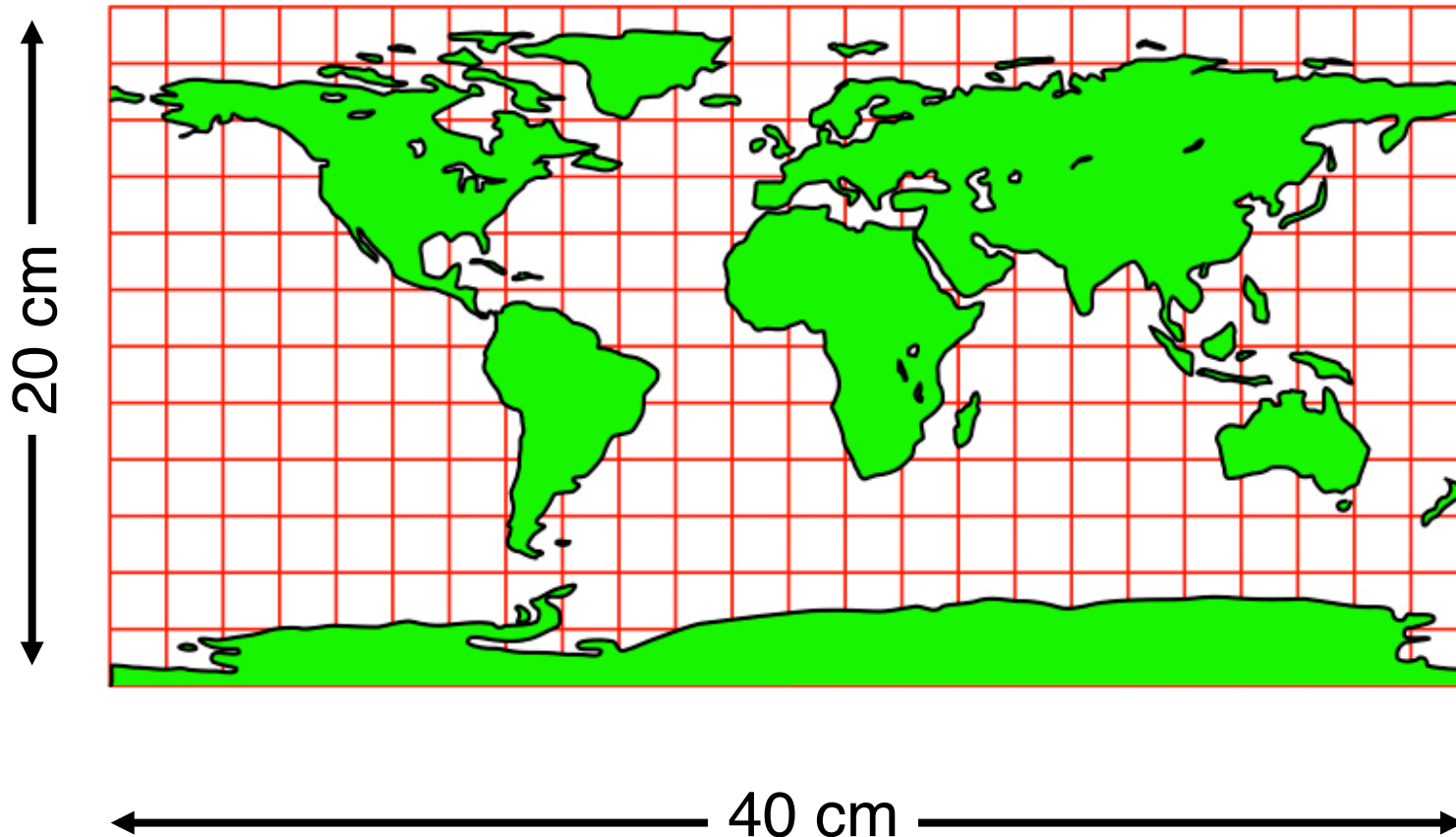


# Großkreis als Gerade auf der Karte? Gnomonische Projektion



Handout Würfelwelt

Maßstab eins zu eins?

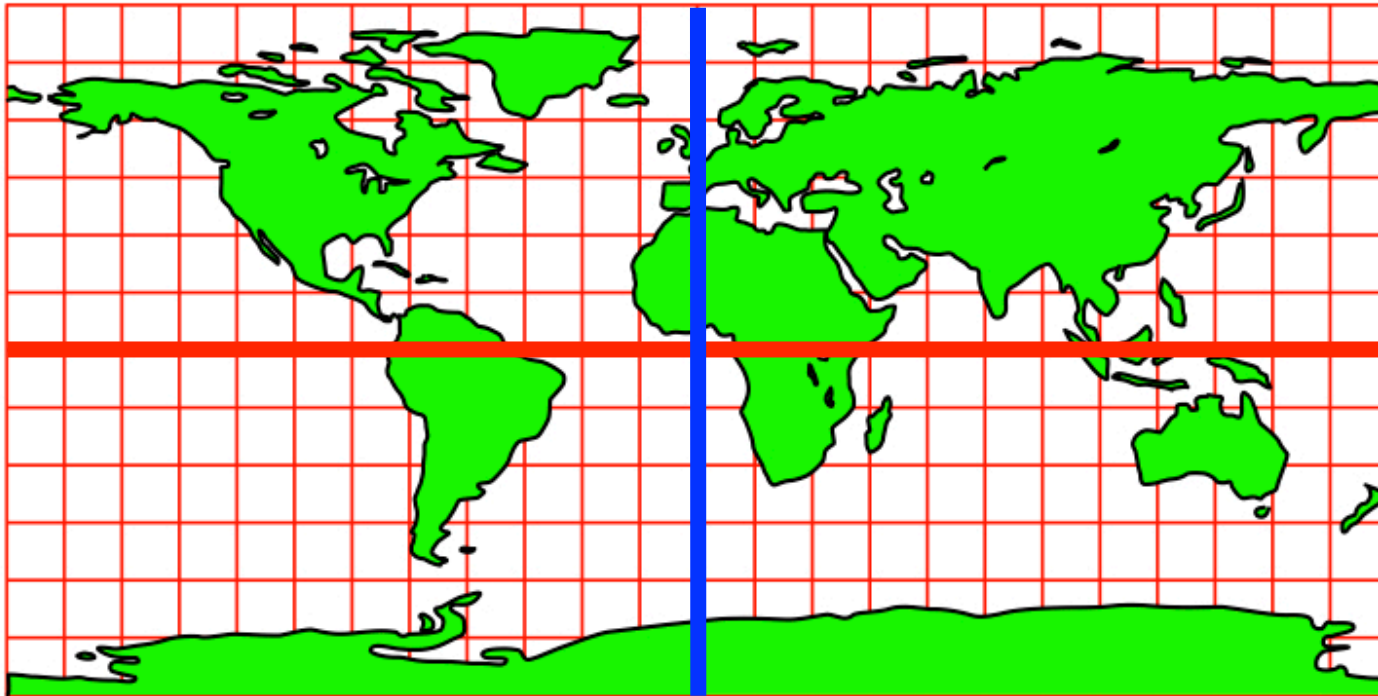




Maßstab eins zu eins?

