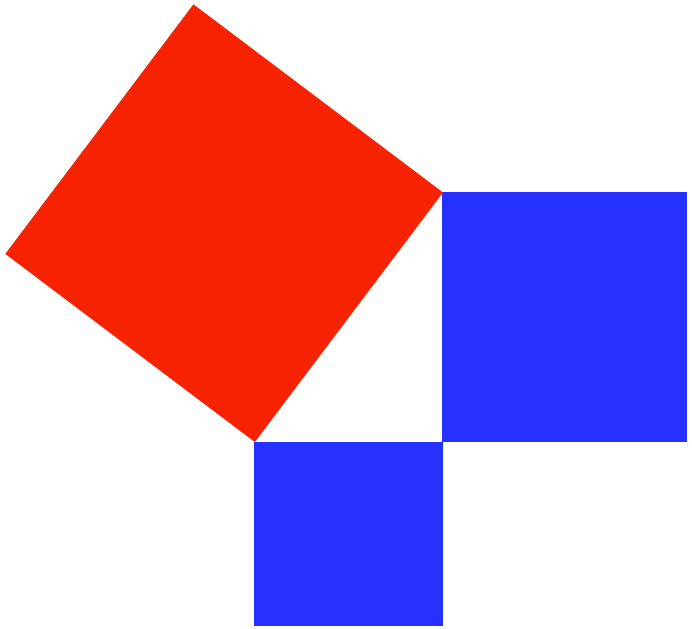
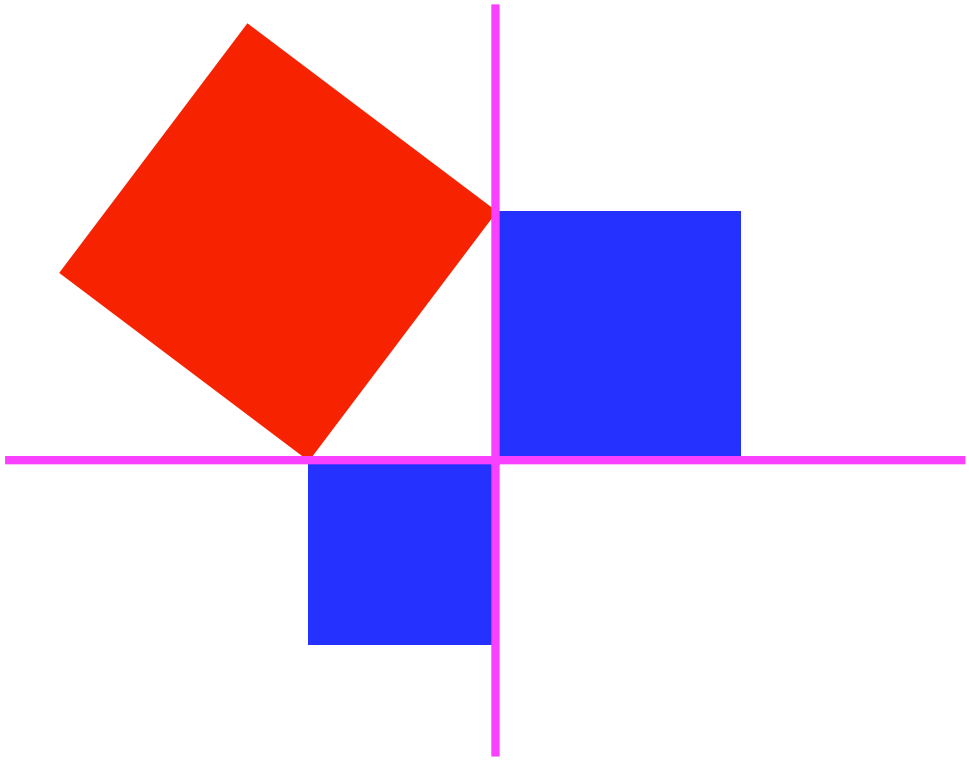
Erinnerung



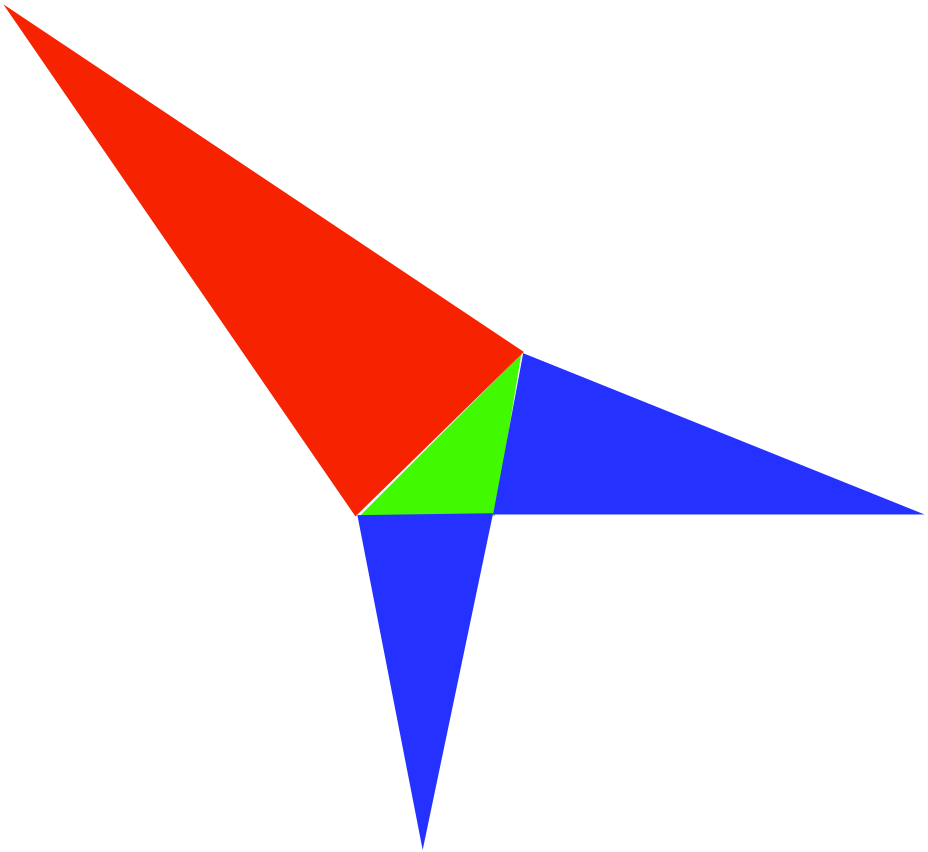
blau + blau = rot

Erinnerung



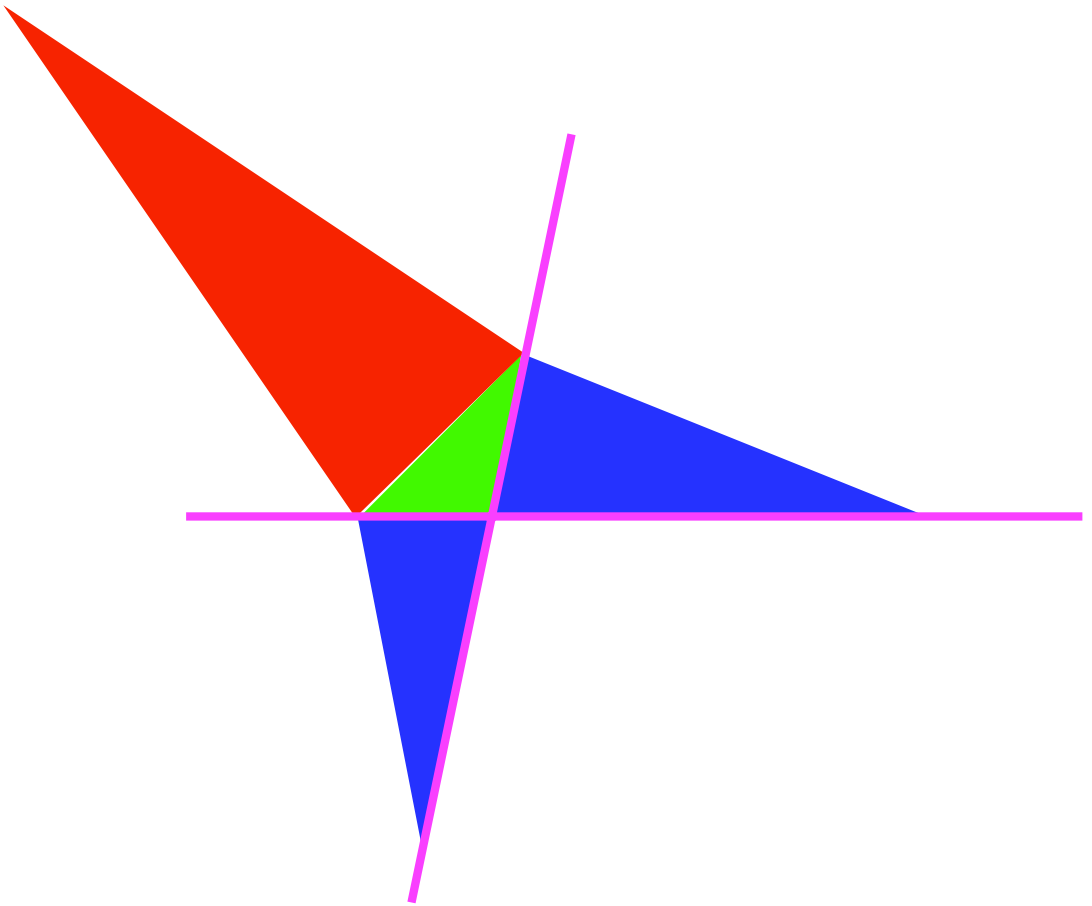
blau + blau = rot

Neu



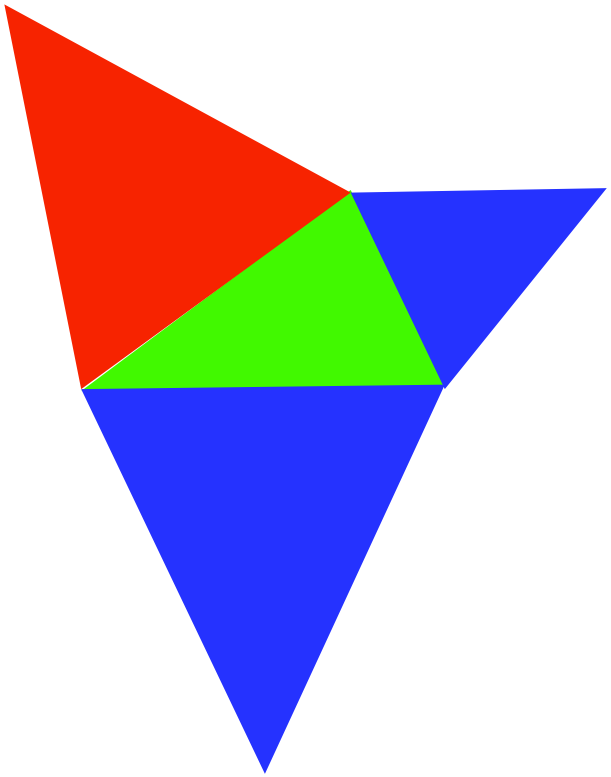
blau + blau + grün = rot

Neu



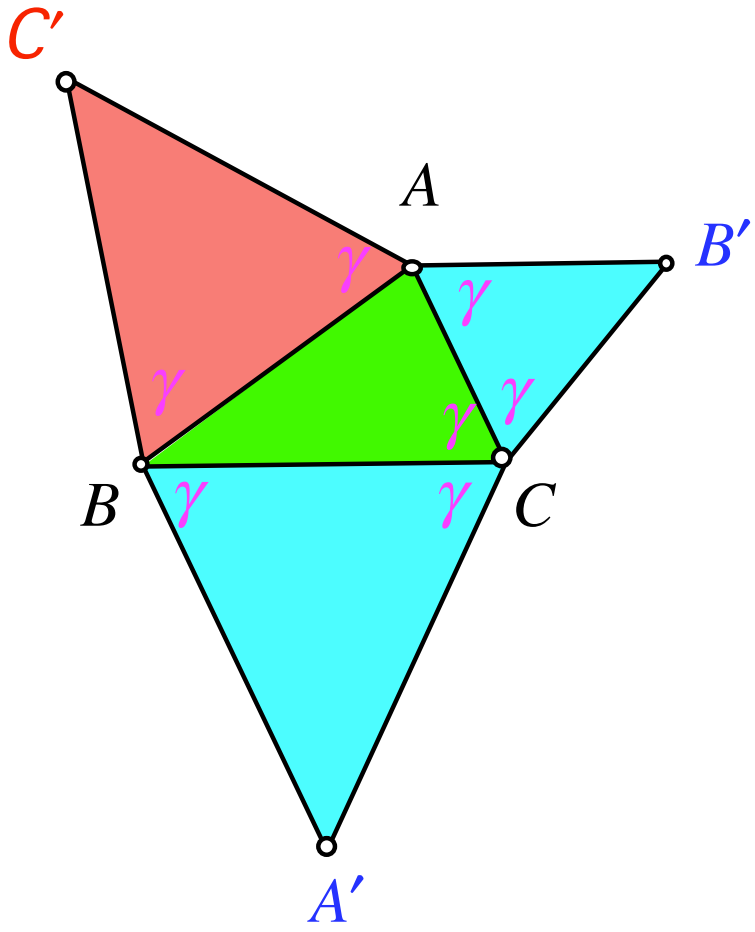
blau + blau + grün = rot

Neu



blau + blau = grün + rot

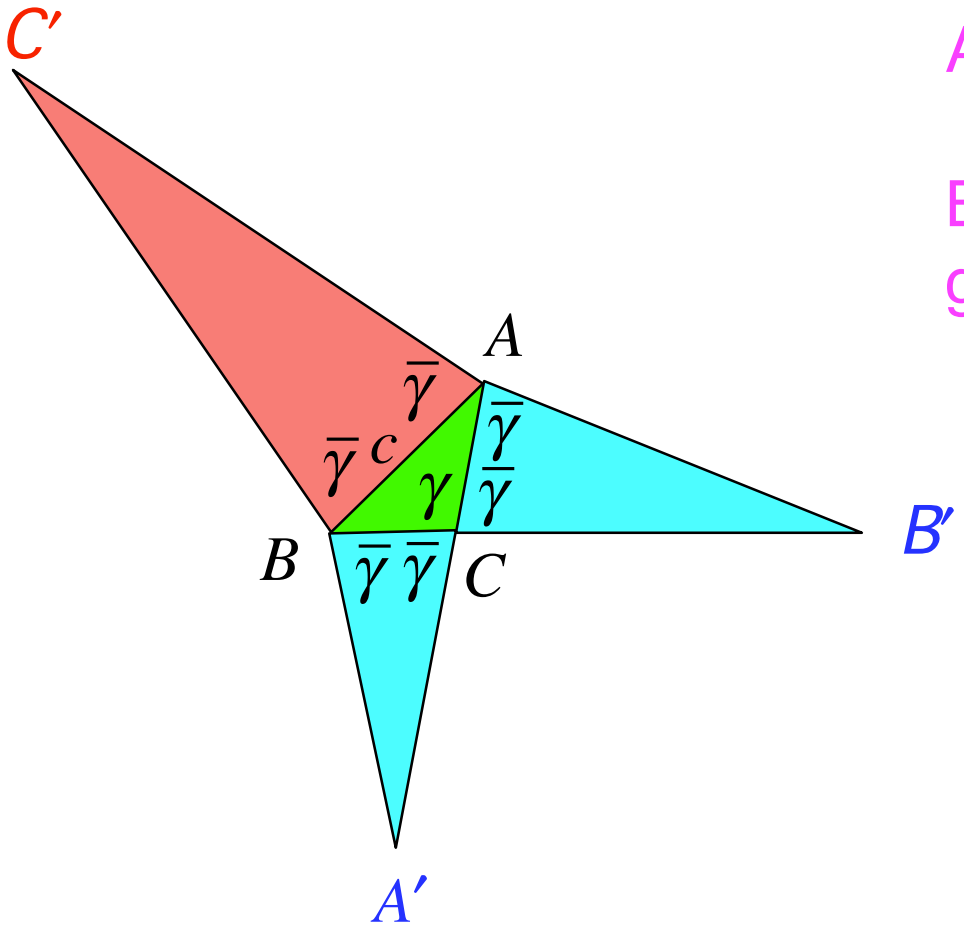
Neu



blau + blau = grün + rot

Beweis rechnerisch

$$\overline{\gamma} = 180° - \gamma$$



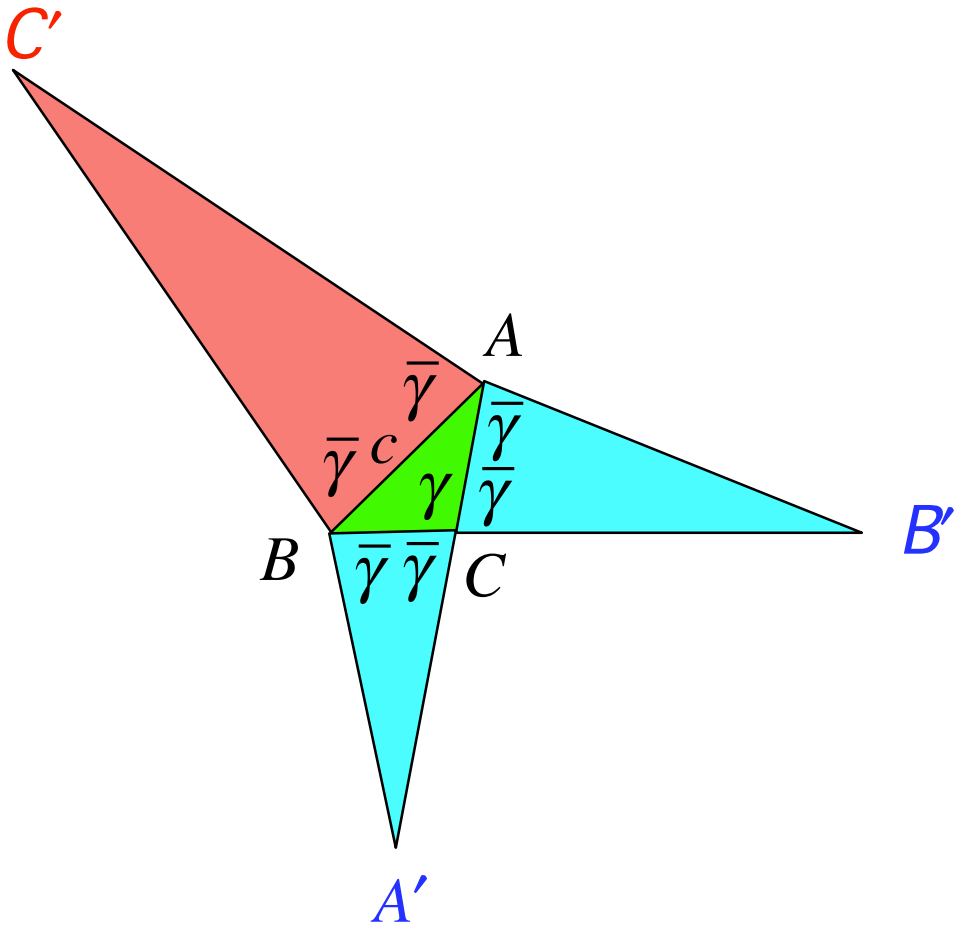
Außenwinkel im Dreieck ABC

Basiswinkel in den gleichschenkligen Dreiecken

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$

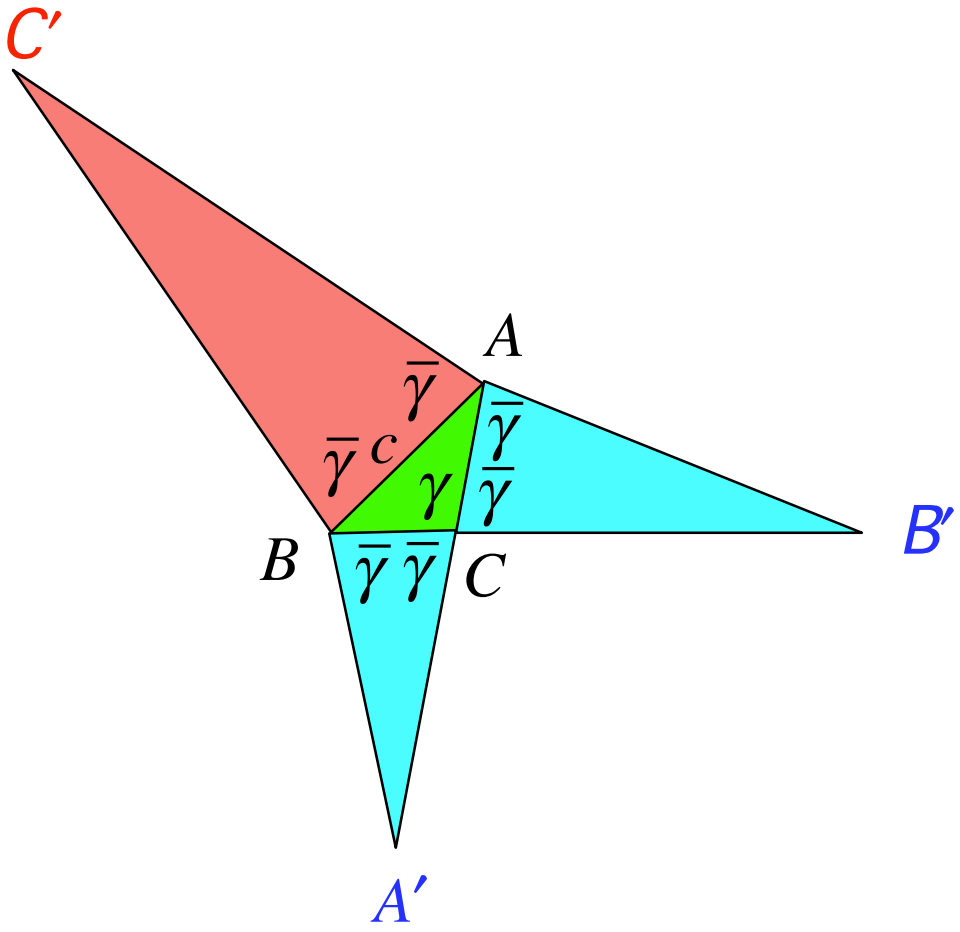


$$A_{\Delta A'CB} = \frac{a^2}{4}\tan(\bar{\gamma})$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$



$$A_{\Delta A'CB} = \frac{a^2}{4}\tan(\bar{\gamma})$$

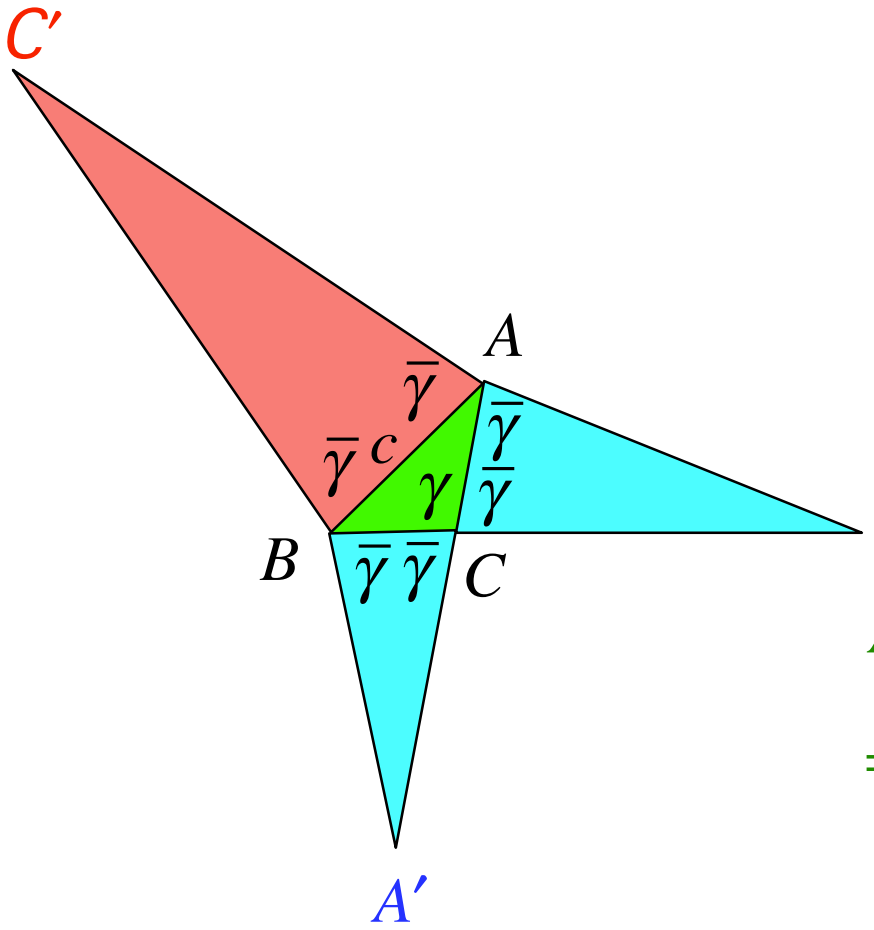
$$A_{\Delta B'AC} = \frac{b^2}{4}\tan(\bar{\gamma})$$

$$A_{\Delta C'BA} = \frac{c^2}{4}\tan(\bar{\gamma})$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$



$$A_{\Delta A'CB} = \frac{a^2}{4}\tan(\bar{\gamma})$$

$$A_{\Delta B'AC} = \frac{b^2}{4}\tan(\bar{\gamma})$$

$$A_{\Delta C'BA} = \frac{c^2}{4}\tan(\bar{\gamma})$$

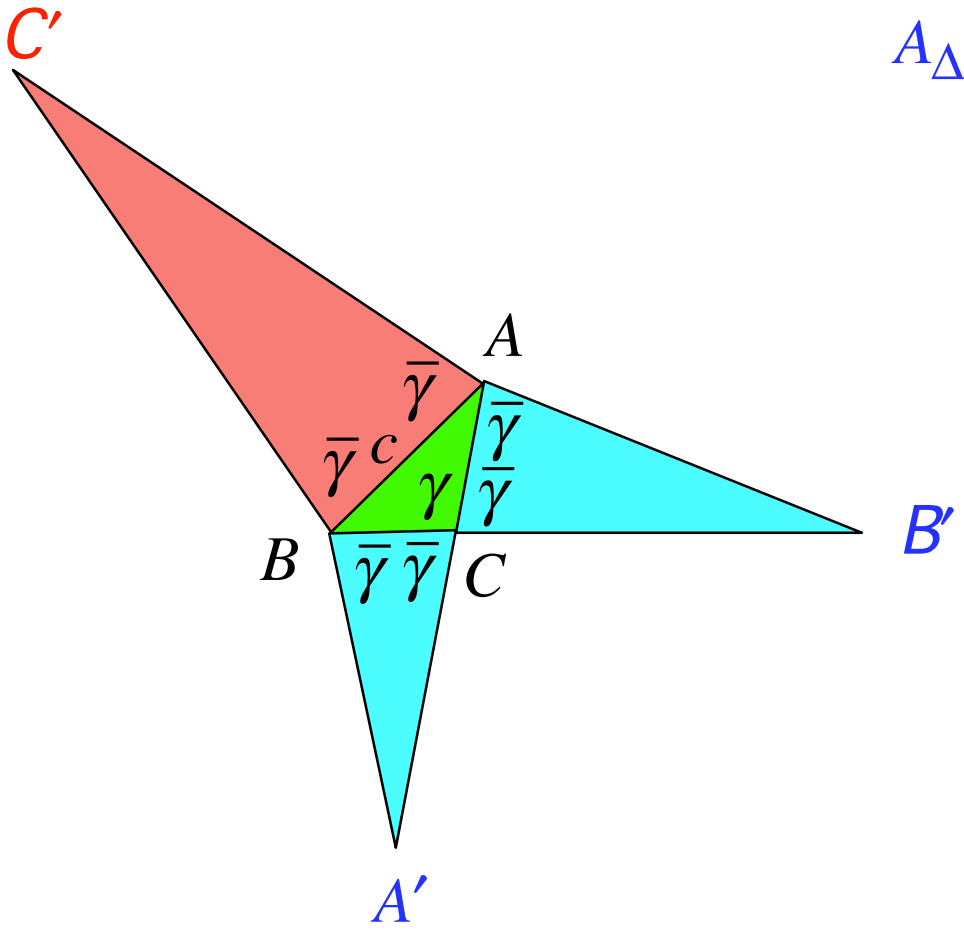
$$A_{\Delta ABC} = \frac{1}{2}ab\sin(\gamma) = \frac{1}{2}ab\sin(\bar{\gamma})$$

$$= \frac{2ab}{4}\tan(\bar{\gamma})\cos(\bar{\gamma})$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$

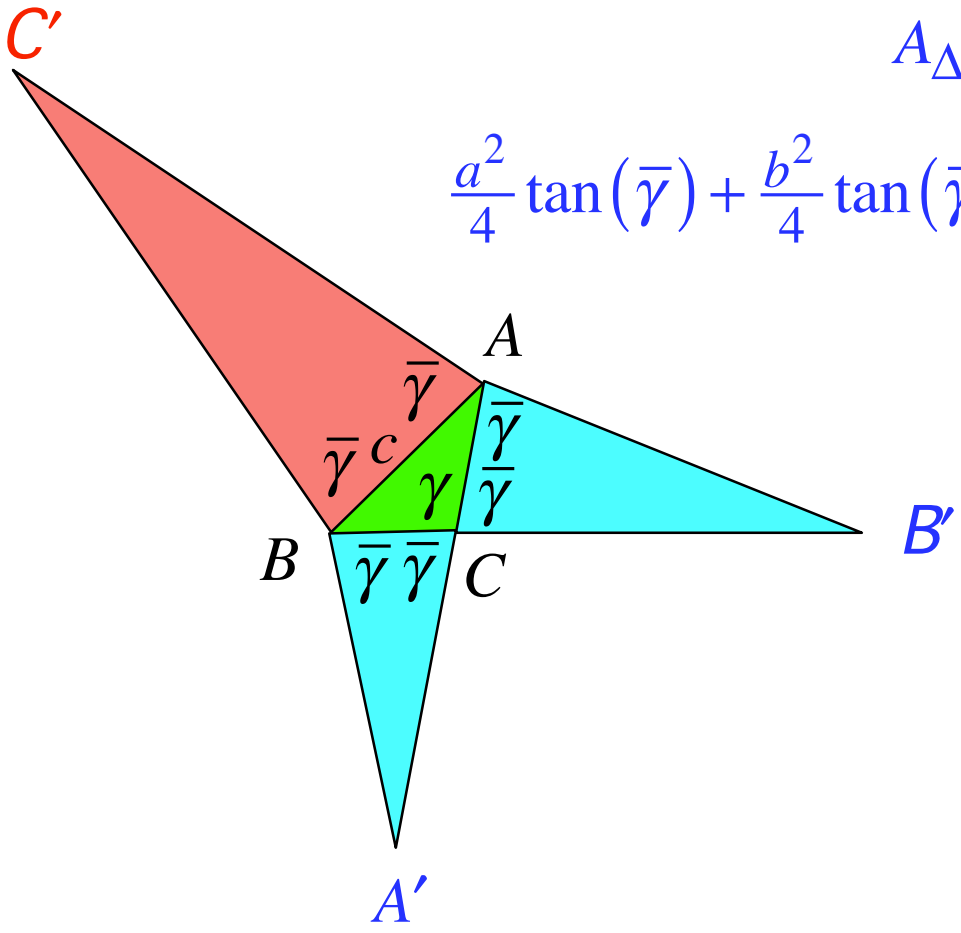


$$A_{\Delta A'CB} + A_{\Delta B'AC} + A_{\Delta ABC} \overset{?}{=} A_{\Delta C'BA}$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$



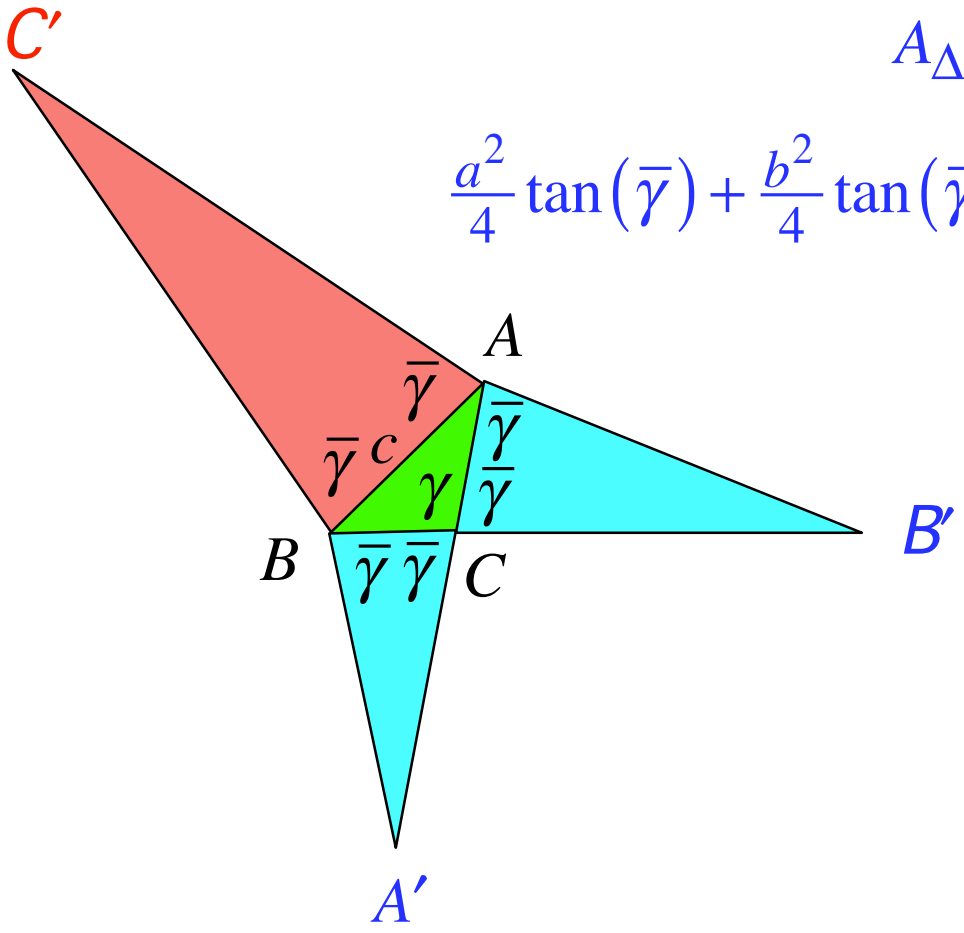
$$A_{\Delta A'CB} + A_{\Delta B'AC} + A_{\Delta ABC} \overset{?}{=} A_{\Delta C'BA}$$

$$\frac{a^2}{4}\tan(\bar{\gamma}) + \frac{b^2}{4}\tan(\bar{\gamma}) + \frac{2ab}{4}\tan(\bar{\gamma})\cos(\bar{\gamma}) \overset{?}{=} \frac{c^2}{4}\tan(\bar{\gamma})$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$



$$A_{\Delta A'CB} + A_{\Delta B'AC} + A_{\Delta ABC} \overset{?}{=} A_{\Delta C'BA}$$

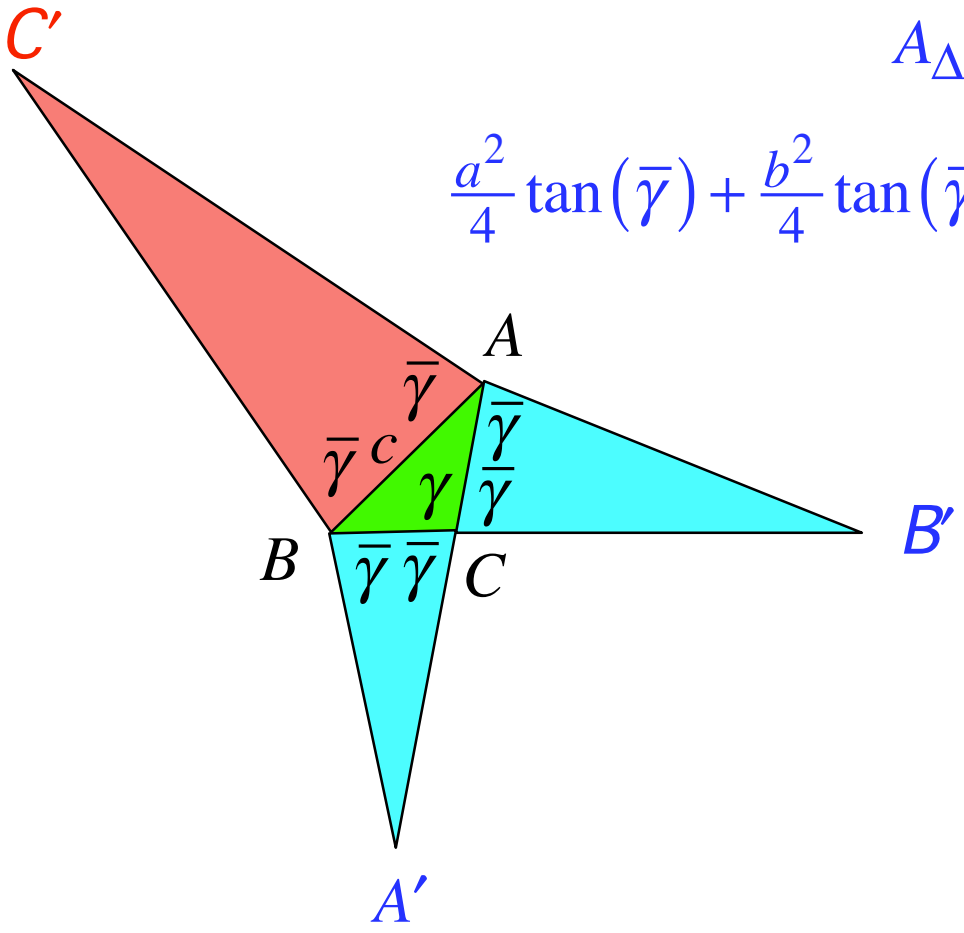
$$\frac{a^2}{4}\tan(\bar{\gamma}) + \frac{b^2}{4}\tan(\bar{\gamma}) + \frac{2ab}{4}\tan(\bar{\gamma})\cos(\bar{\gamma}) \overset{?}{=} \frac{c^2}{4}\tan(\bar{\gamma})$$

$$a^2 + b^2 + 2ab\cos(\bar{\gamma}) \overset{?}{=} c^2$$

blau + blau + grün = rot

Beweis rechnerisch

$$\bar{\gamma} = 180° - \gamma$$



$$A_{\Delta A'CB} + A_{\Delta B'AC} + A_{\Delta ABC} \overset{?}{=} A_{\Delta C'BA}$$

$$\frac{a^2}{4}\tan(\bar{\gamma}) + \frac{b^2}{4}\tan(\bar{\gamma}) + \frac{2ab}{4}\tan(\bar{\gamma})\cos(\bar{\gamma}) \overset{?}{=} \frac{c^2}{4}\tan(\bar{\gamma})$$