$$\frac{1}{100\,000\,000}$$

20 cm

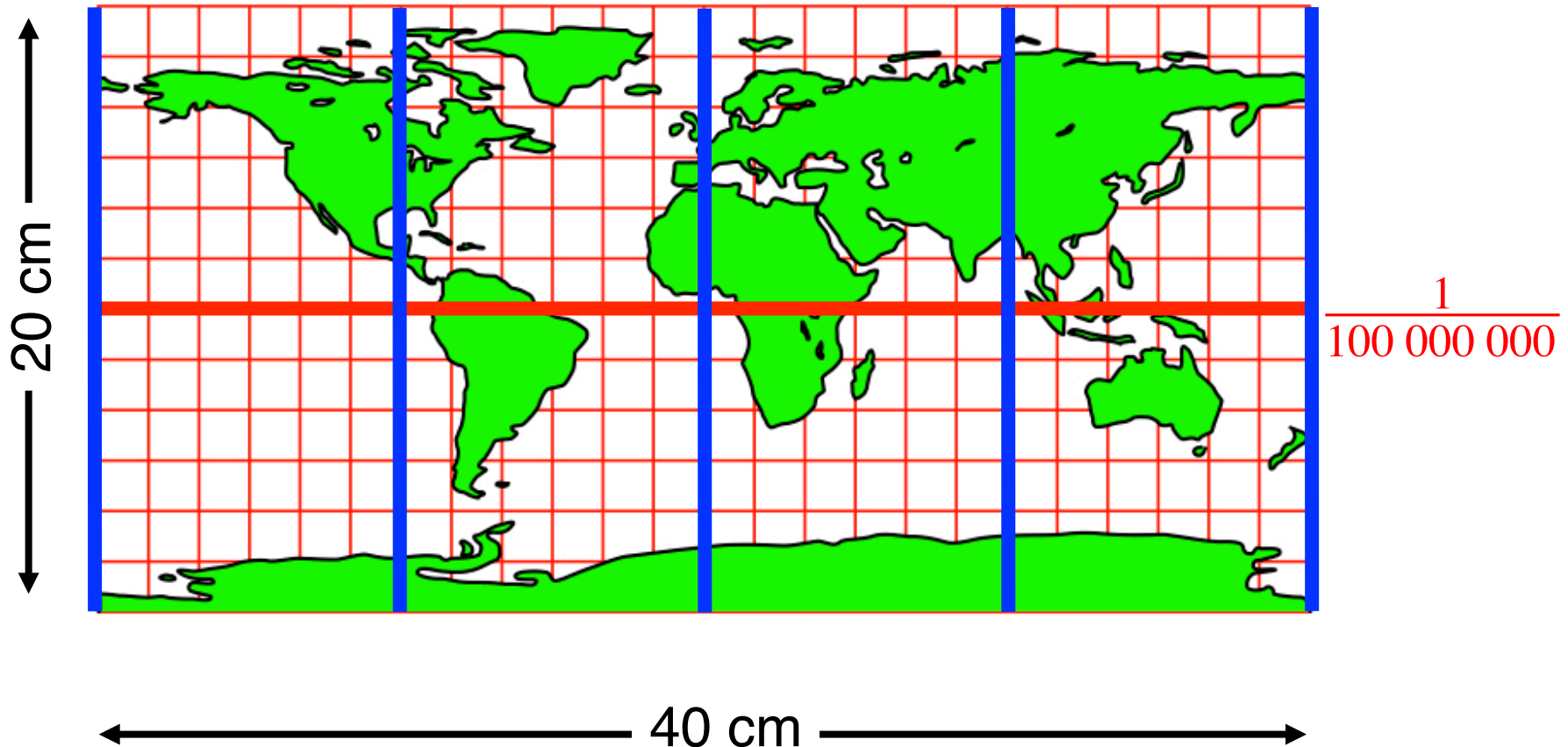


$$\frac{1}{100\,000\,000}$$

40 cm

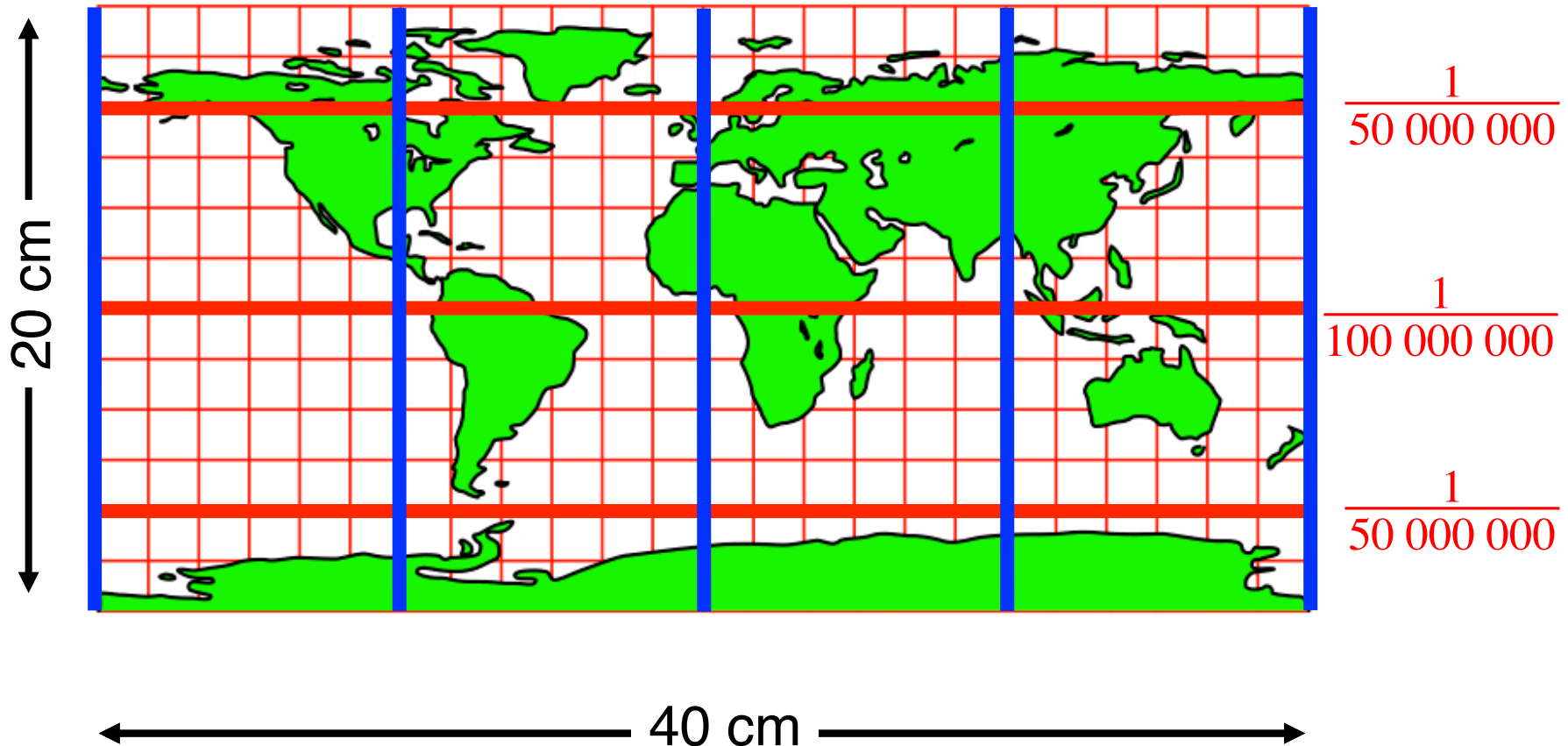
# Maßstab eins zu eins?

$\frac{1}{100\ 000\ 000}$     $\frac{1}{100\ 000\ 000}$     $\frac{1}{100\ 000\ 000}$     $\frac{1}{100\ 000\ 000}$     $\frac{1}{100\ 000\ 000}$



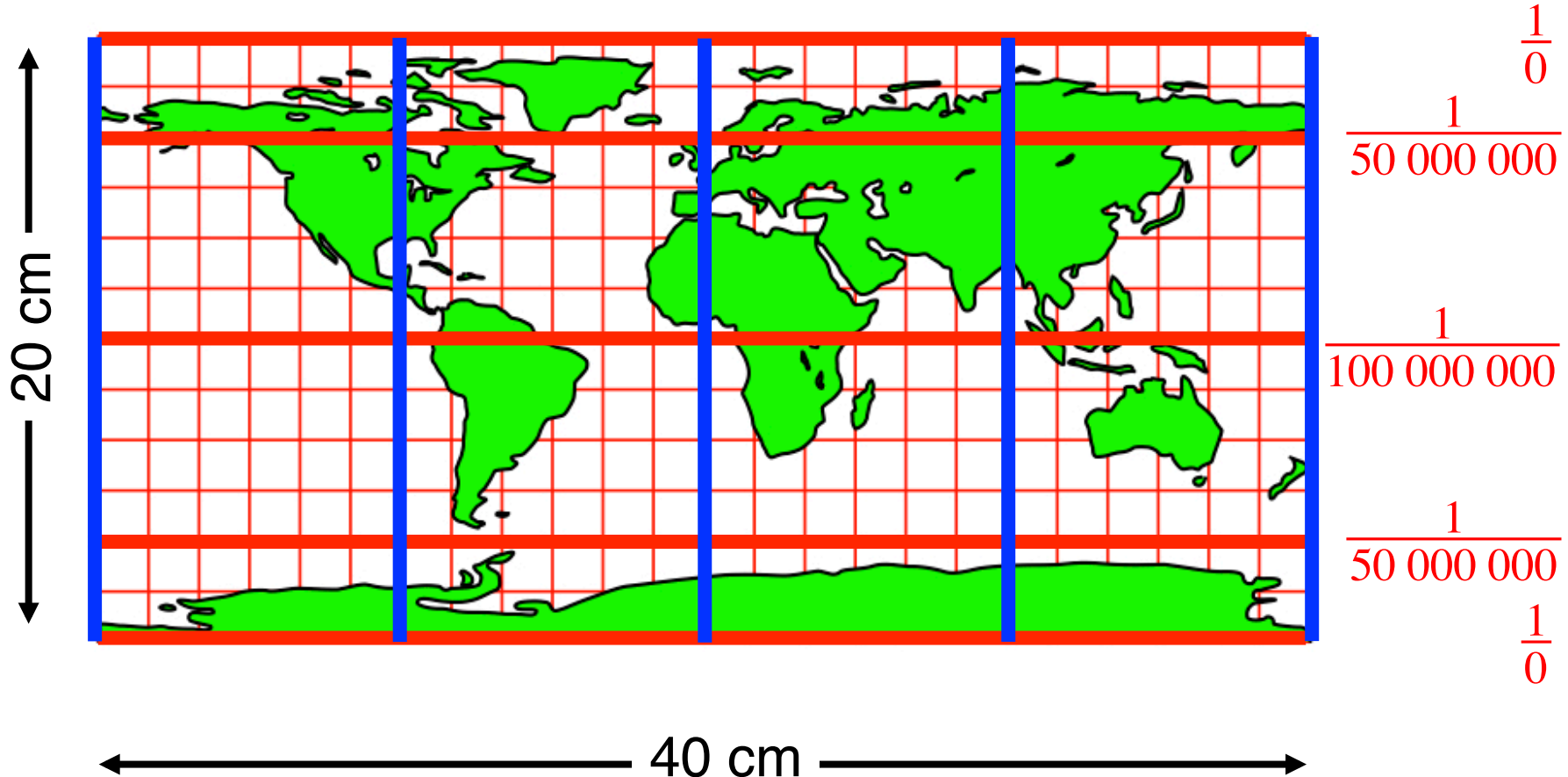
# Maßstab eins zu eins?