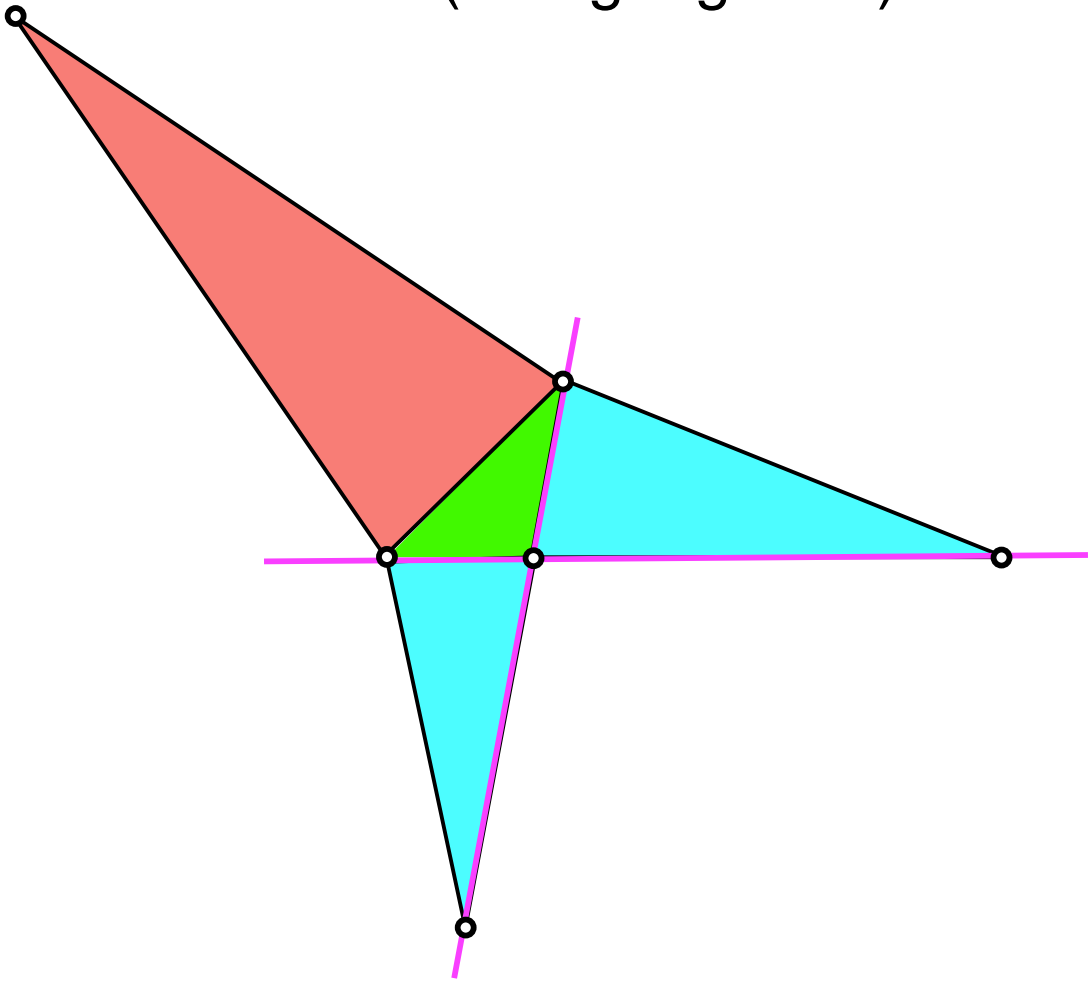
$$a^2 + b^2 + 2ab\cos(\bar{\gamma}) \overset{?}{=} c^2$$

$$\cos(\bar{\gamma}) = -\cos(\gamma)$$

Kosinussatz

blau + blau + grün = rot

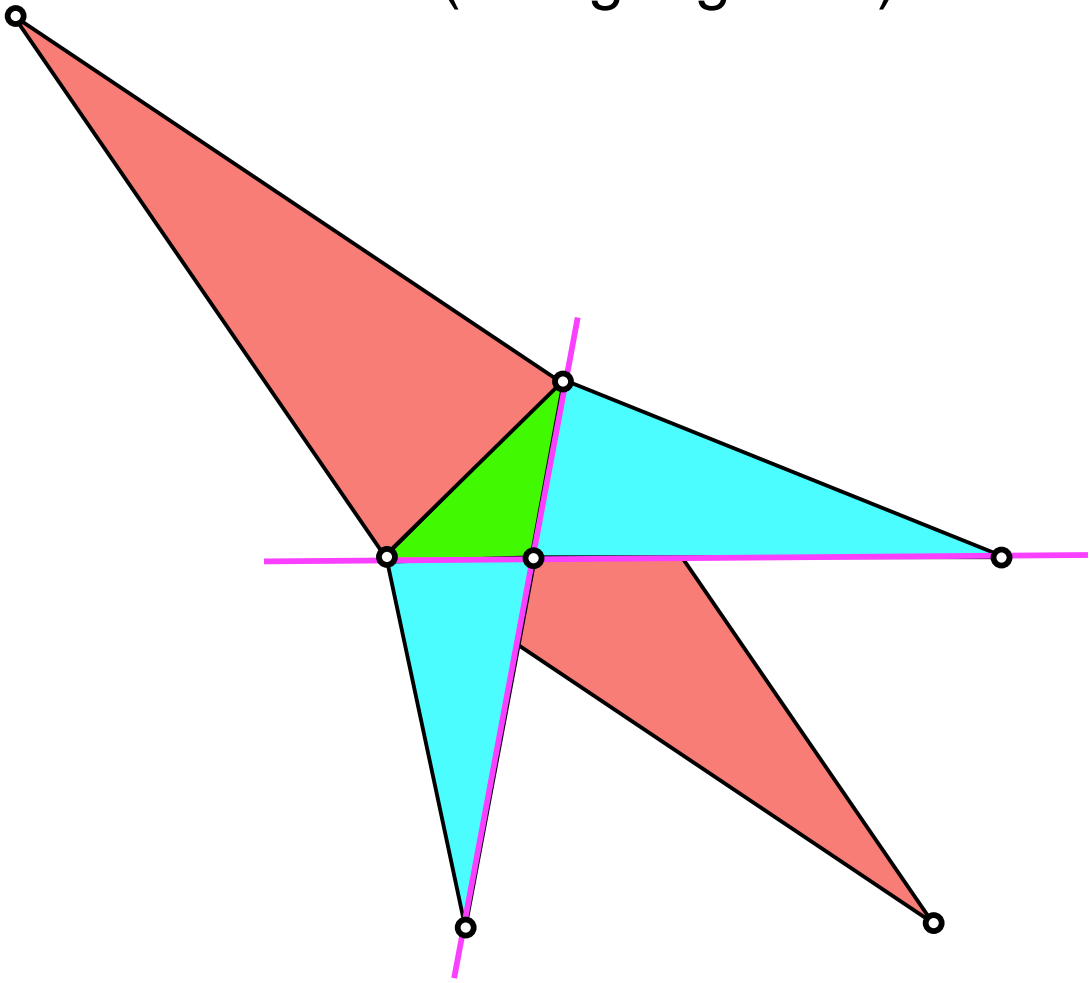
Beweis schön (Wolfgang Kroll)



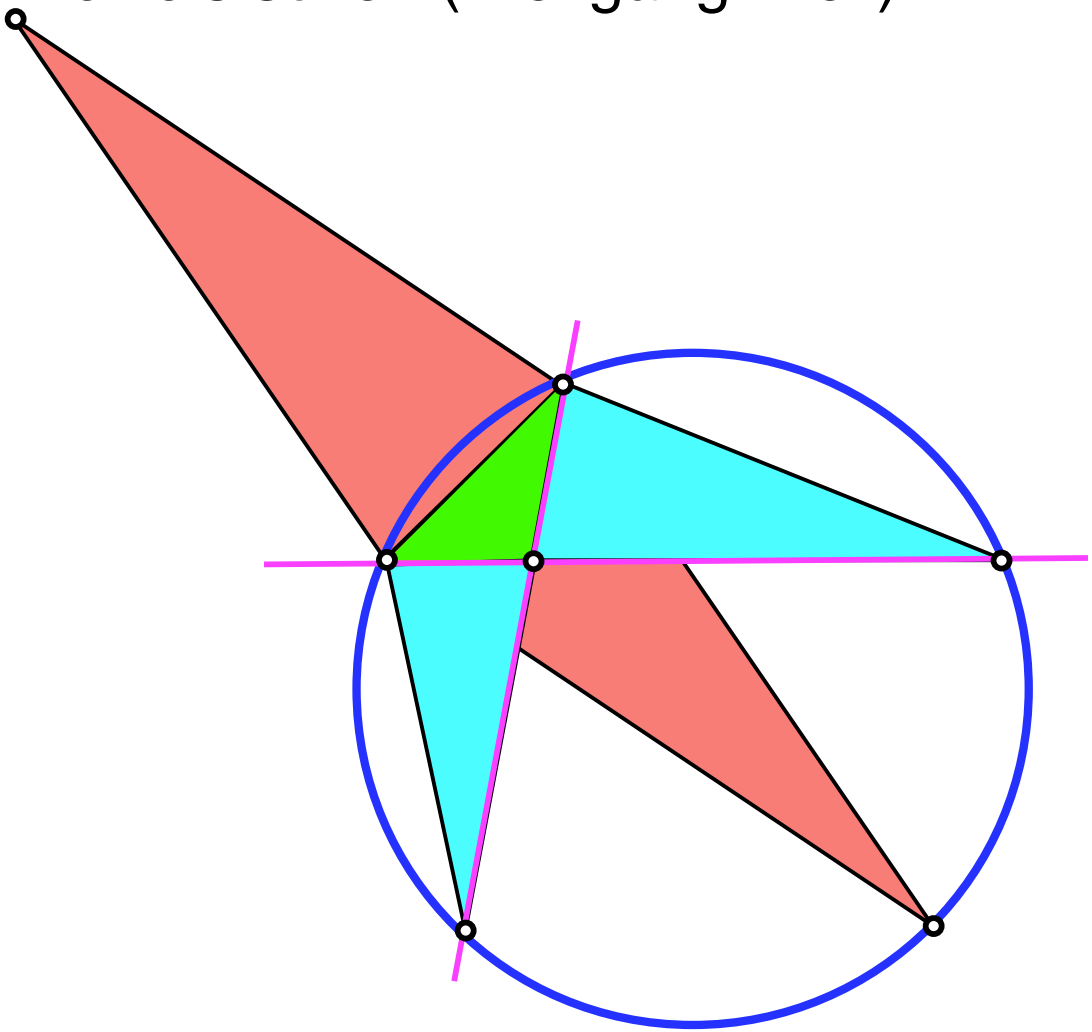
blau + blau + grün = rot

Beweis schön (Wolfgang Kroll)

Rotes Dreieck spiegeln



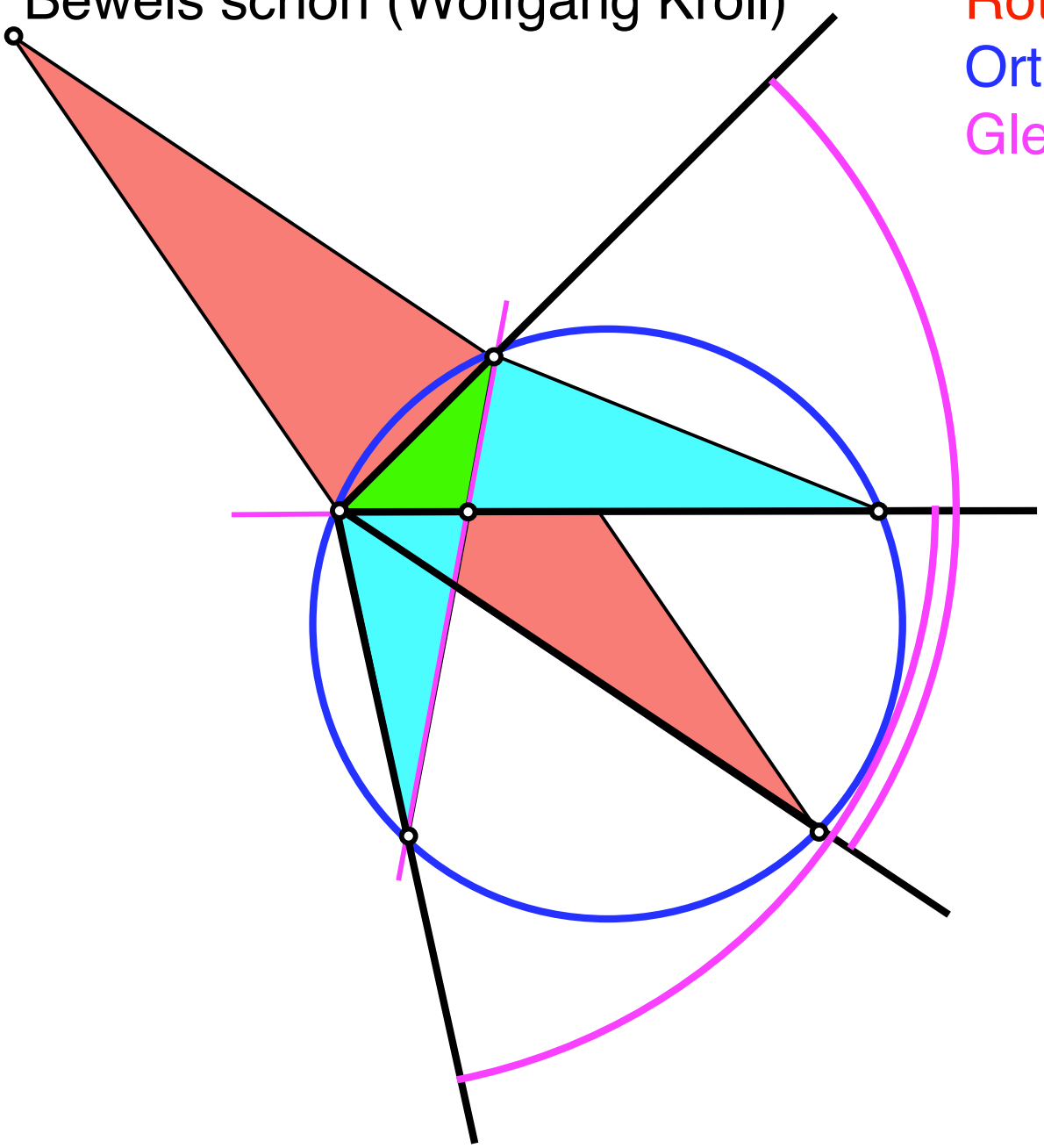
Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Beweis schön (Wolfgang Kroll)

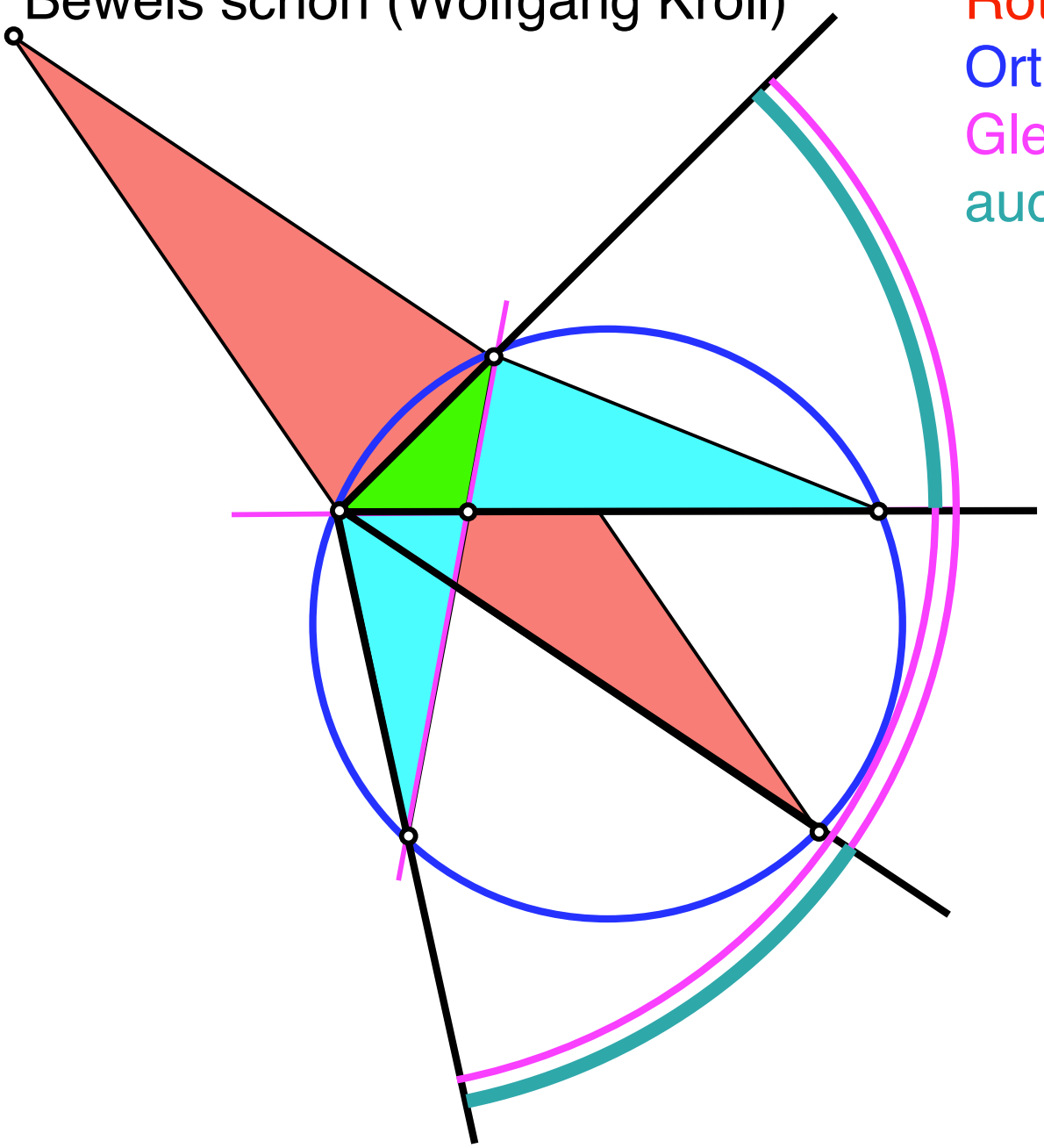


Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

Beweis schön (Wolfgang Kroll)



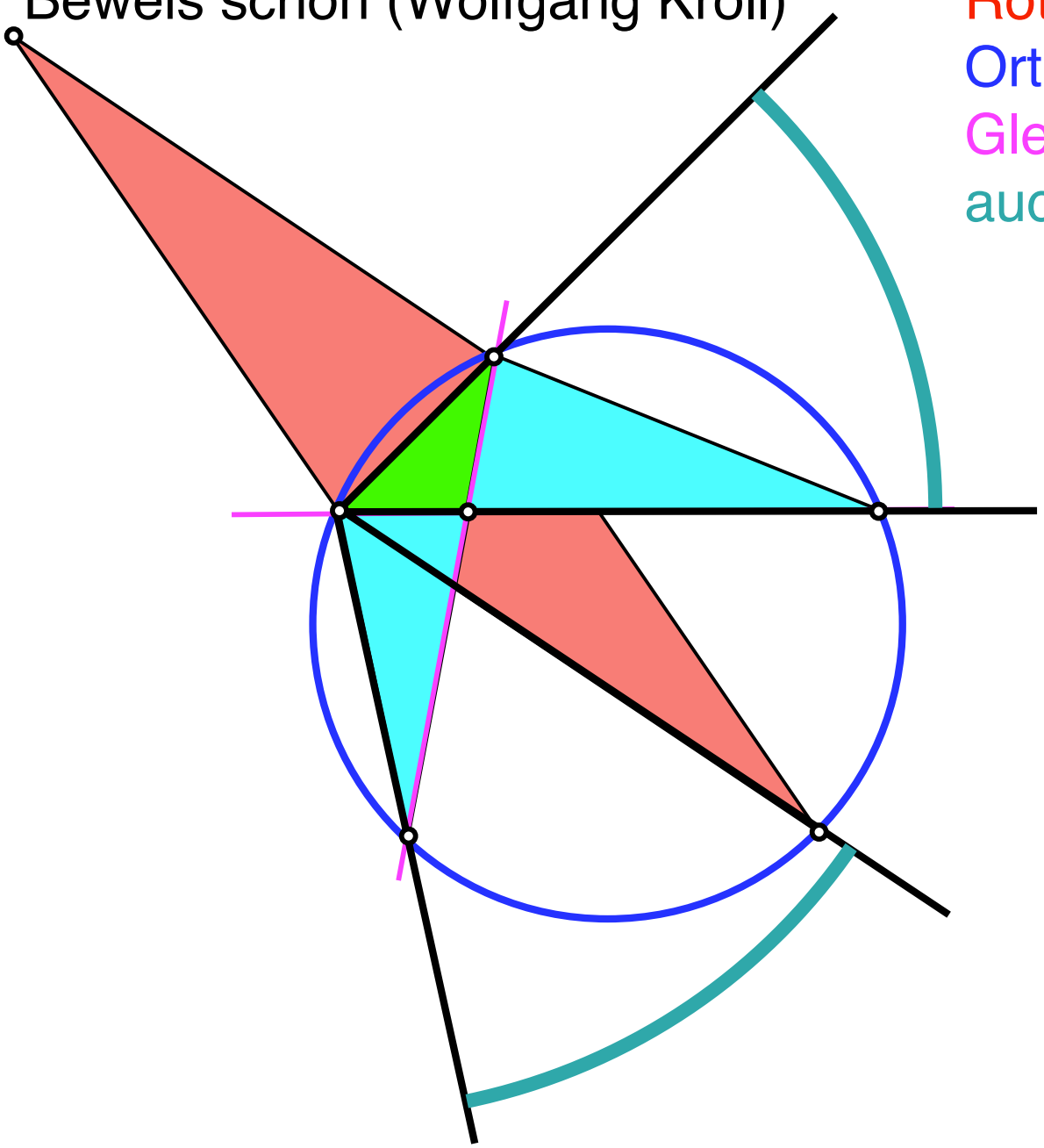
Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

Beweis schön (Wolfgang Kroll)



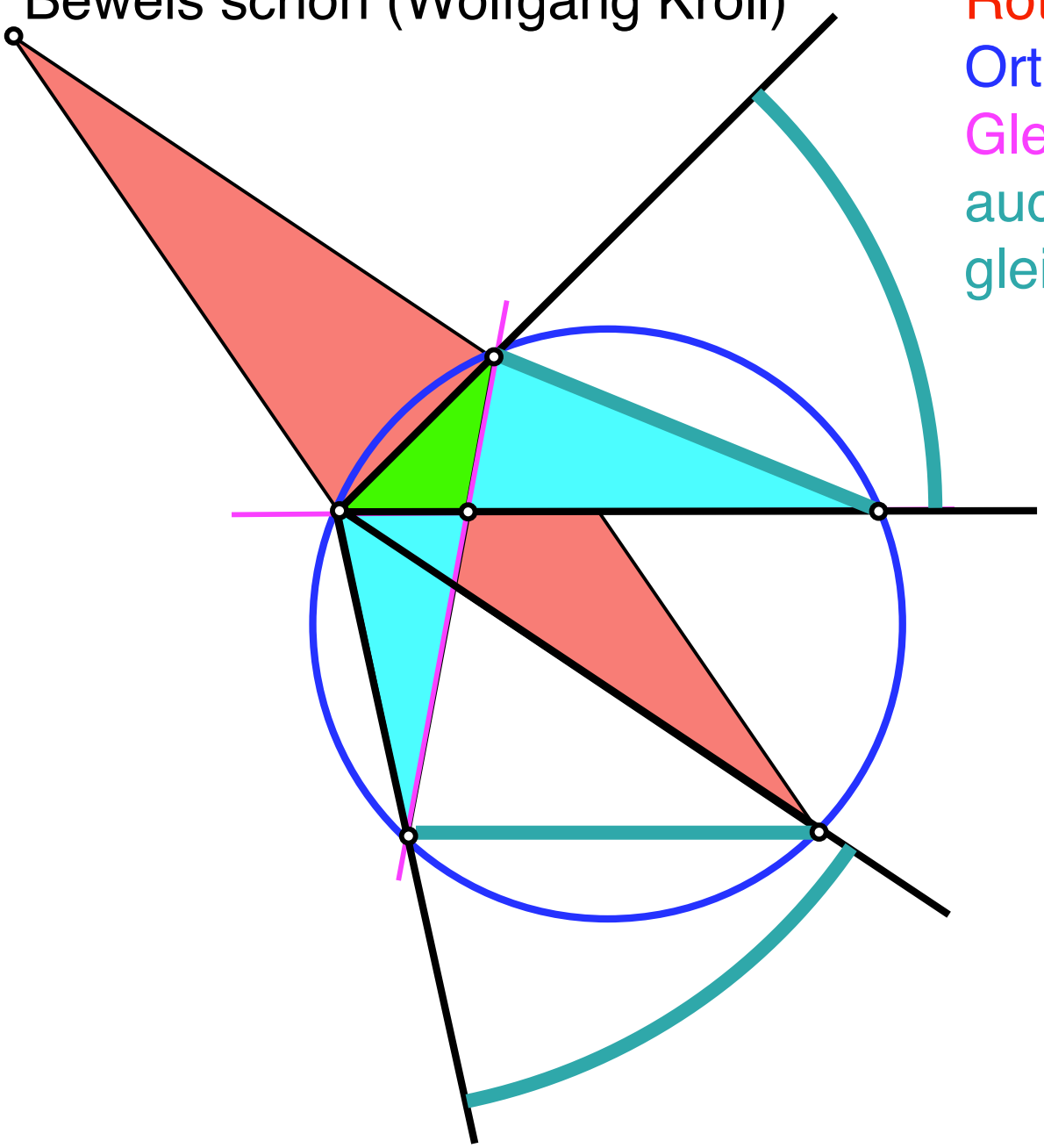
Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

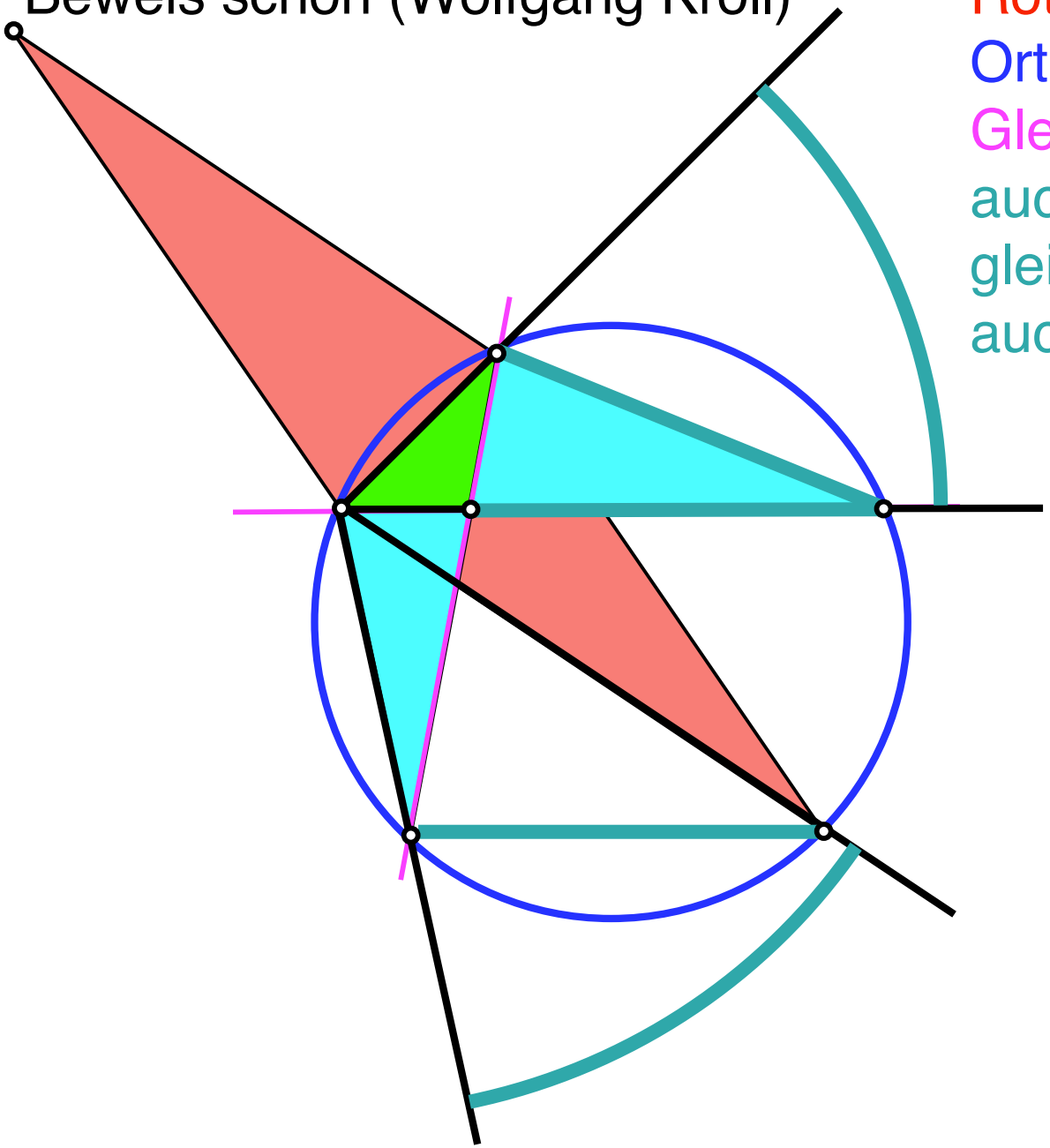
Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

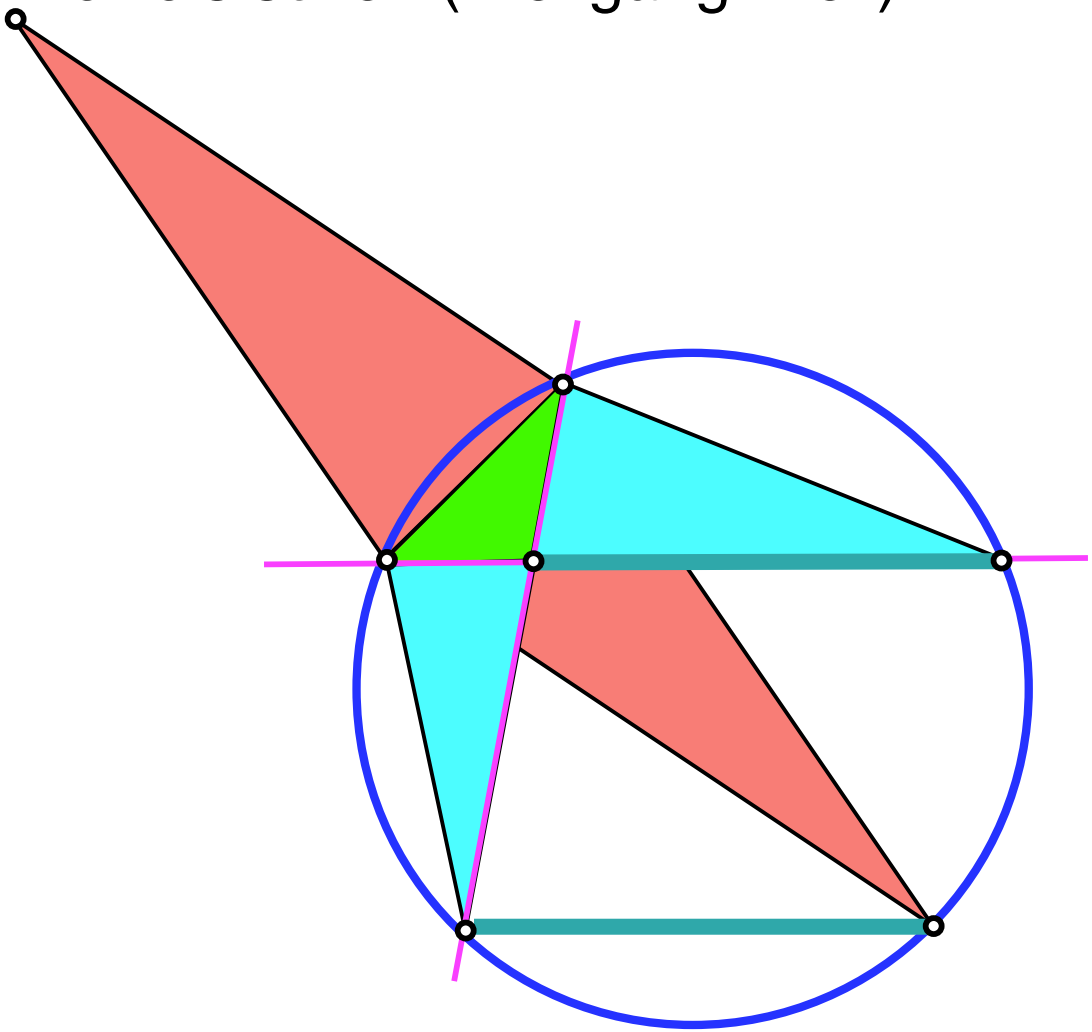
Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

auch gleich lang

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

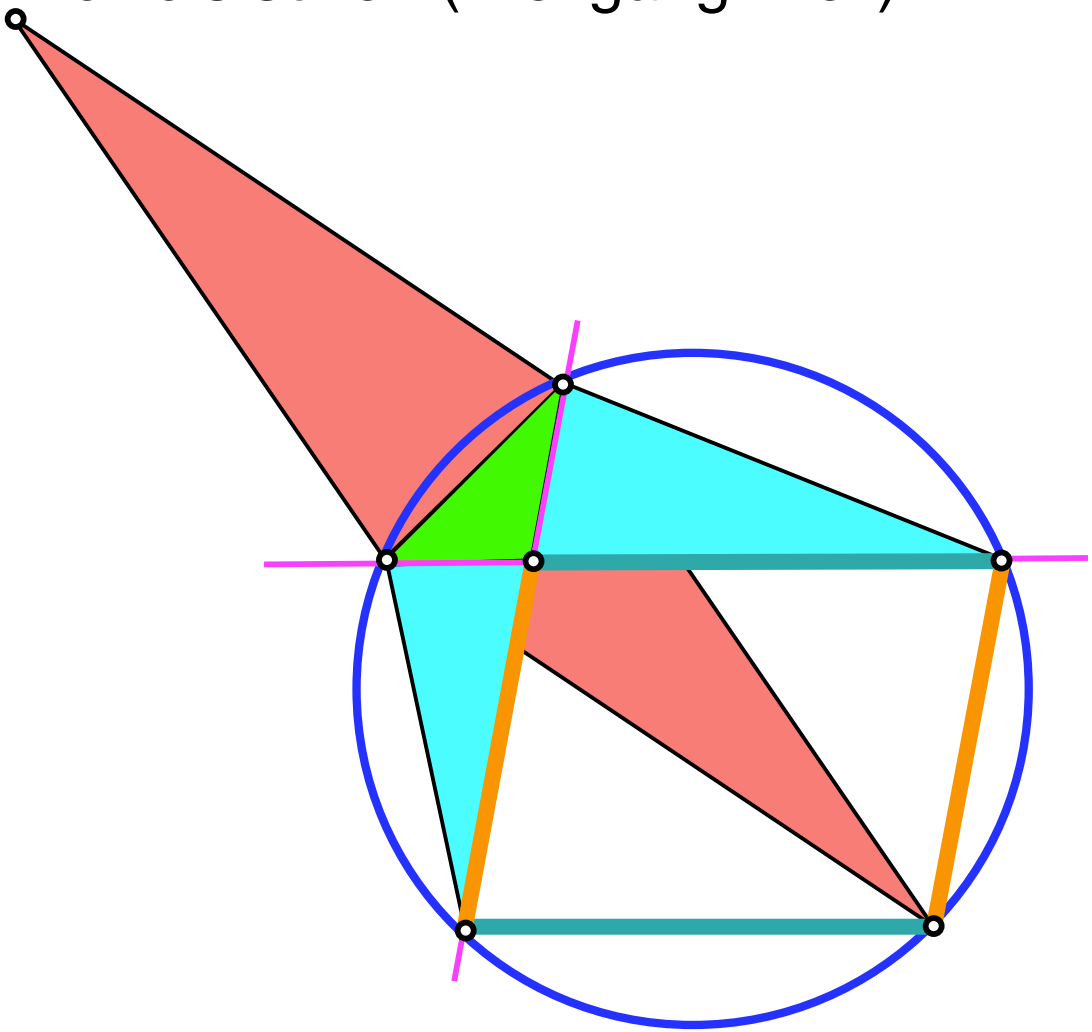
Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