$\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$



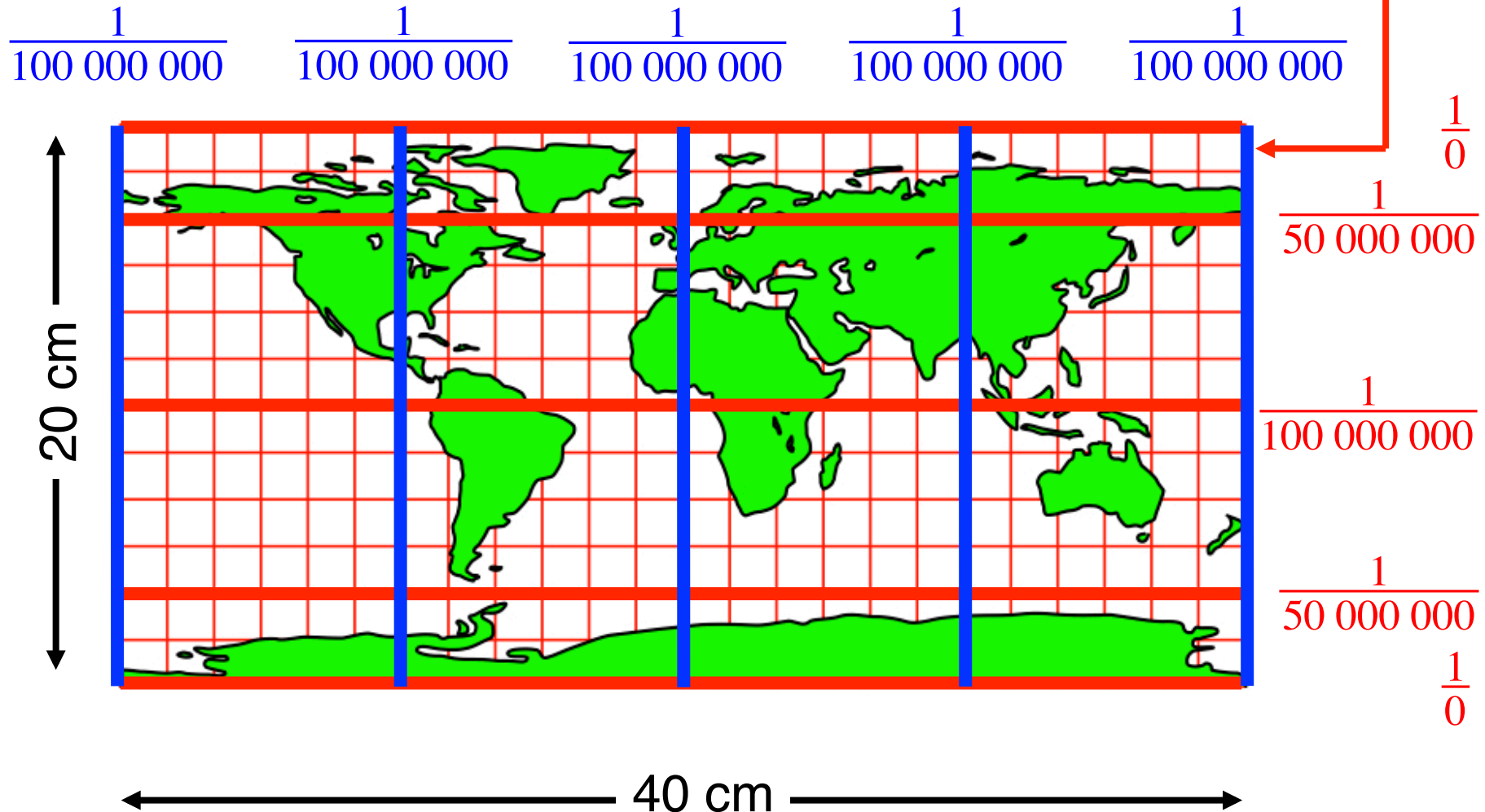
# Maßstab eins zu eins?

$\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$      $\frac{1}{100\ 000\ 000}$



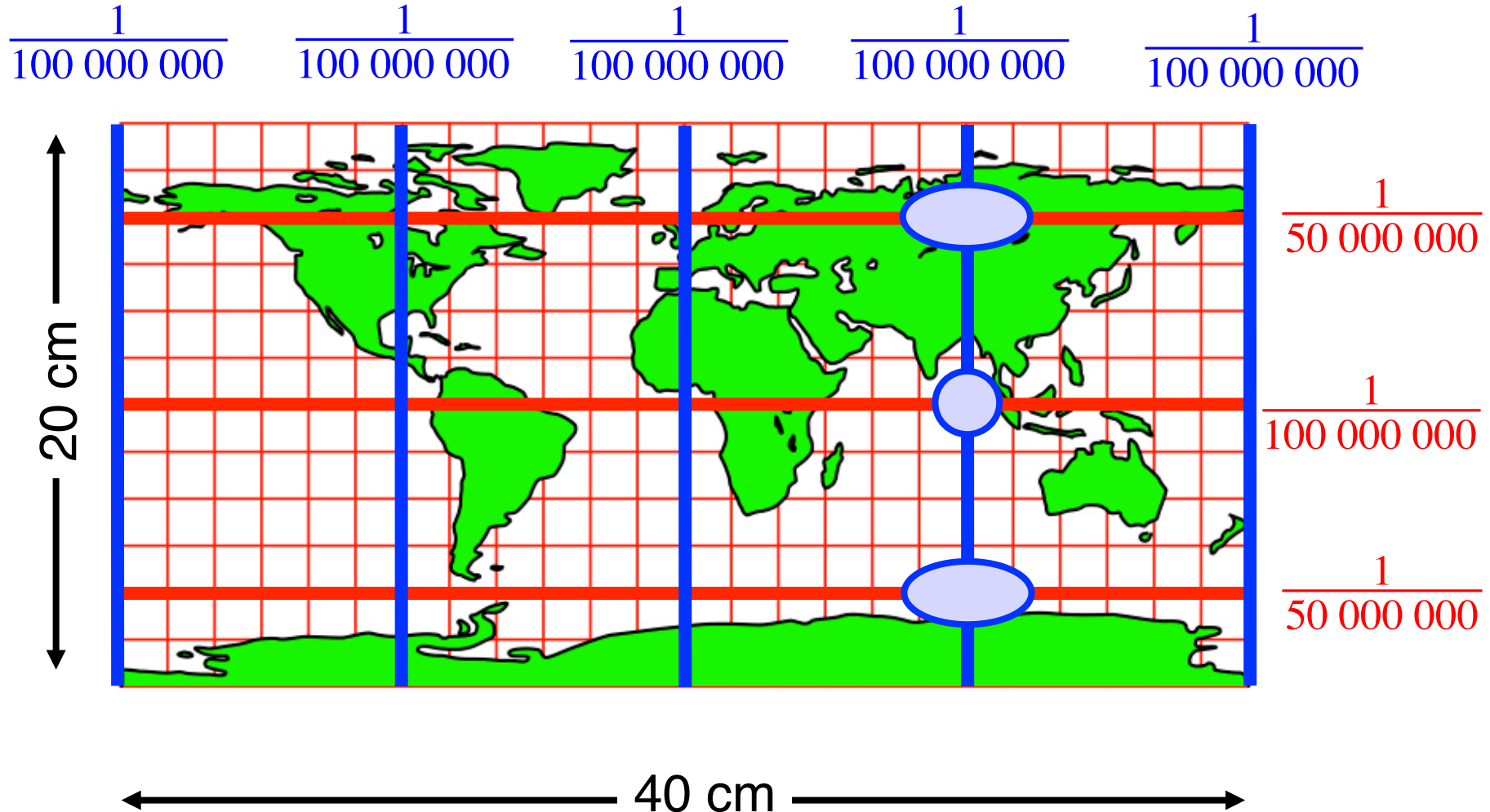
Maßstab eins zu eins?

6.37 cm vom Nordpol entfernt

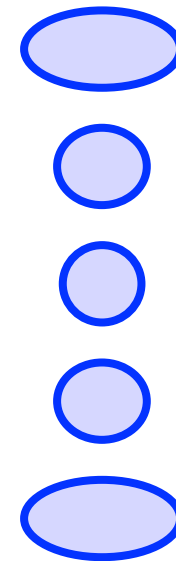
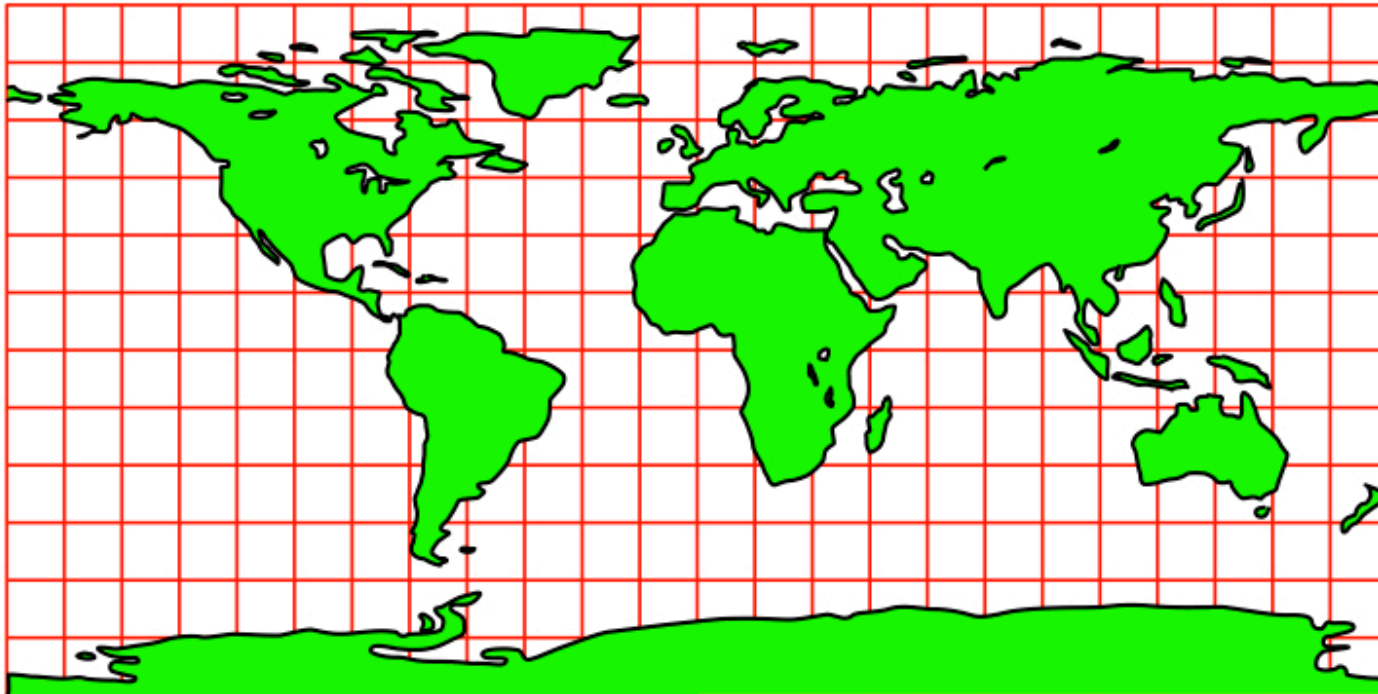


$$\phi = \pm 89.9999994^\circ = \pm 89^\circ 59' 59.998''$$

# Verzerrungsellipse, Tissotsche Indikatrix



Verzerrungsellipse, Tissotsche Indikatrix  
Bild des Swimming Pools





Carl Friedrich Gauß  
1777 - 1855

verzerrungsfreie,  
maßstäbliche

Theorema egregium:

Es gibt **keine isometrische** Abbildung  
von der Kugel auf die Ebene.