auch gleich lang

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

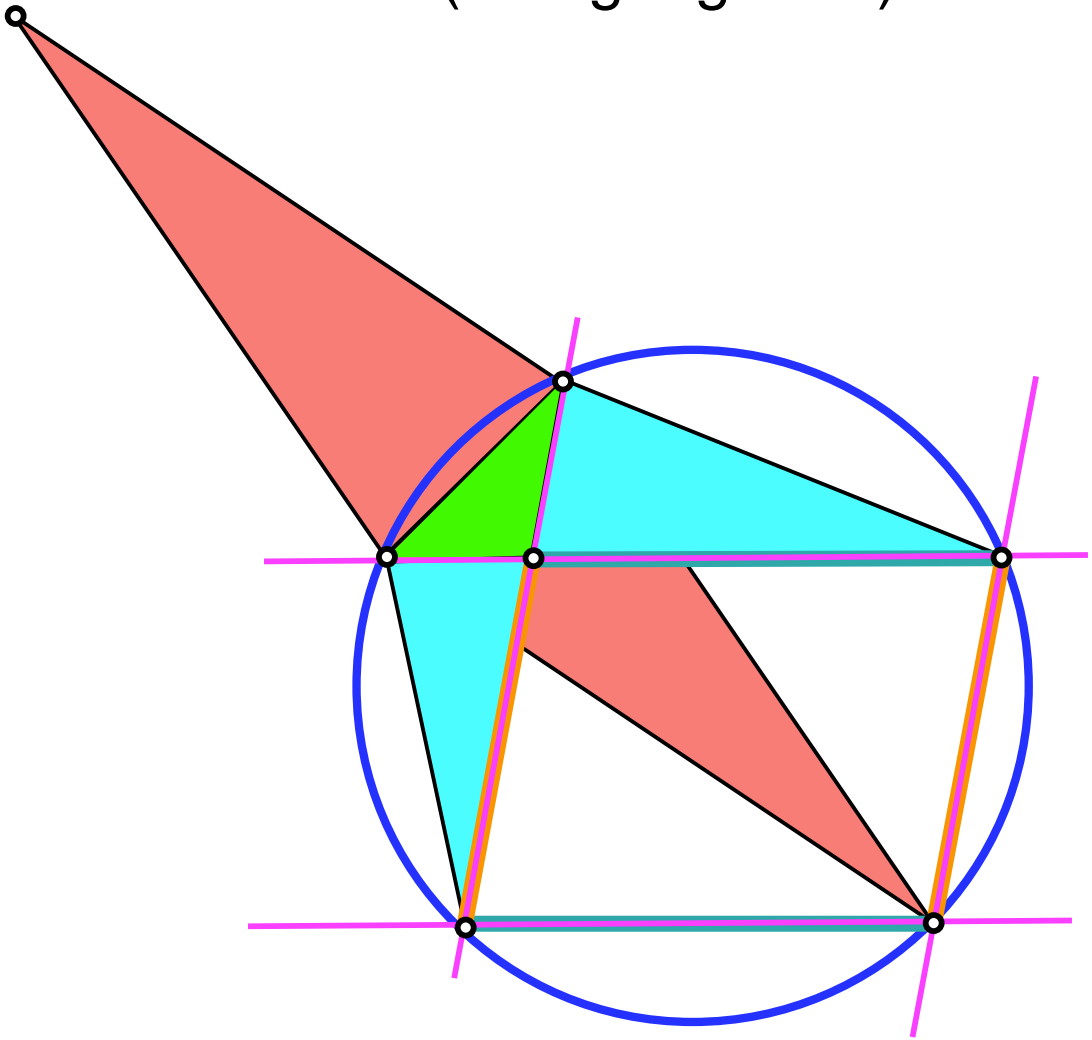
auch gleiche Winkel

gleiche Sehnen

auch gleich lang

analog

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

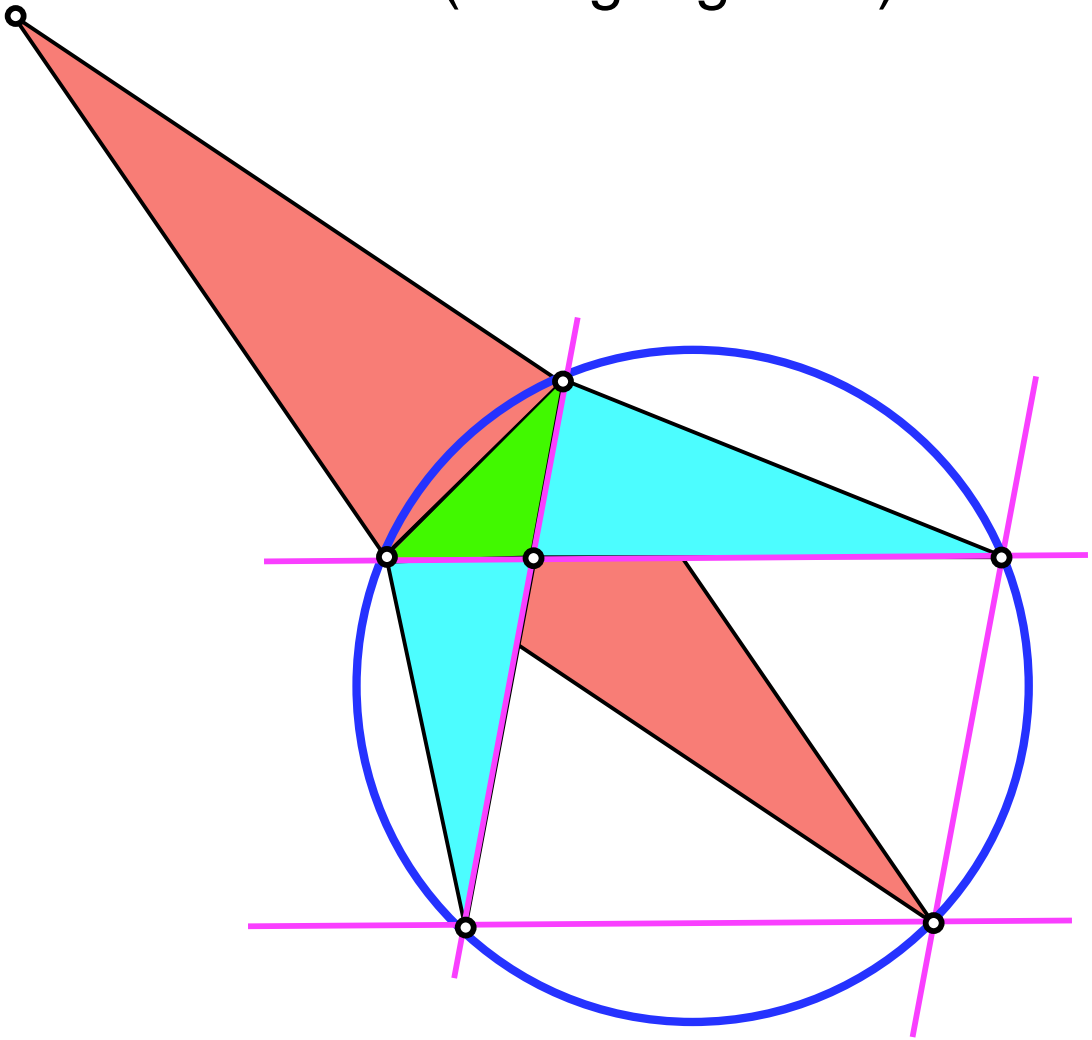
gleiche Sehnen

auch gleich lang

analog

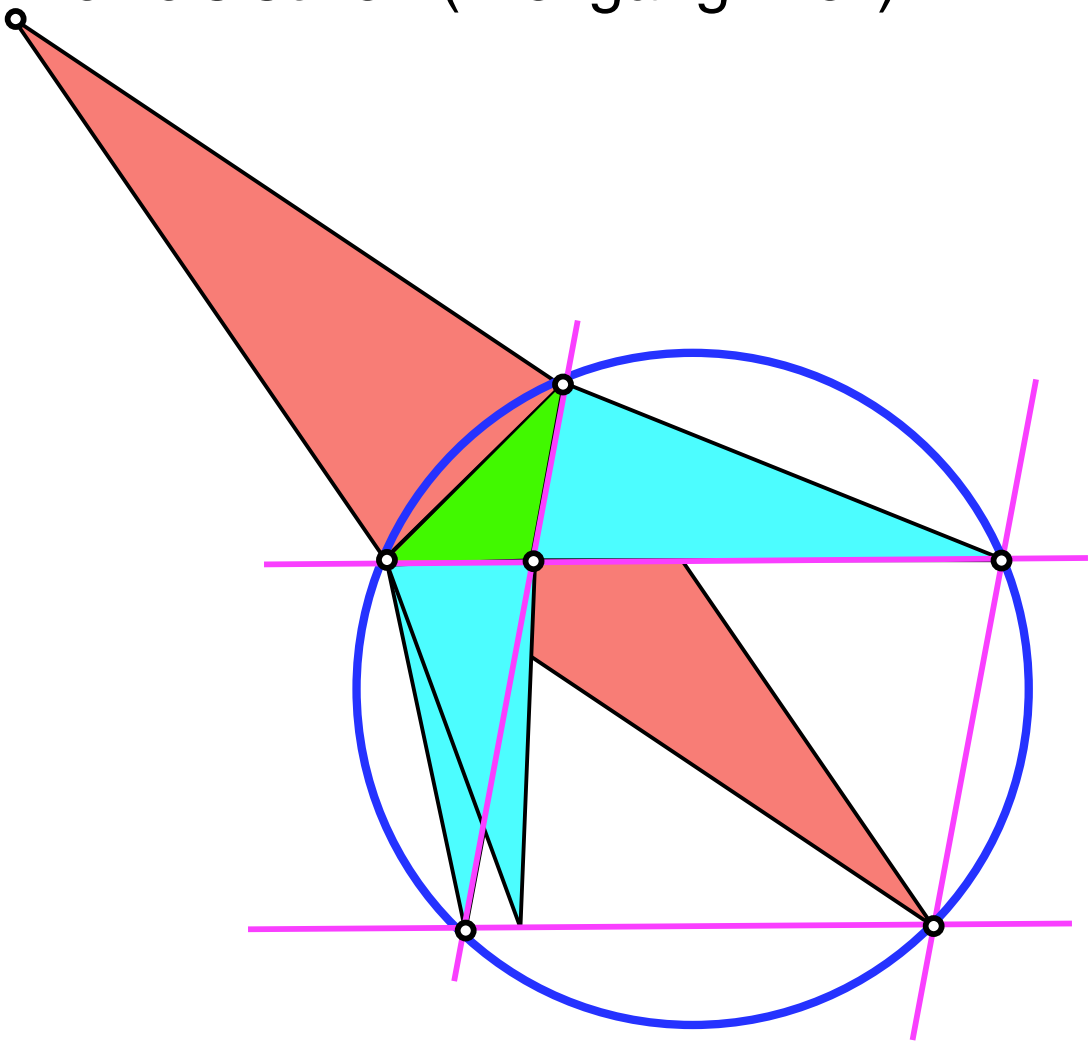
Parallelogramm

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\overline{\gamma}$
auch gleiche Winkel
gleiche Sehnen
auch gleich lang
analog
Parallelogramm

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

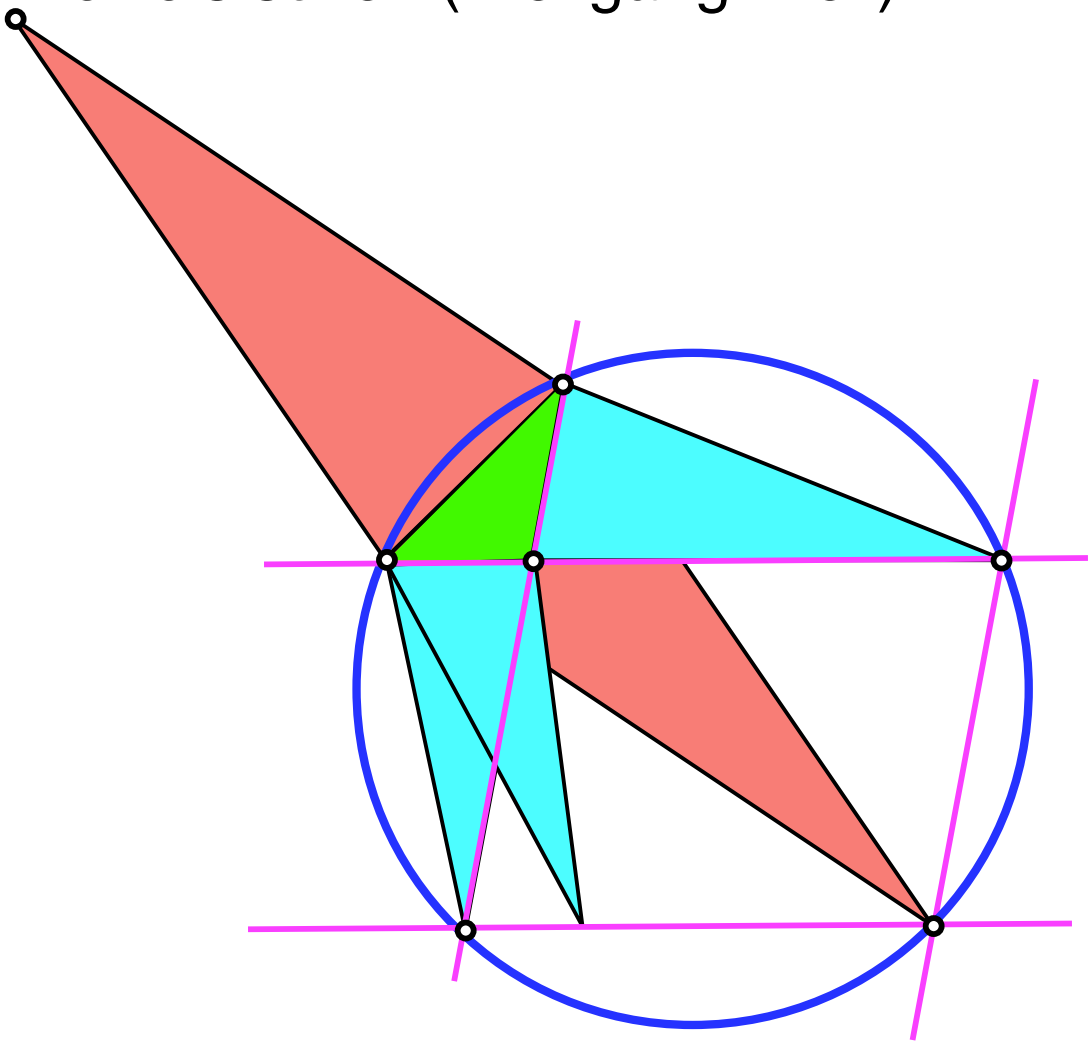
auch gleich lang

analog

Parallelogramm

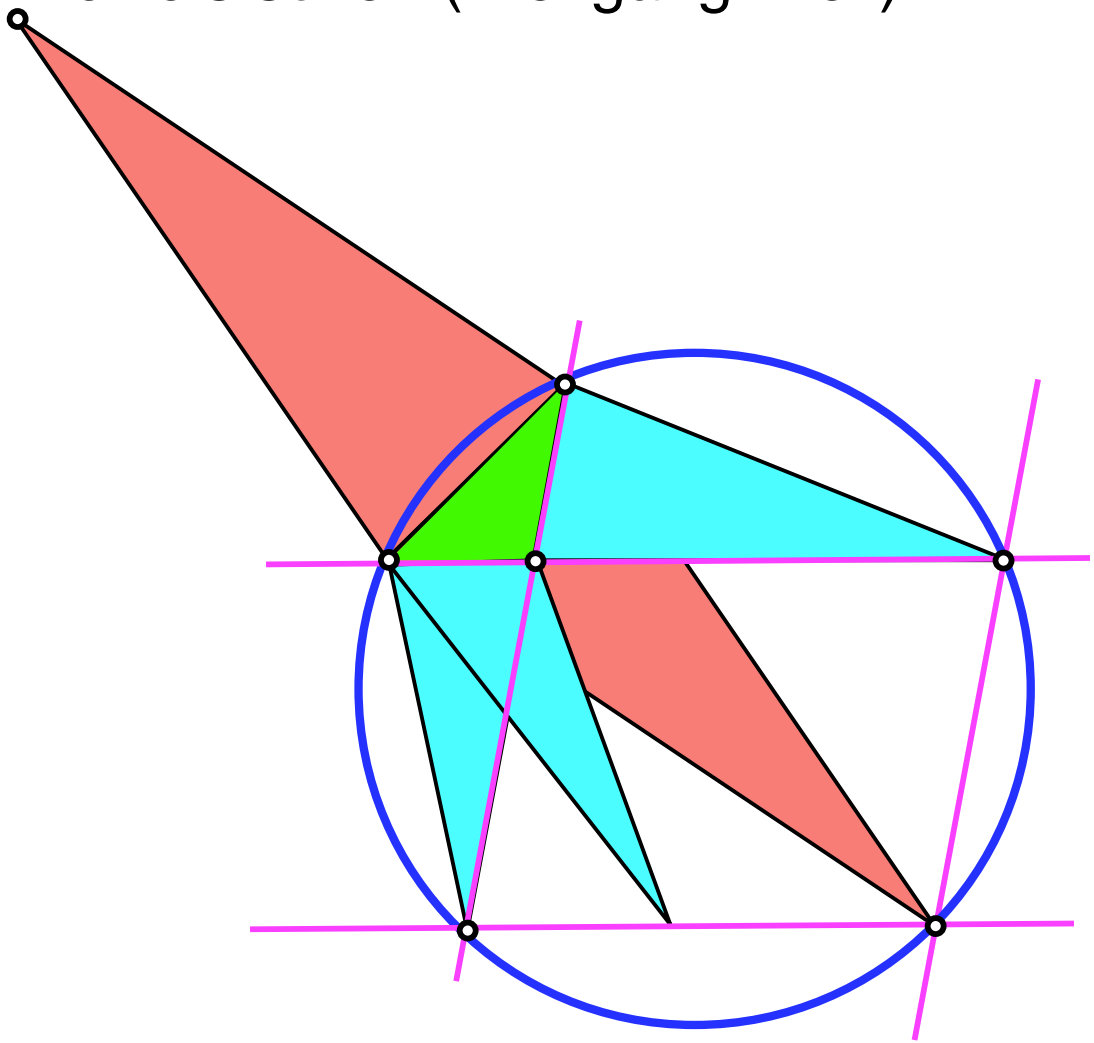
flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\overline{\gamma}$
auch gleiche Winkel
gleiche Sehnen
auch gleich lang
analog
Parallelogramm
flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
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auch gleiche Winkel
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Parallelogramm
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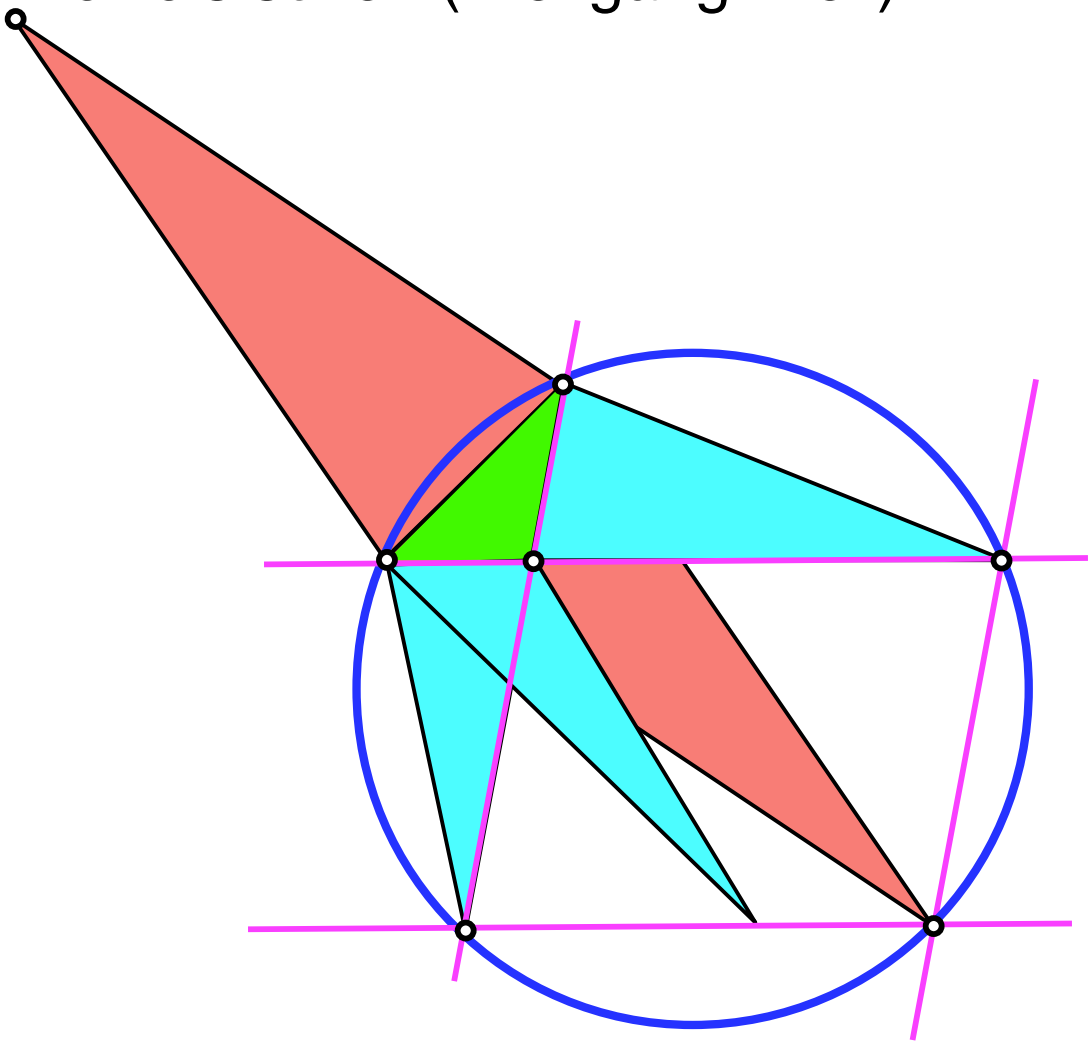
Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

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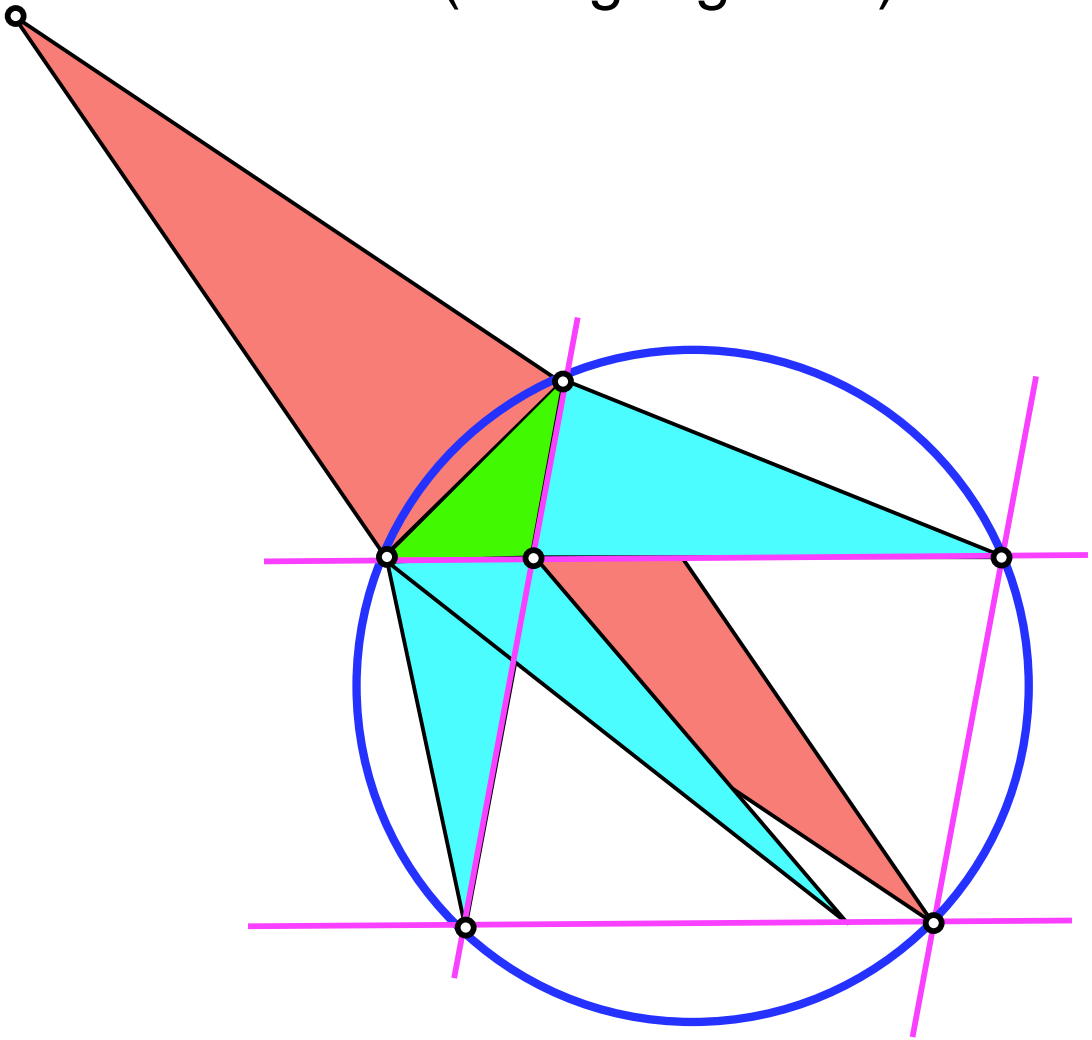
Parallelogramm

flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\overline{\gamma}$
auch gleiche Winkel
gleiche Sehnen
auch gleich lang
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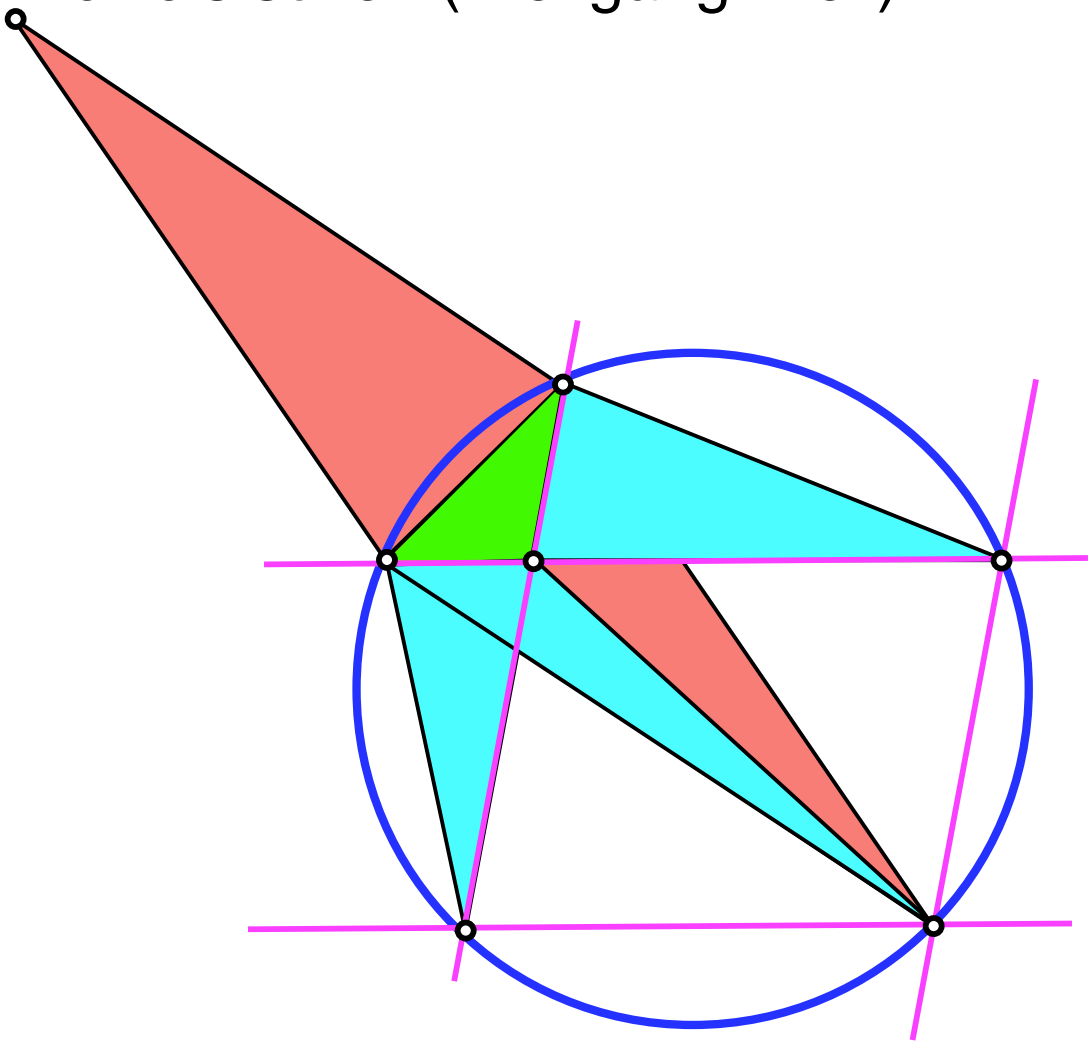
Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

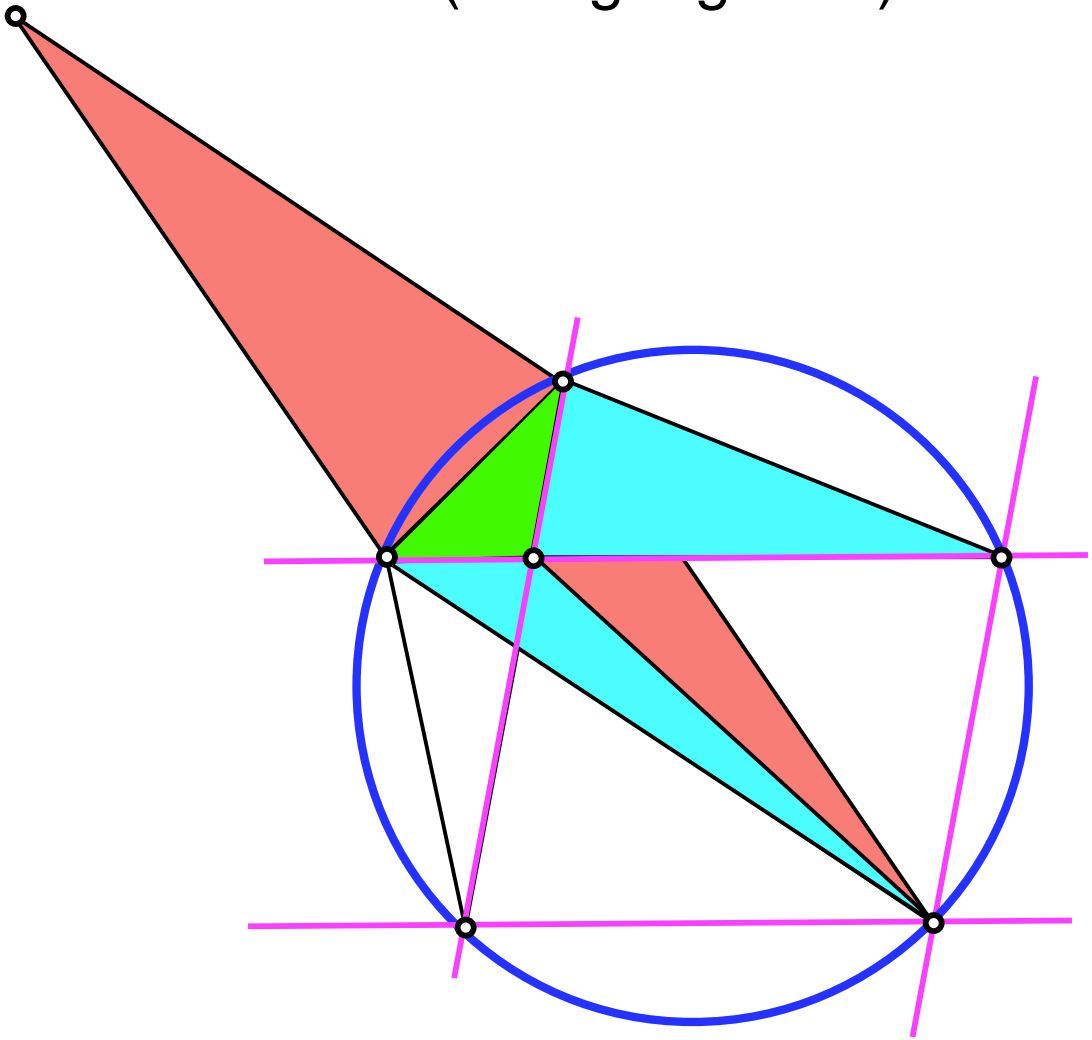
auch gleich lang

analog

Parallelogramm

flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln

Ortsbogen

Gleiche Winkel $\overline{\gamma}$

auch gleiche Winkel

gleiche Sehnen

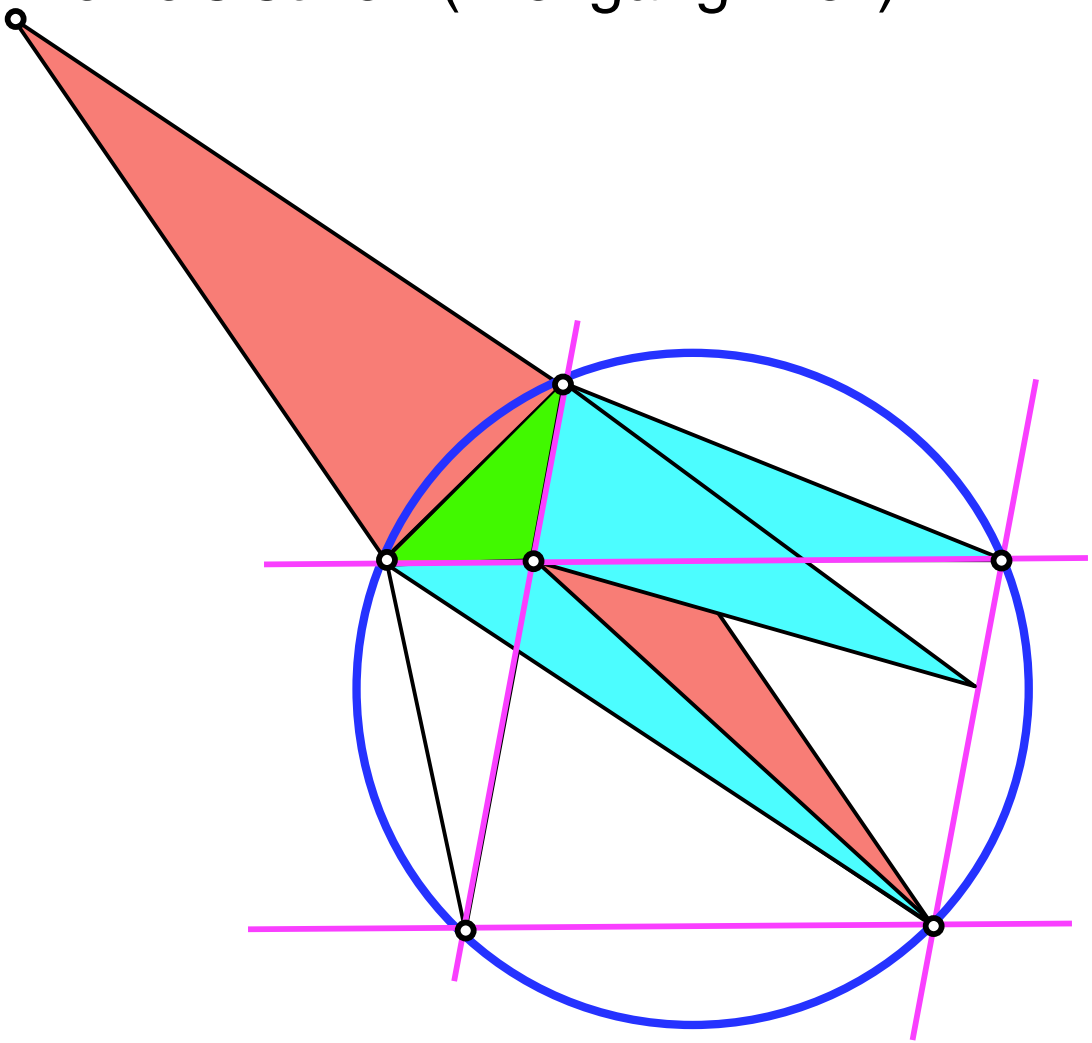
auch gleich lang

analog

Parallelogramm

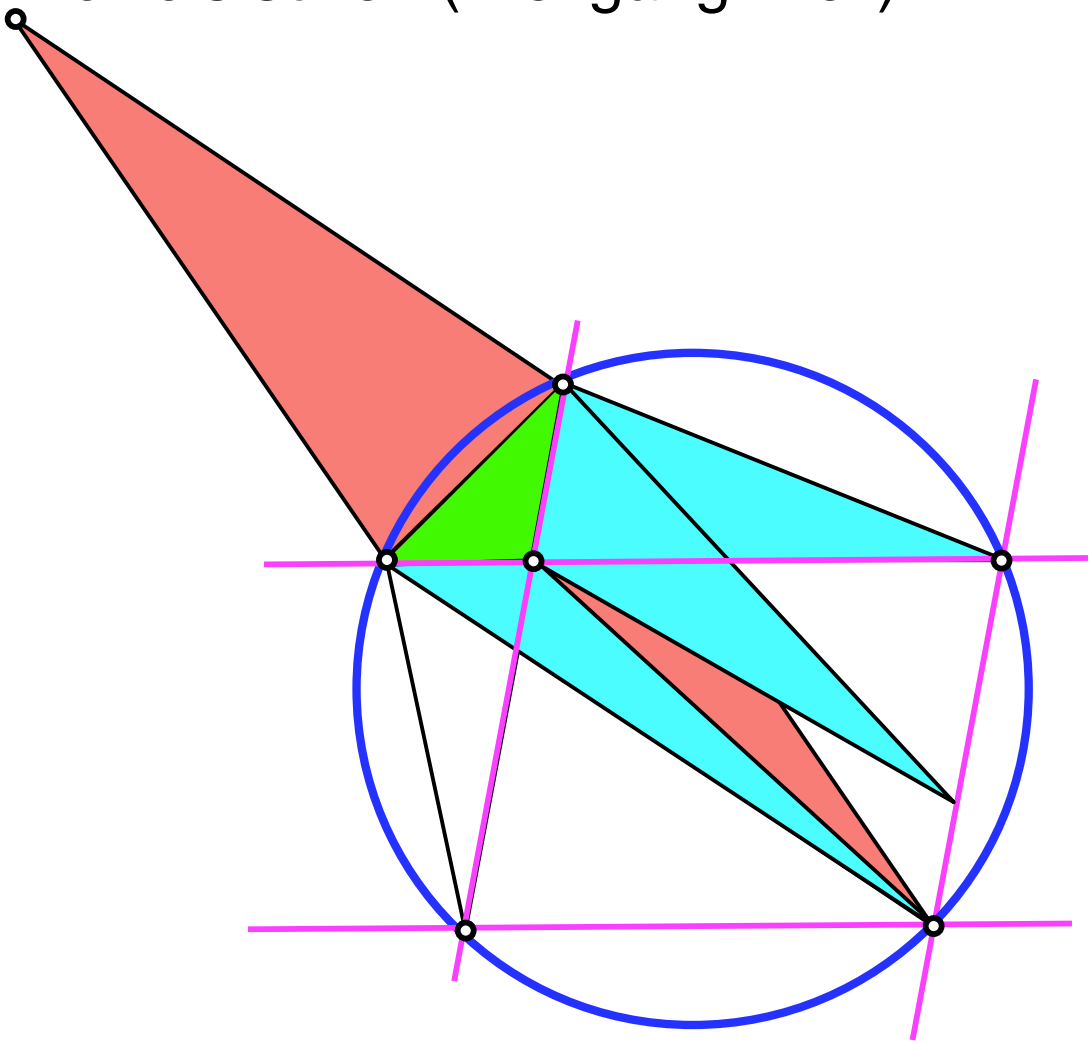
flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\overline{\gamma}$
auch gleiche Winkel
gleiche Sehnen
auch gleich lang
analog
Parallelogramm
flächengleich
flächengleich

Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\overline{\gamma}$
auch gleiche Winkel
gleiche Sehnen
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Parallelogramm
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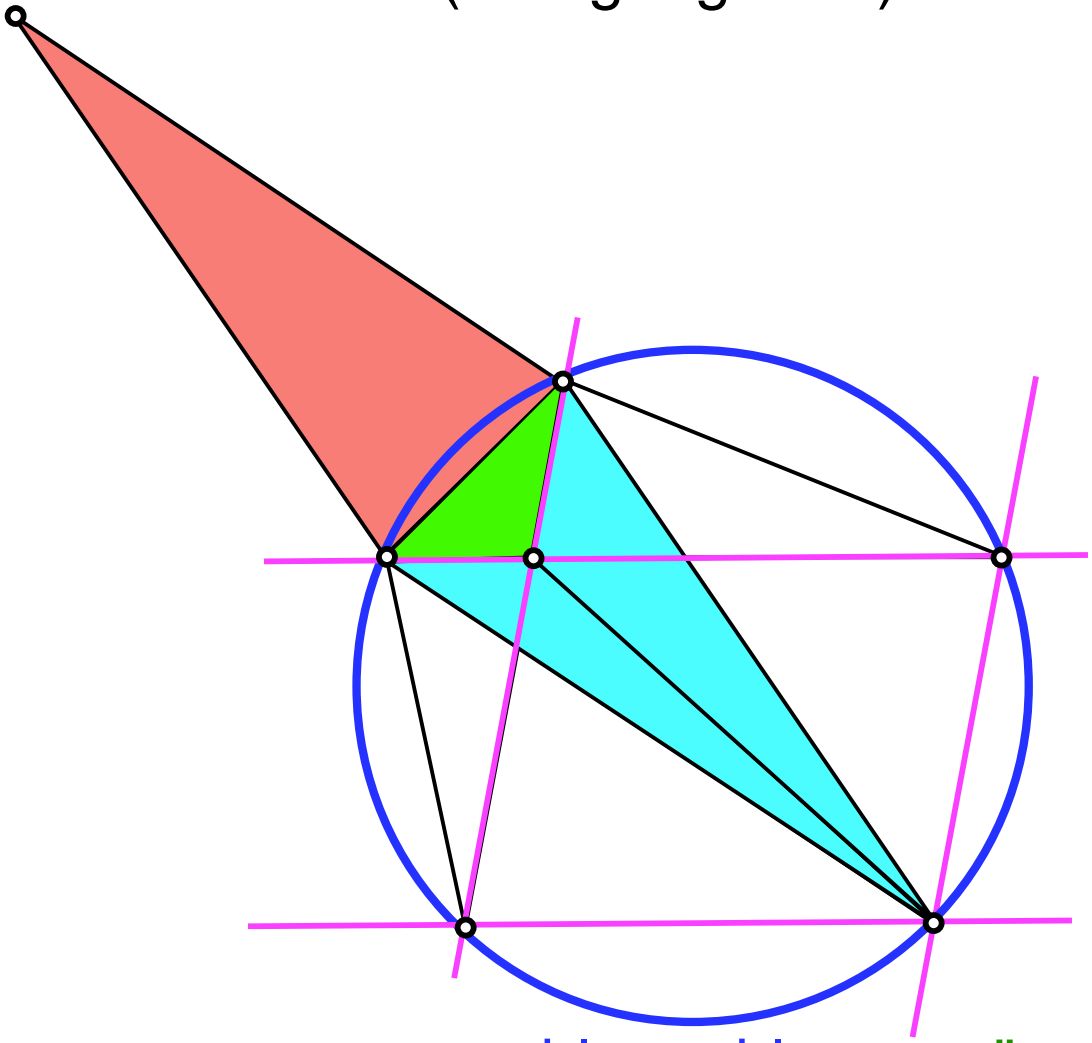
flächengleich

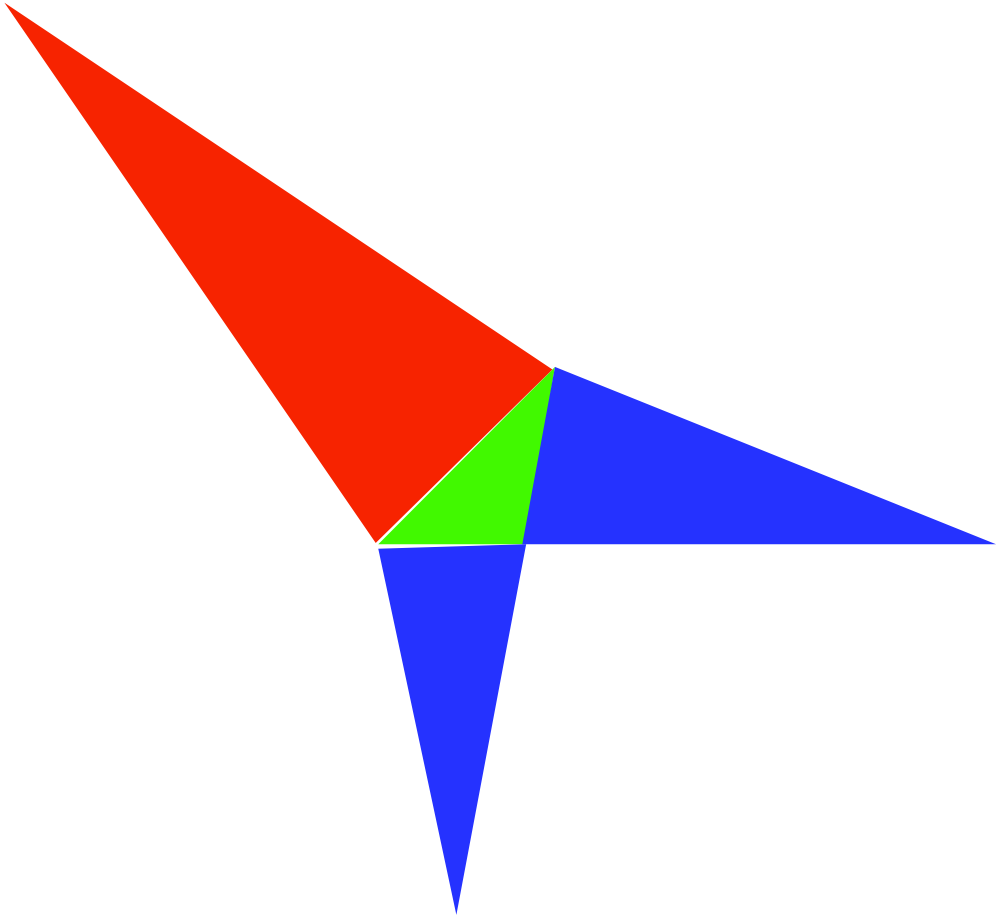
Beweis schön (Wolfgang Kroll)



Rotes Dreieck spiegeln
Ortsbogen
Gleiche Winkel $\bar{\gamma}$
auch gleiche Winkel
gleiche Sehnen
auch gleich lang
analog
Parallelogramm
flächengleich
flächengleich

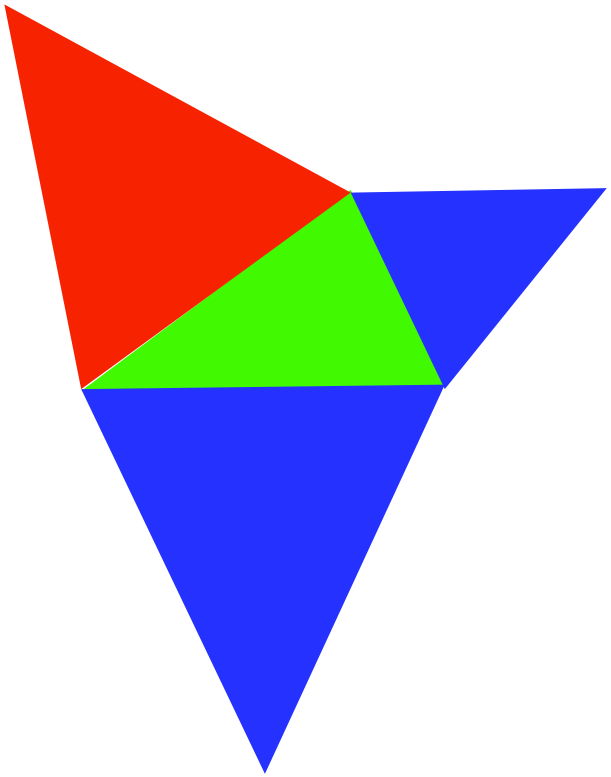
was zu beweisen war

blau + blau + grün = rot



blau + blau + grün = rot

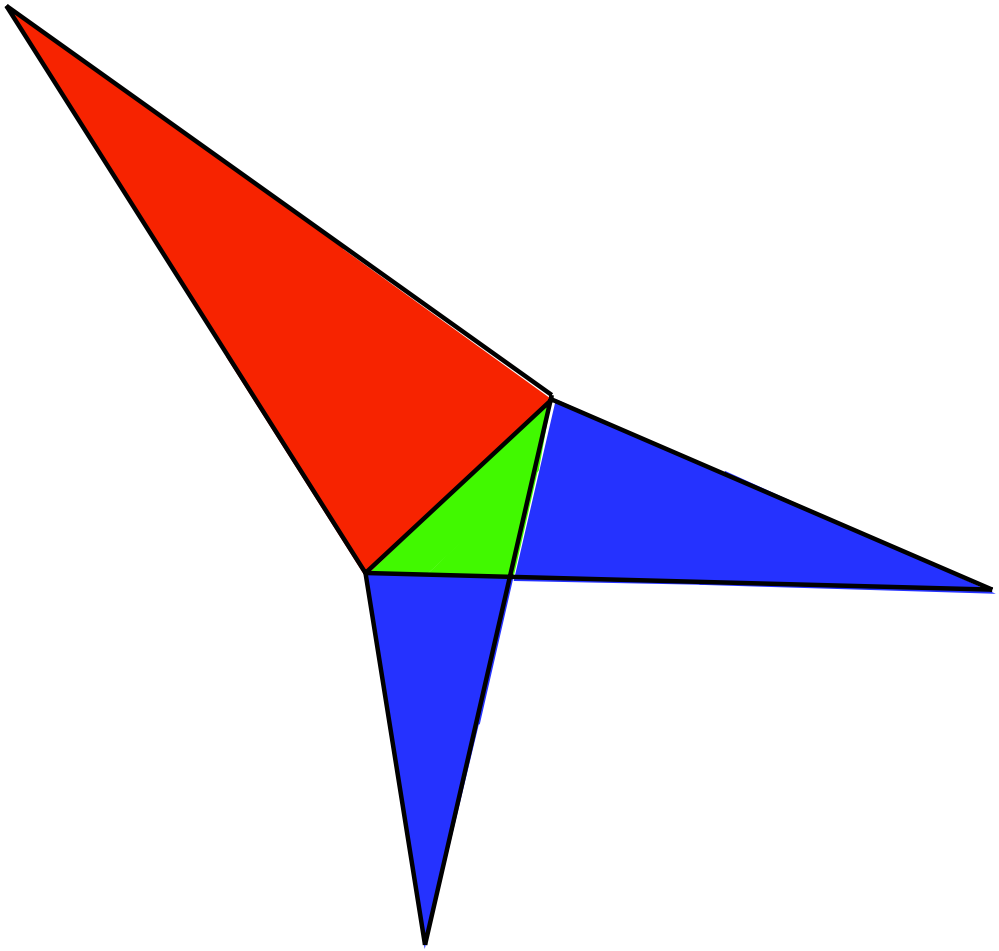
Analoge Beweise:



blau + blau = grün + rot

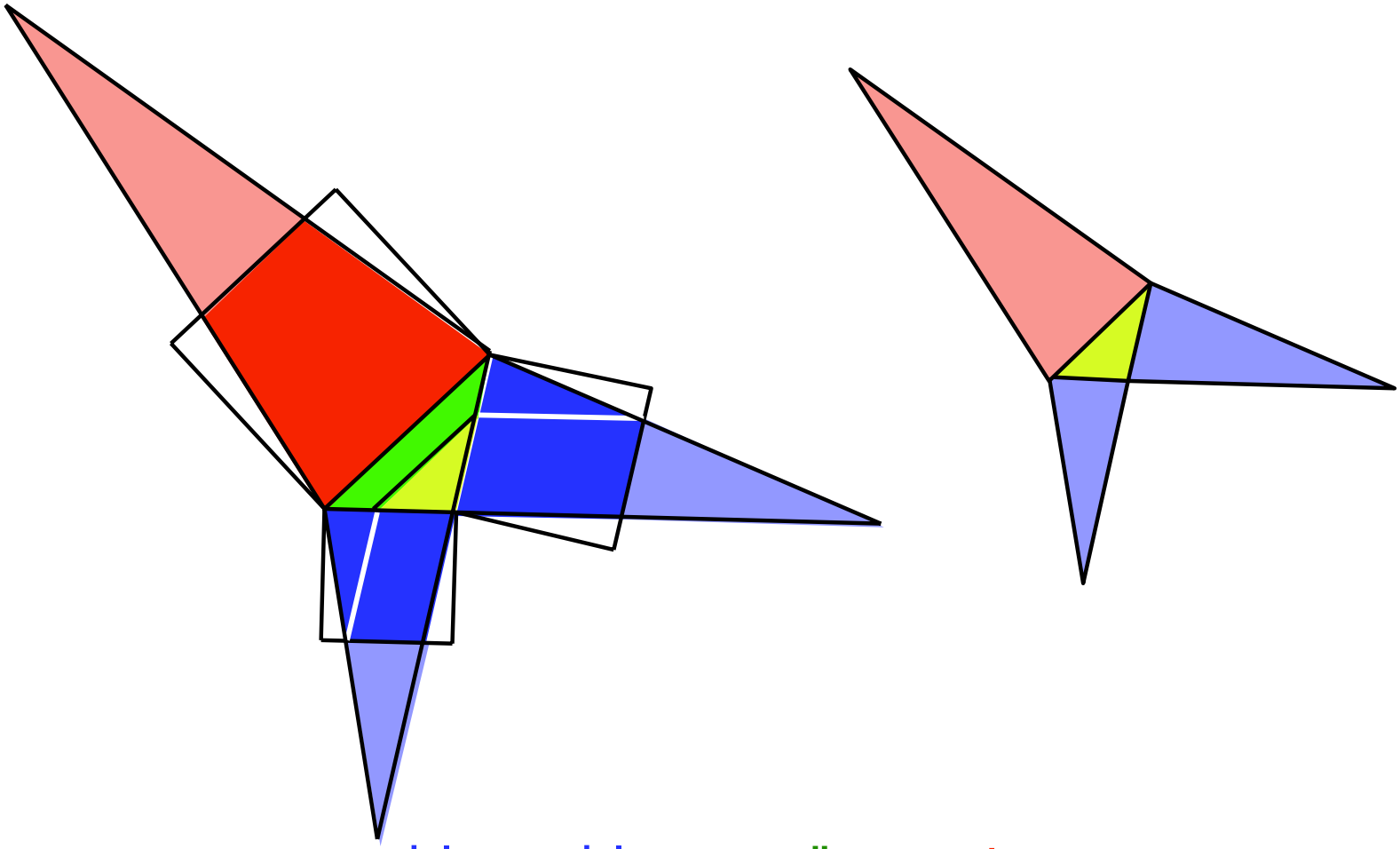
Link mit Pythagoras

Link mit Pythagoras



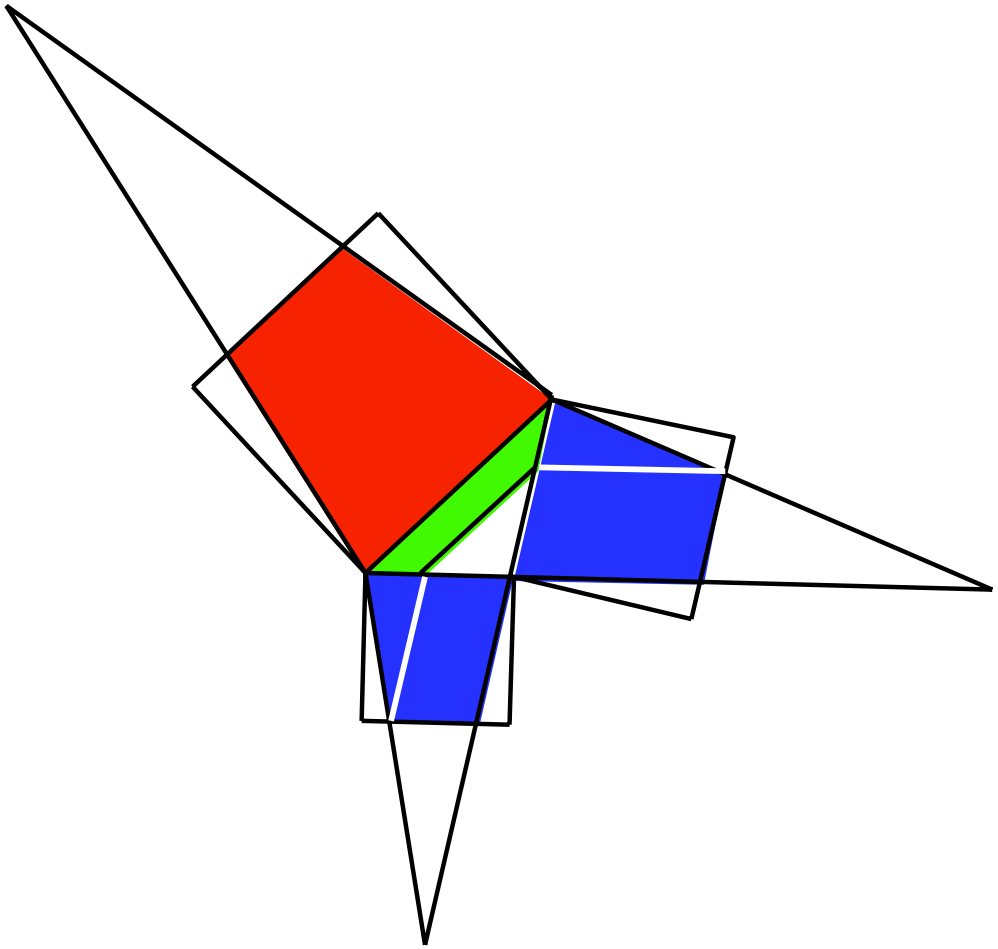
blau + blau + grün = rot

Link mit Pythagoras



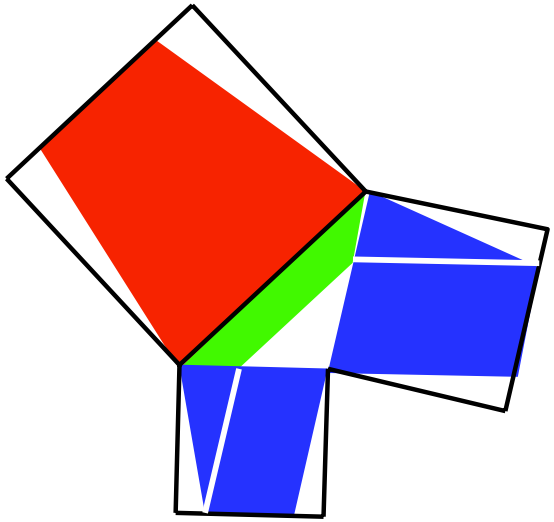
blau + blau + grün = rot

Link mit Pythagoras



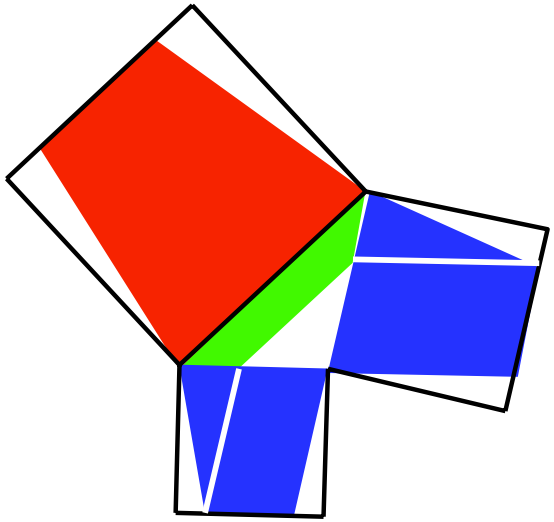
blau + blau + grün = rot

Link mit Pythagoras



blau + blau + grün = rot

Link mit Pythagoras



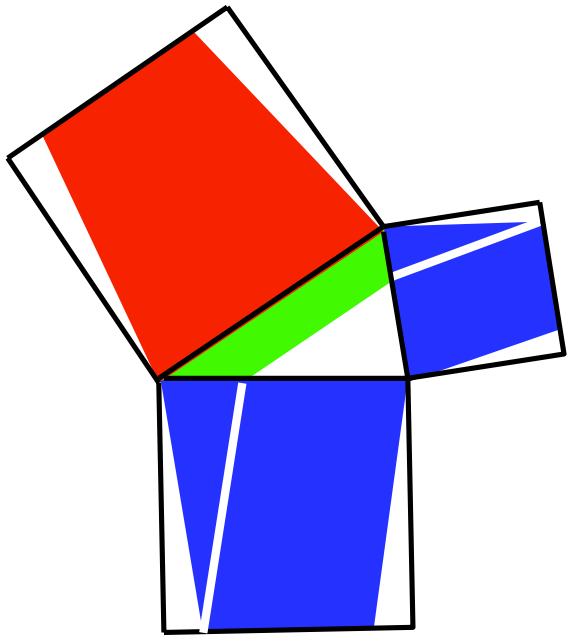
Was passiert bei

$\gamma \searrow 90°$?