Carl Friedrich Gauß  
1777 - 1855

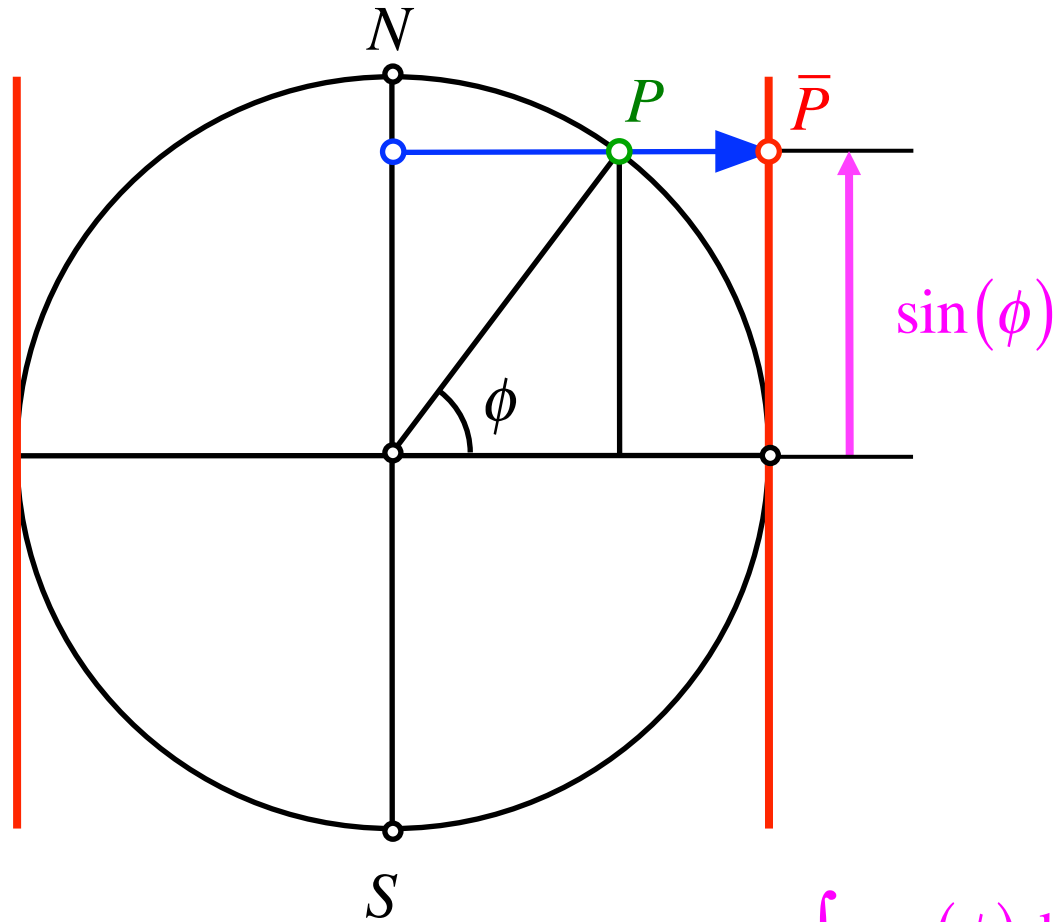
Theorema egregium:

Es gibt **keine** isometrische Abbildung von der Kugel auf die Ebene.

Hingegen gibt es:

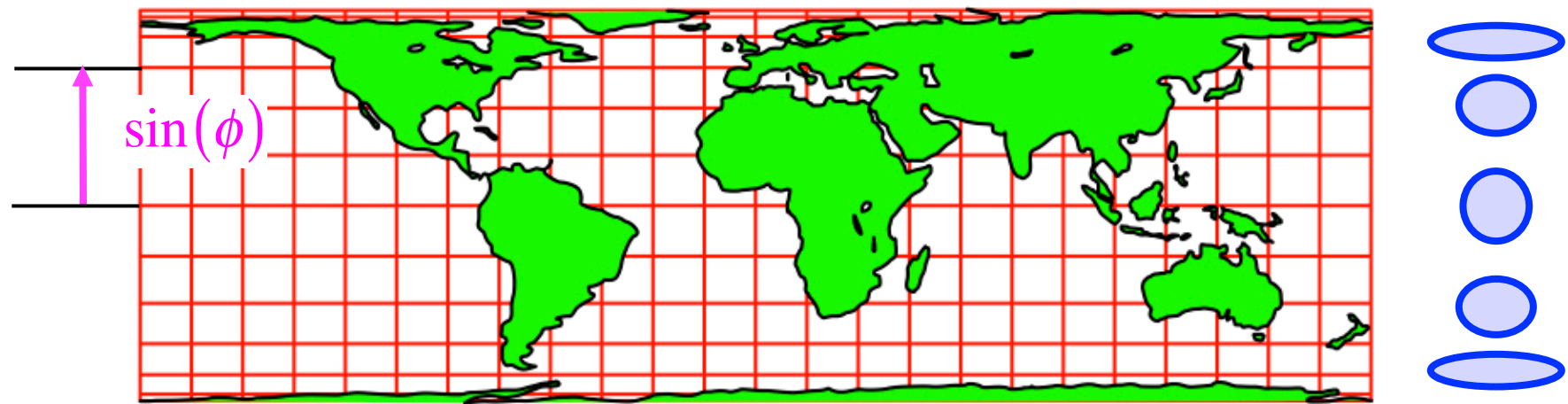
- flächentreue Karten (equivalent)
- winkeltreue Karten (conformal)

# Flächentreu (equivalent), Archimedes / Lambert



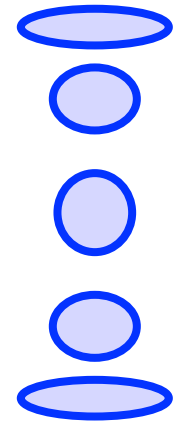
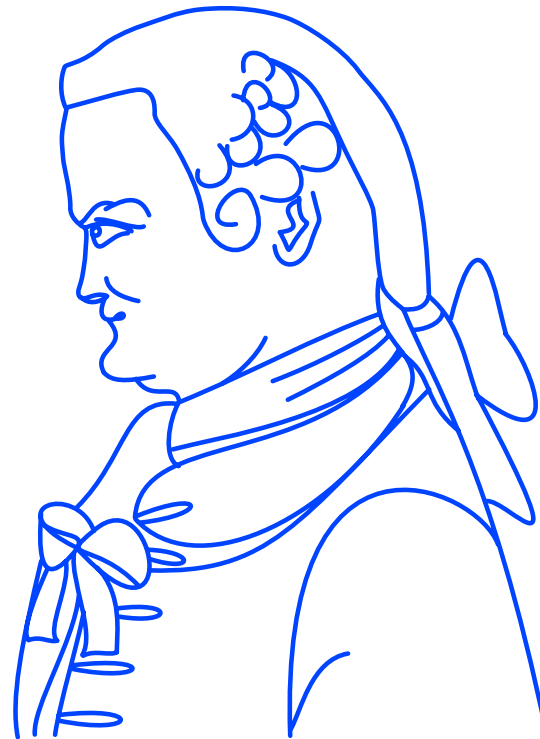
$$\int \cos(\phi) d\phi = \sin(\phi)$$

Flächentreu (equivalent), Archimedes / Lambert  
Bild des Swimming Pools



$$\int \cos(\phi) d\phi = \sin(\phi)$$

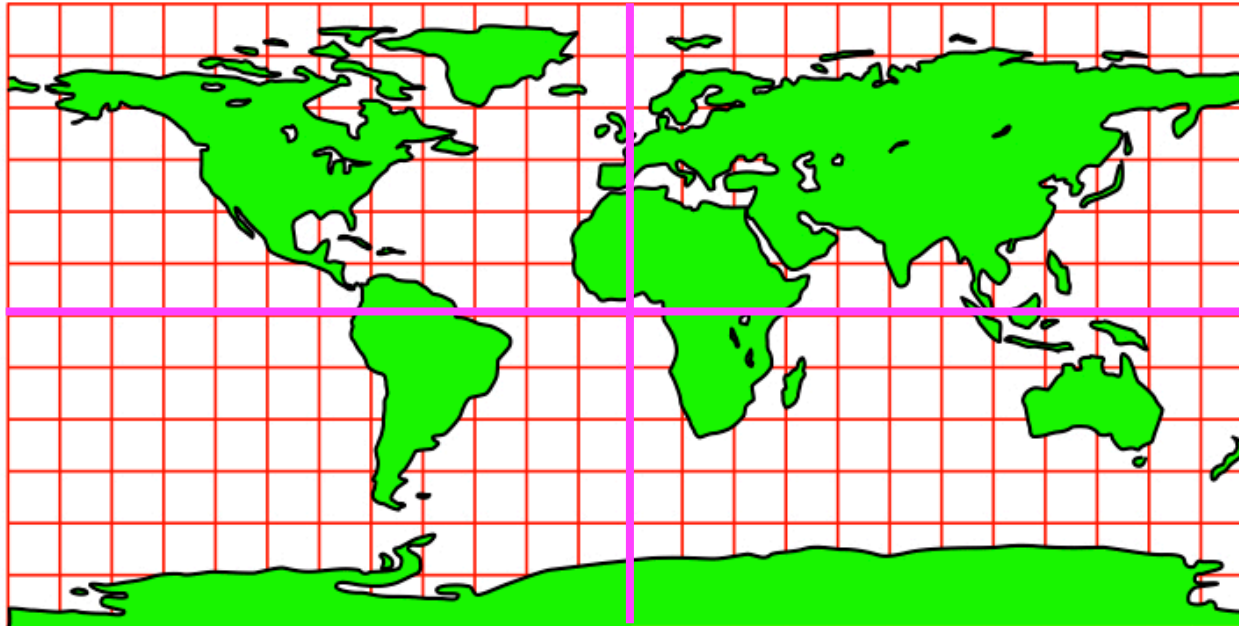
# Flächentreu (equivalent), Archimedes / Lambert



Johann Heinrich Lambert  
1728-1777

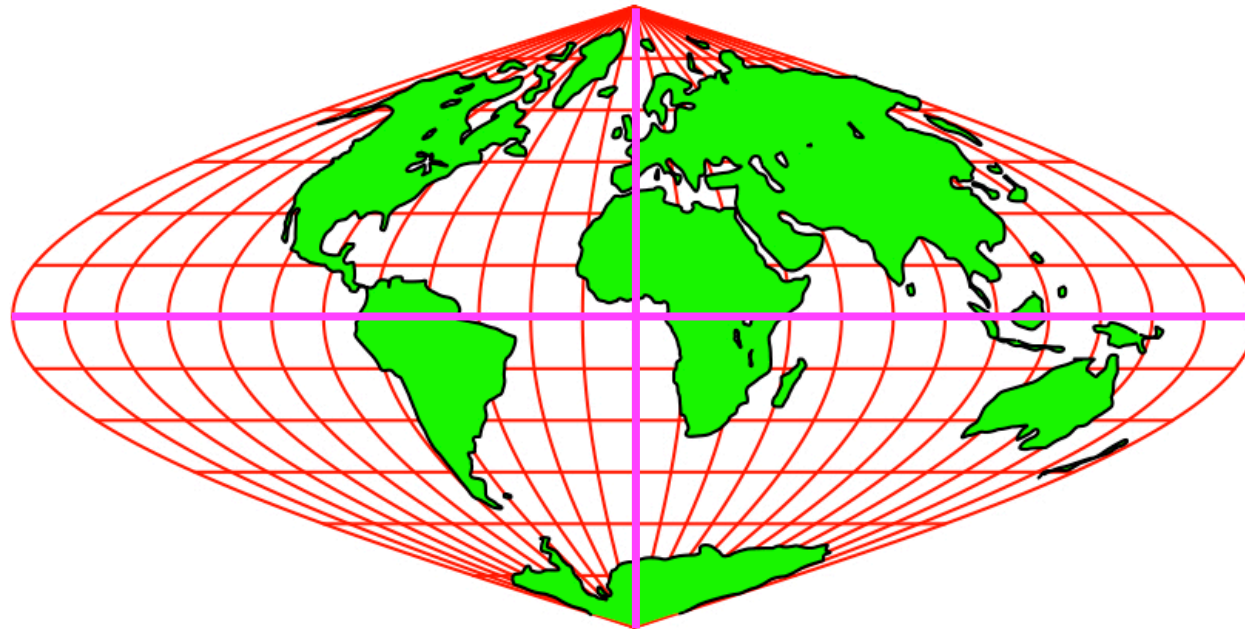
Flächentreu (equivalent), Mercator / Sanson

# Flächentreu (equivalent), Mercator / Sanson



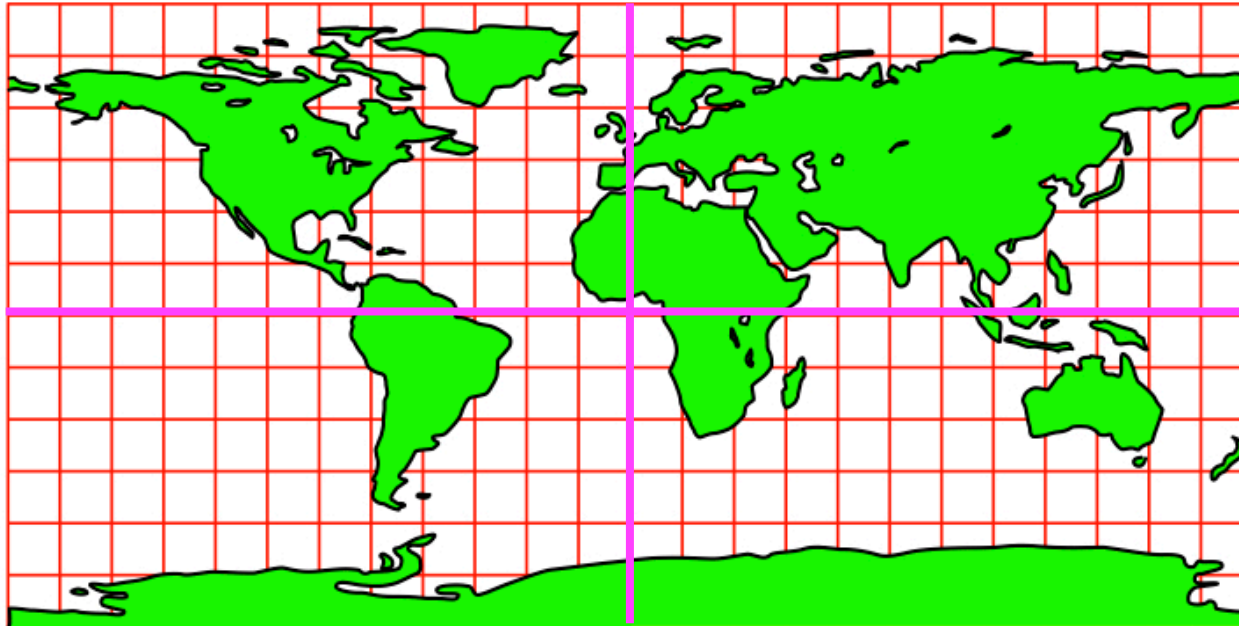
Idee: an den Polen zu Punkt einbrutzeln

# Flächentreu (equivalent), Mercator / Sanson



Idee: an den Polen zu Punkt einbrutzeln

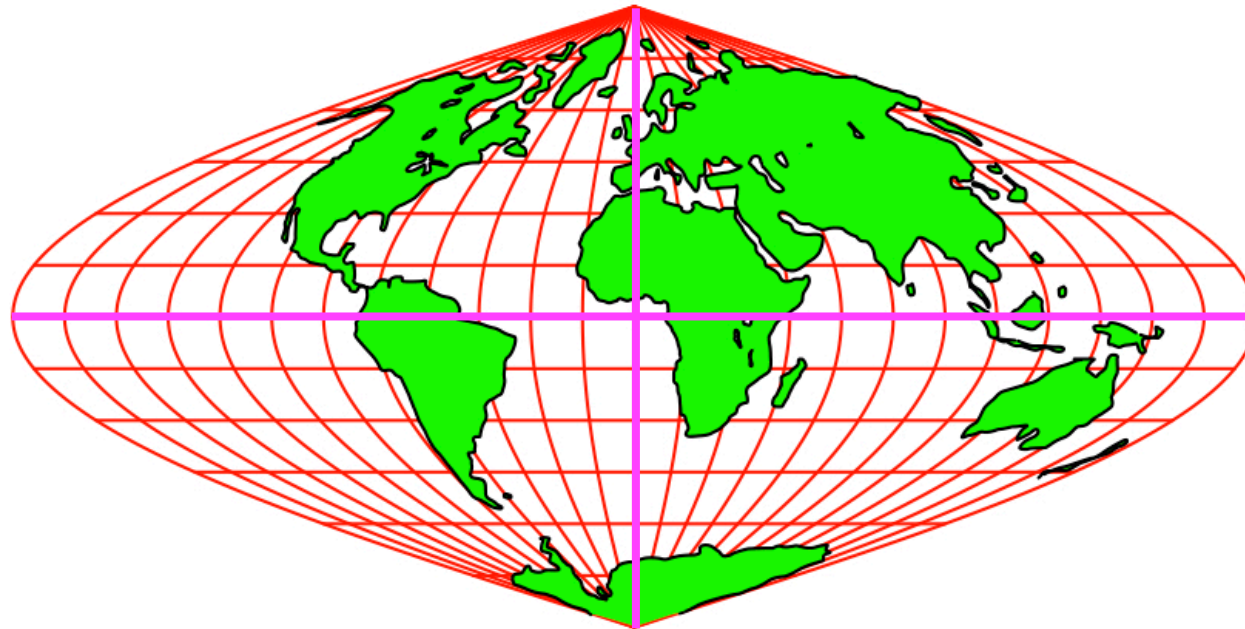
# Flächentreu (equivalent), Mercator / Sanson



Idee: an den Polen zu Punkt einbrutzeln

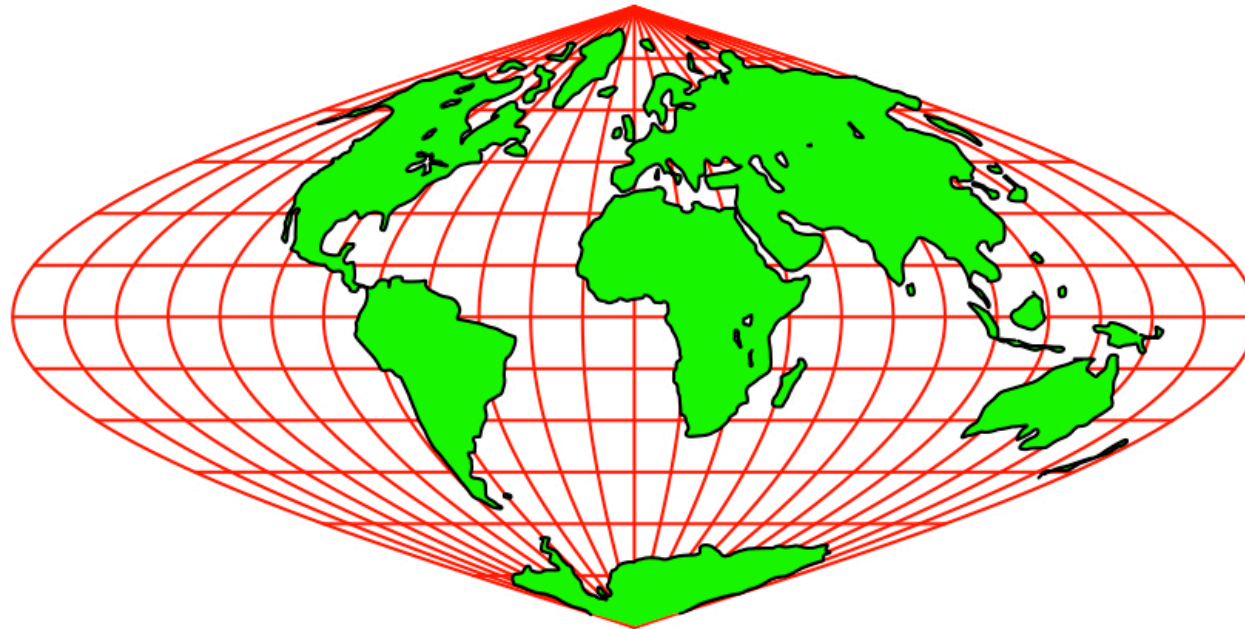


# Flächentreu (equivalent), Mercator / Sanson



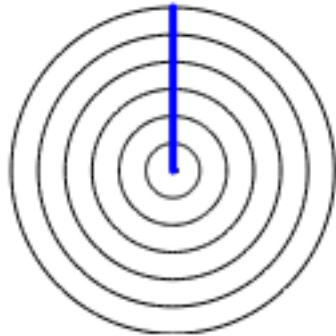
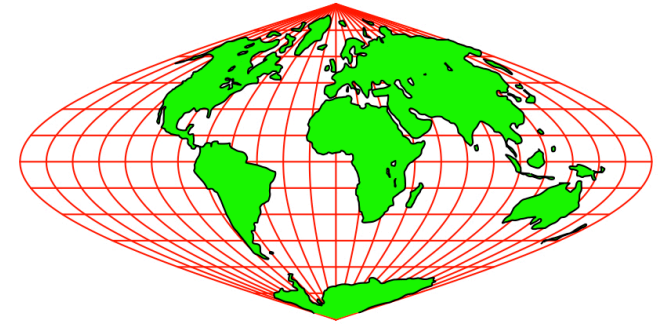
Idee: an den Polen zu Punkt einbrutzeln

# Flächentreu (equivalent), Mercator / Sanson



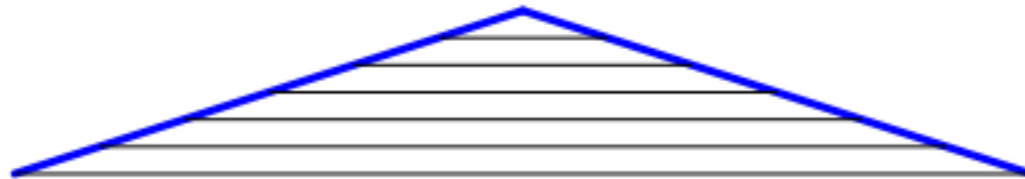
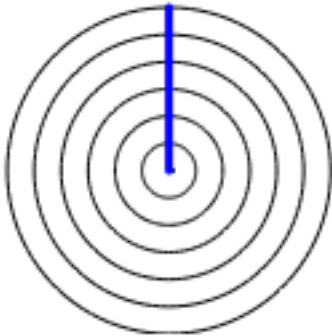
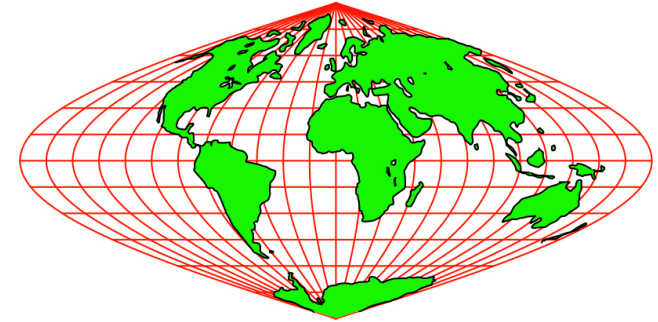
Erinnerung an die Schule

Kreis und Dreieck



Erinnerung an die Schule

Kreis und Dreieck



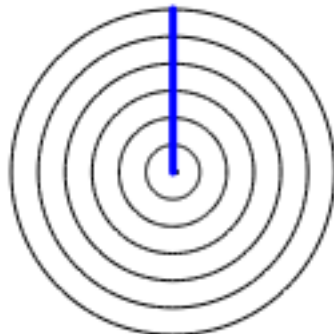
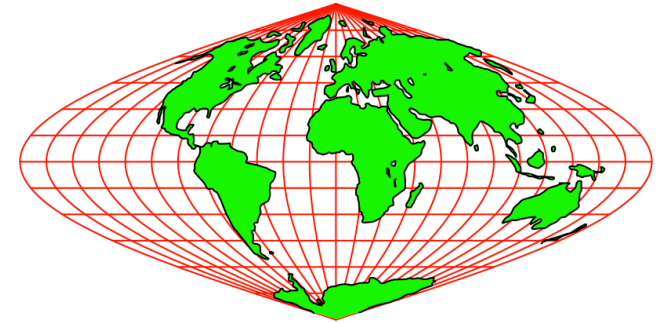
$$\text{Grundlinie} = 2r\pi$$

$$\text{Höhe} = r$$

$$\text{Flächeninhalt} = \frac{2r\pi \cdot r}{2} = r^2 \pi$$

Erinnerung an die Schule

Kreis und Dreieck



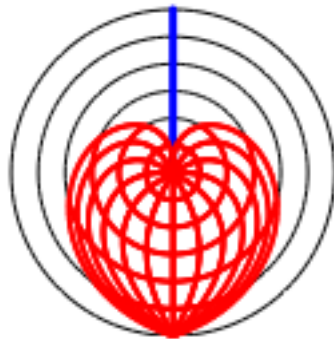
$$\text{Grundlinie} = 2r\pi$$

$$\text{Höhe} = r$$

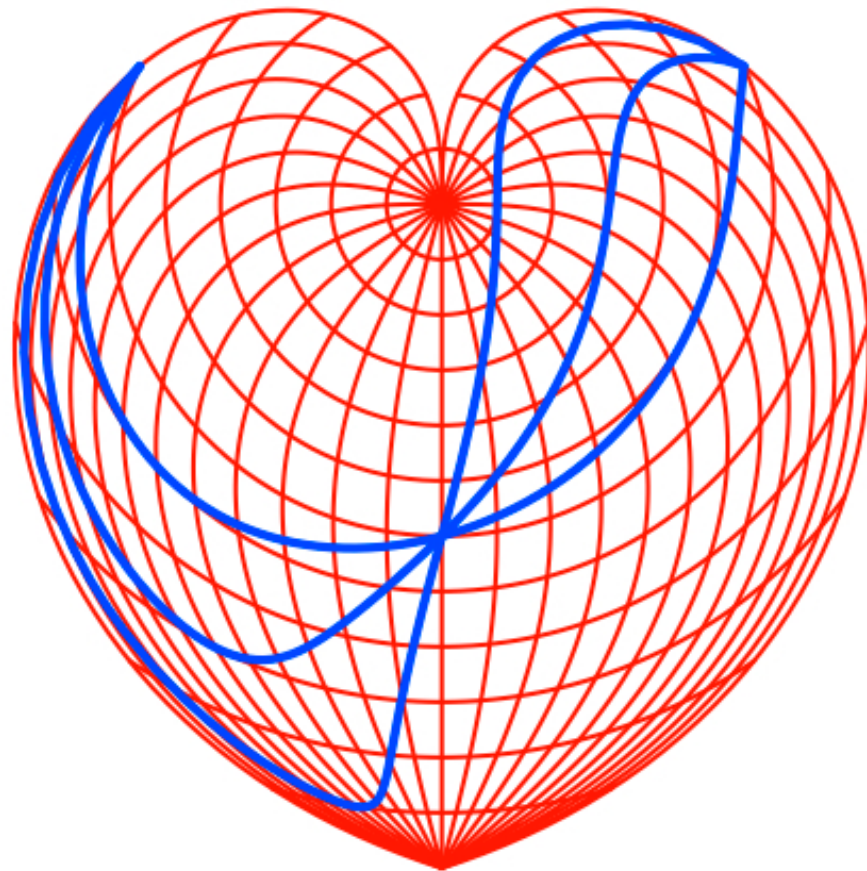
$$\text{Flächeninhalt} = \frac{2r\pi \cdot r}{2} = r^2 \pi$$

Erinnerung an die Schule

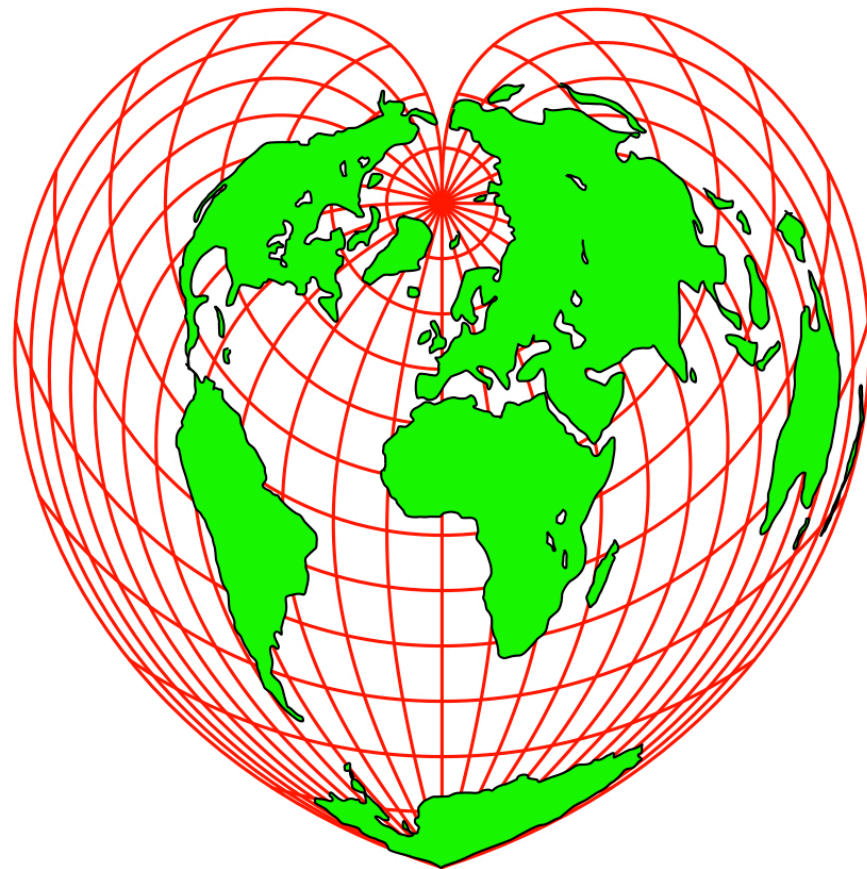
Kreis und Dreieck



Flächentreu (equivalent), Herzkarte von Stab / Werner 1514

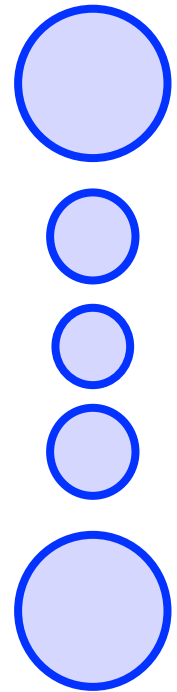


Flächentreu (equivalent), Herzkarte von Stab / Werner 1514





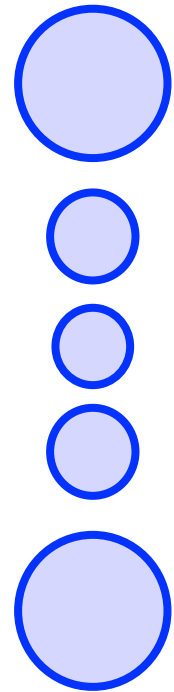
Winkeltreu (conformal), Mercator, 1569



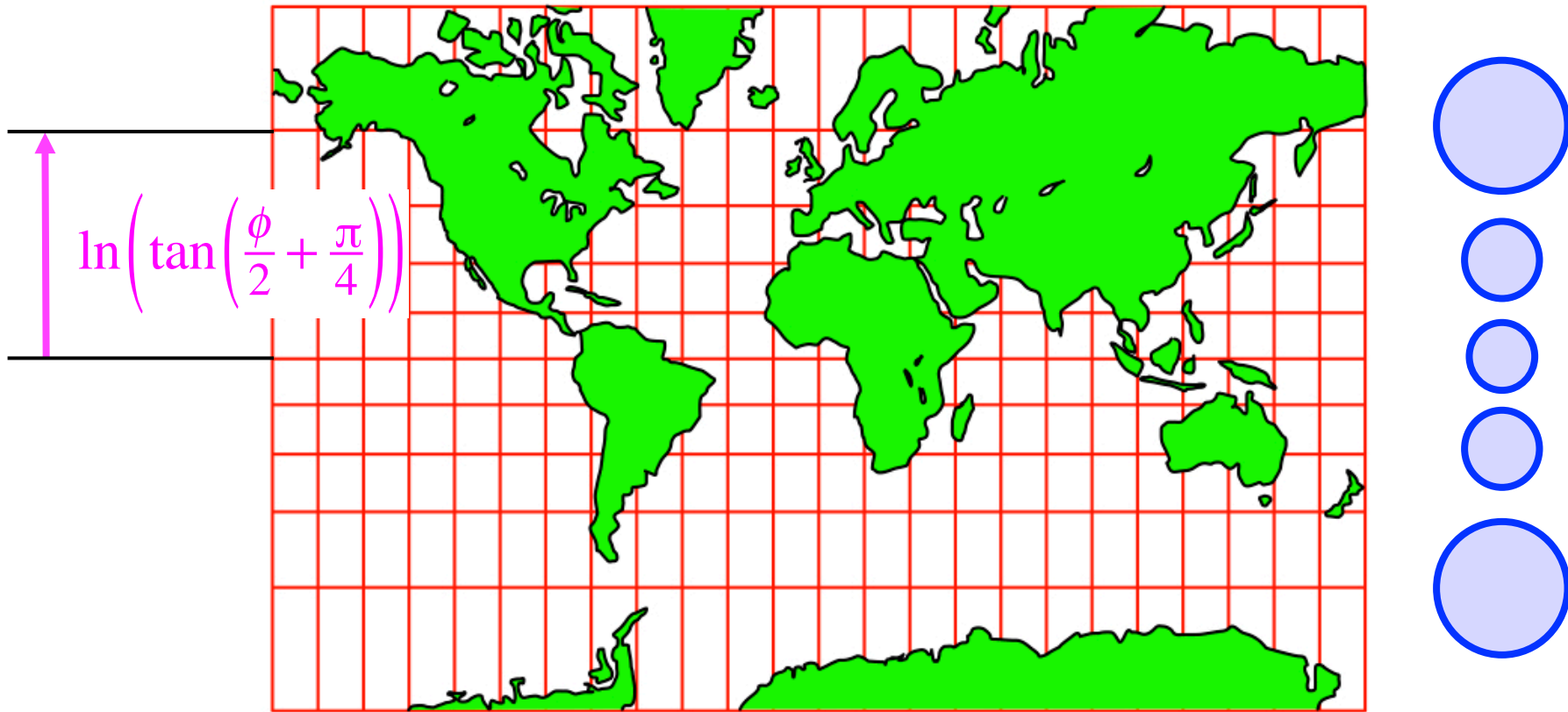
Winkeltreu (conformal), Mercator, 1569



Gerhard Mercator  
1512 - 1594



# Winkeltreu (conformal), Mercator, 1569

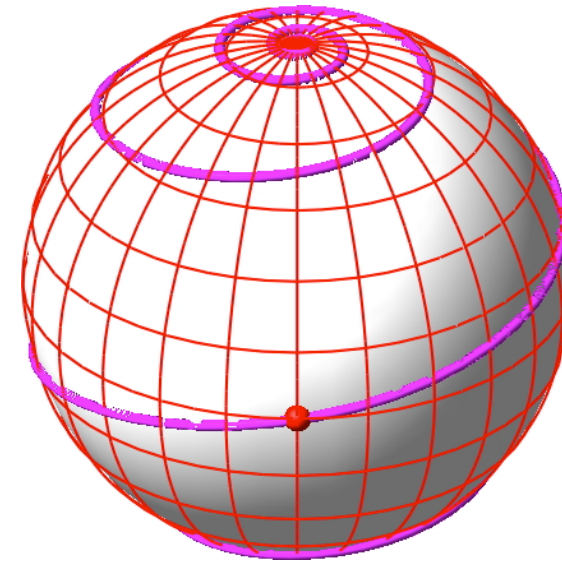
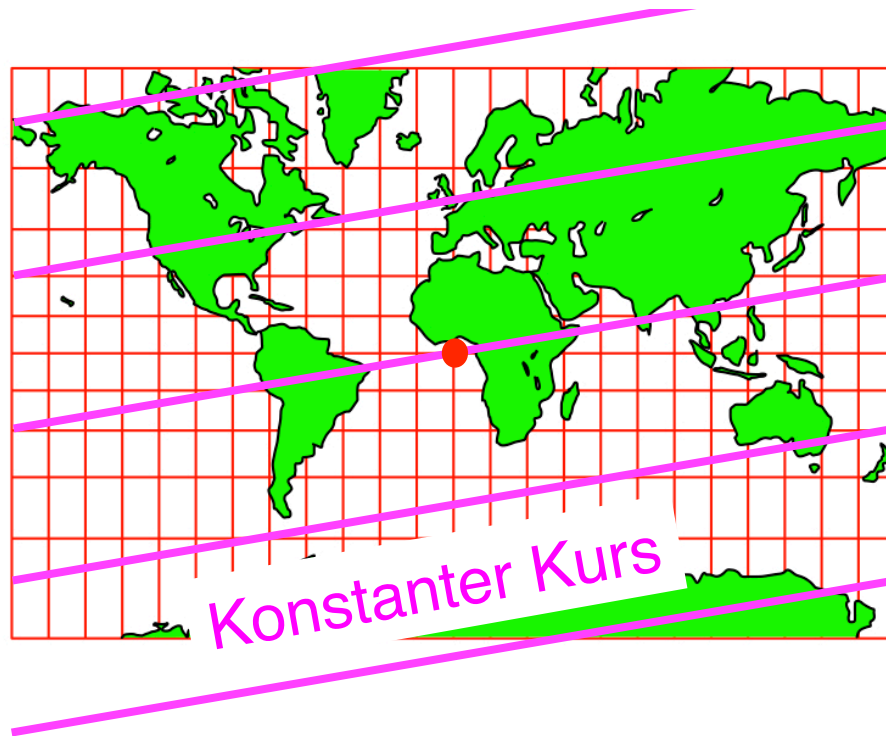


$$\int \frac{1}{\cos(\phi)} d\phi = \ln\left(\tan\left(\frac{\phi}{2} + \frac{\pi}{4}\right)\right)$$

Winkeltreu (conformal), Mercator, 1569

Loxodrome:

Kurve mit konstantem Winkel  $\alpha$  gegenüber Meridianen

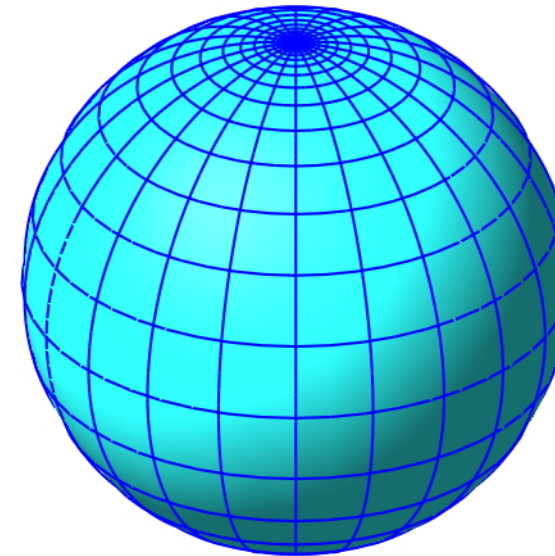
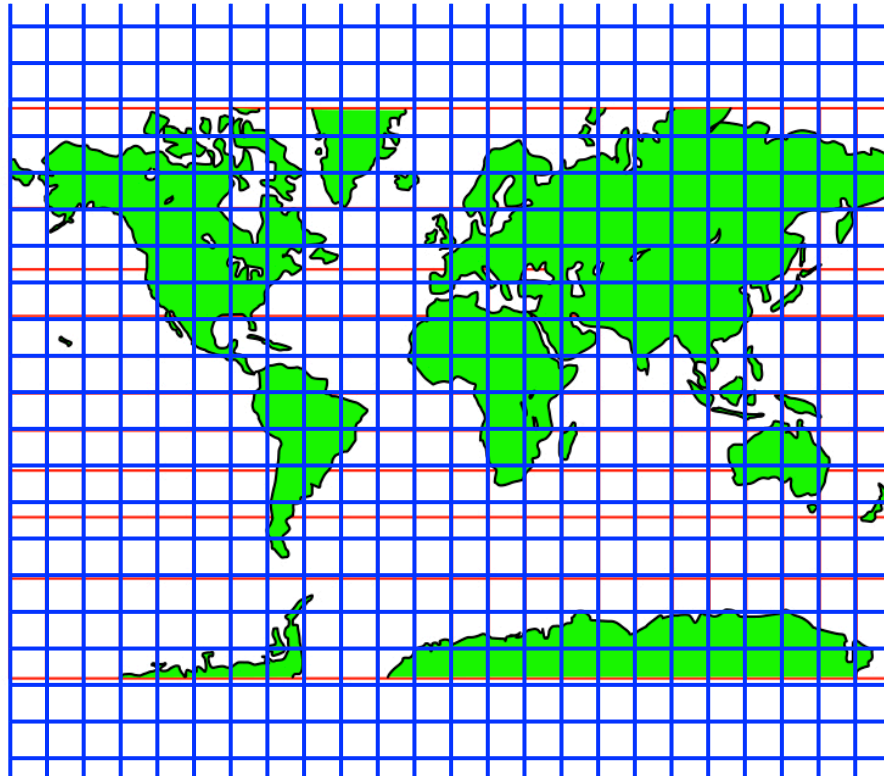


$$\alpha = 80^\circ$$



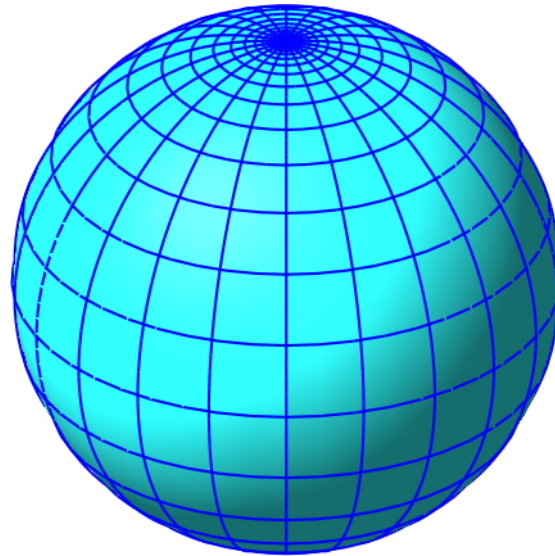
Winkeltreu (conformal), Mercator, 1569

Schöne Kugel: Quadratraster Seekarte —> Kugel

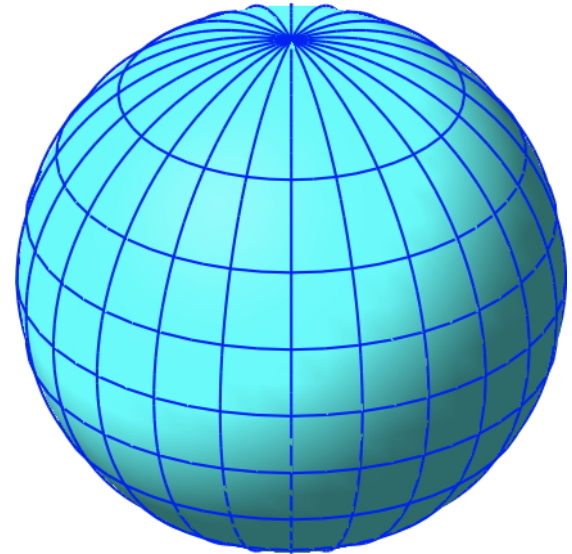
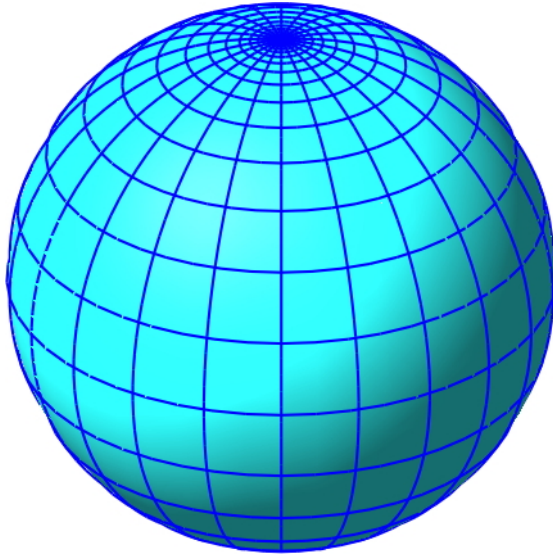
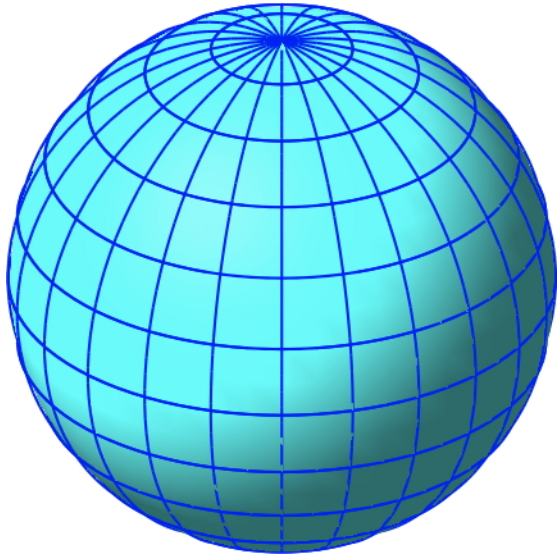


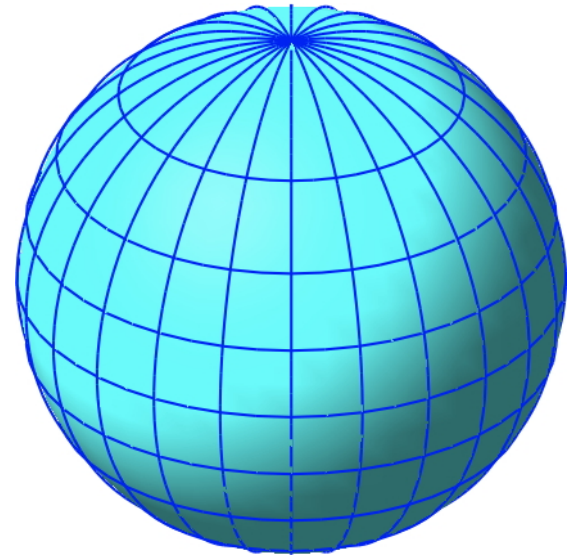
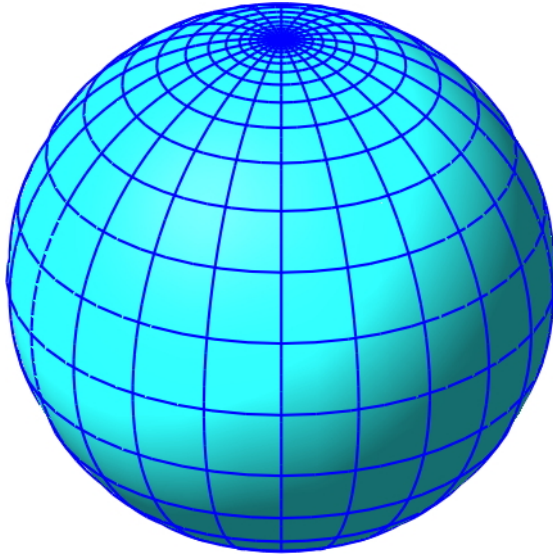
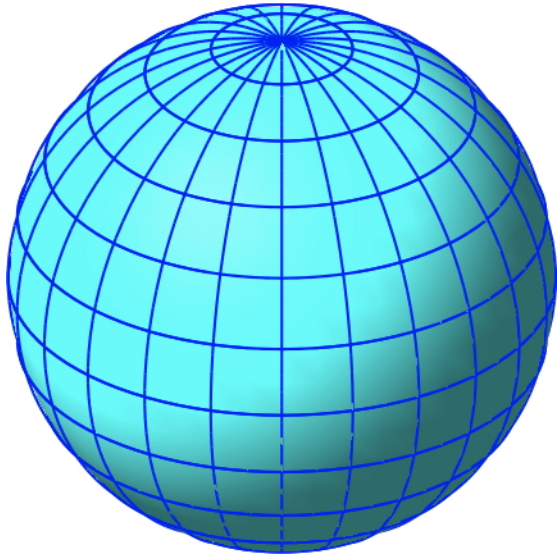
Winkeltreu (conformal), Mercator, 1569

Schöne Kugel: Quadratraster Seekarte → Kugel



Welches ist die schönste Kugel?





Danke