blau + blau + grün = rot

Analog

Link mit Pythagoras

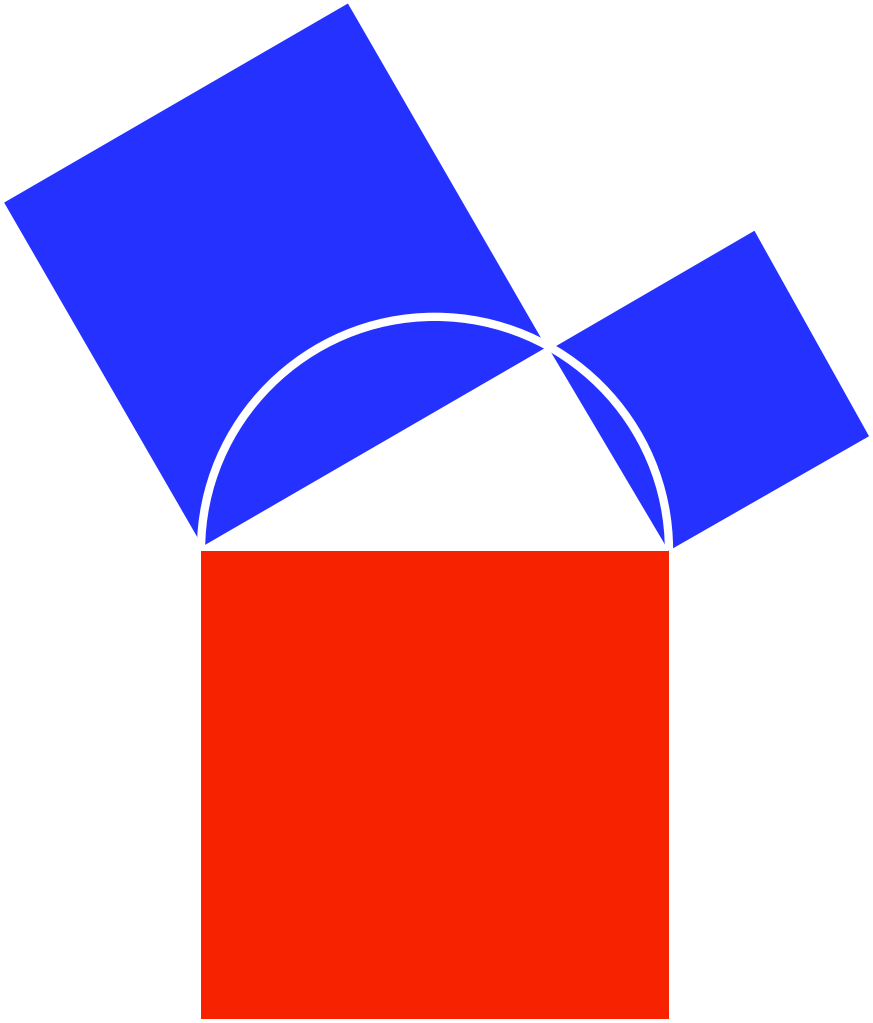


Was passiert bei

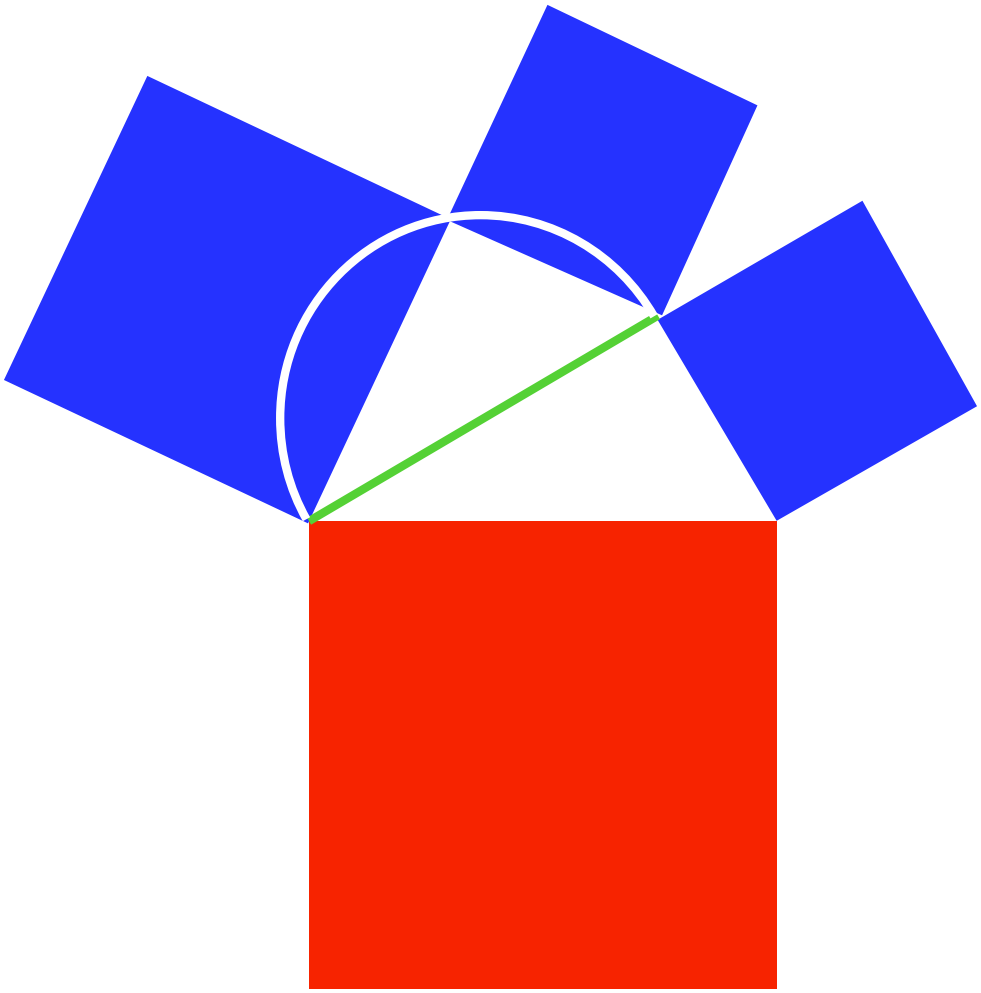
$\gamma \nearrow 90°$?

blau + blau = grün + rot

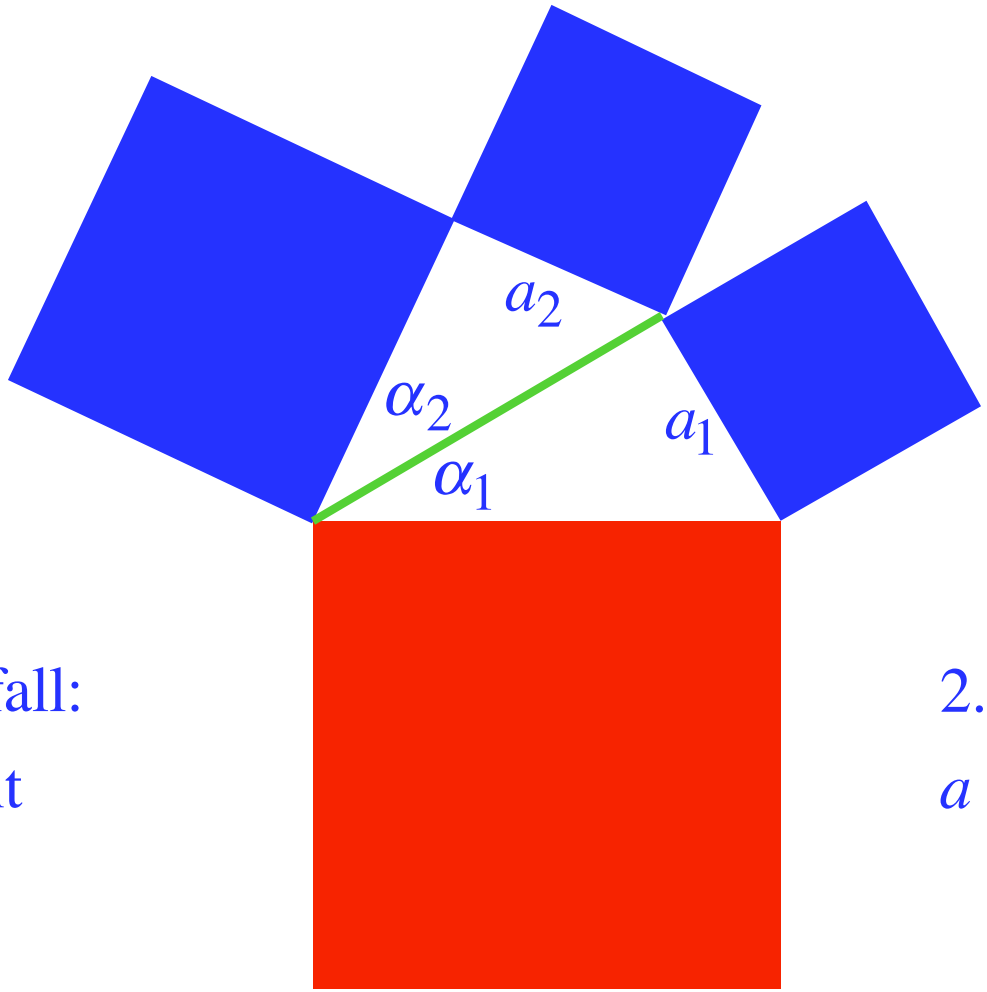
Vergleich mit Pythagoras



blau + blau = rot



blau + blau + blau = rot



1. Sonderfall:

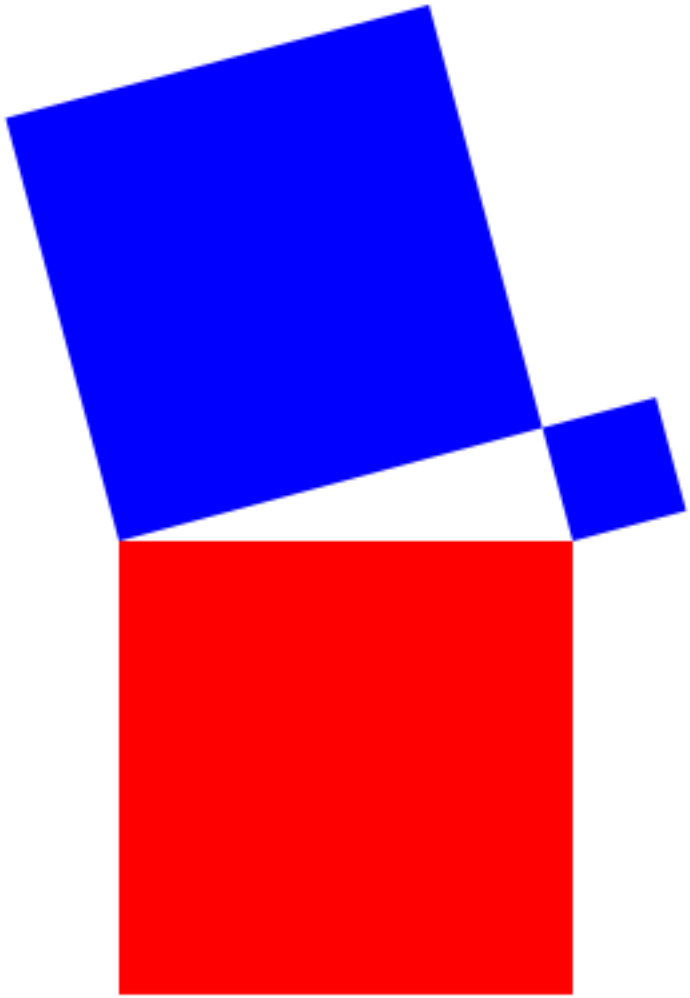
α konstant

2. Sonderfall:

a konstant

blau + blau + blau = rot

Beispiel: $\alpha = 15°$

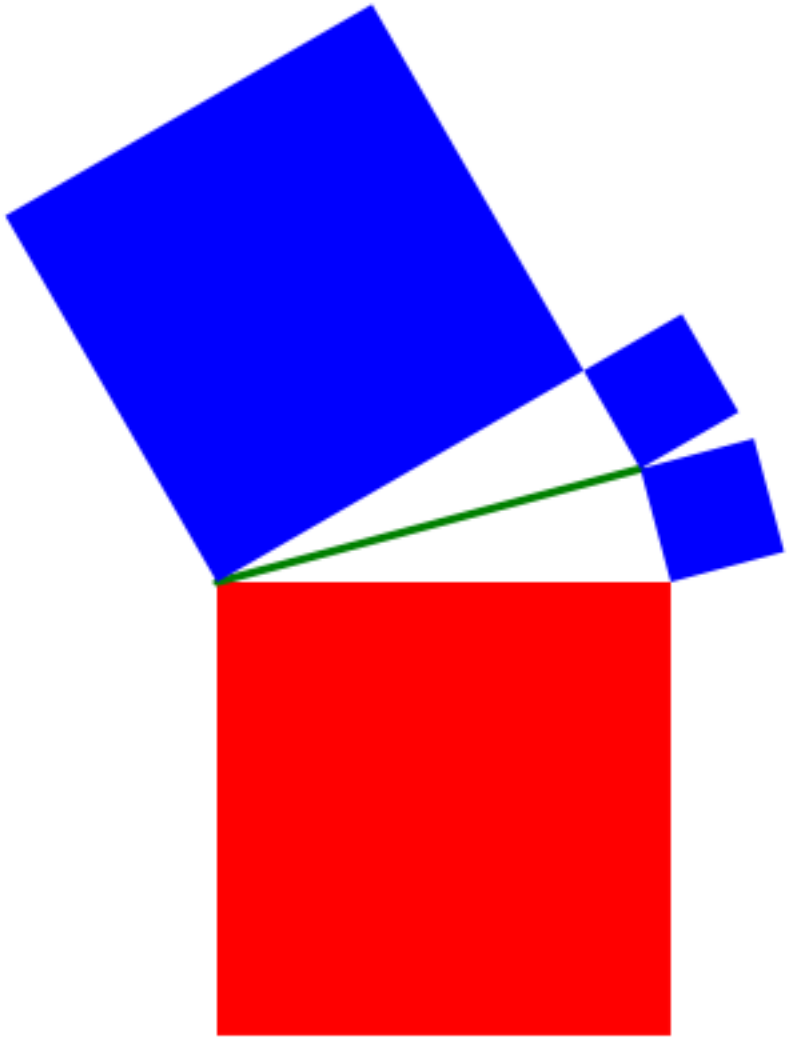


1. Sonderfall:

α konstant

blau + blau = rot

Beispiel: $\alpha = 15°$

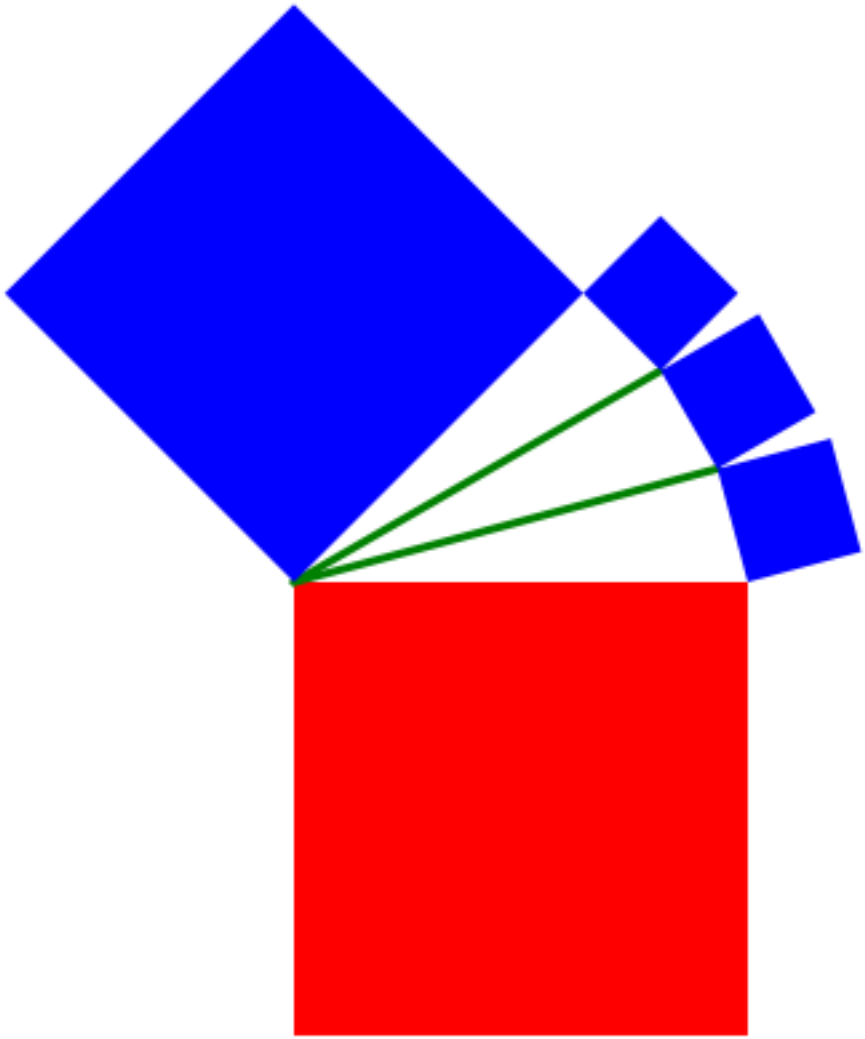


1. Sonderfall:

α konstant

blau + blau + blau = rot

Beispiel: $\alpha = 15°$

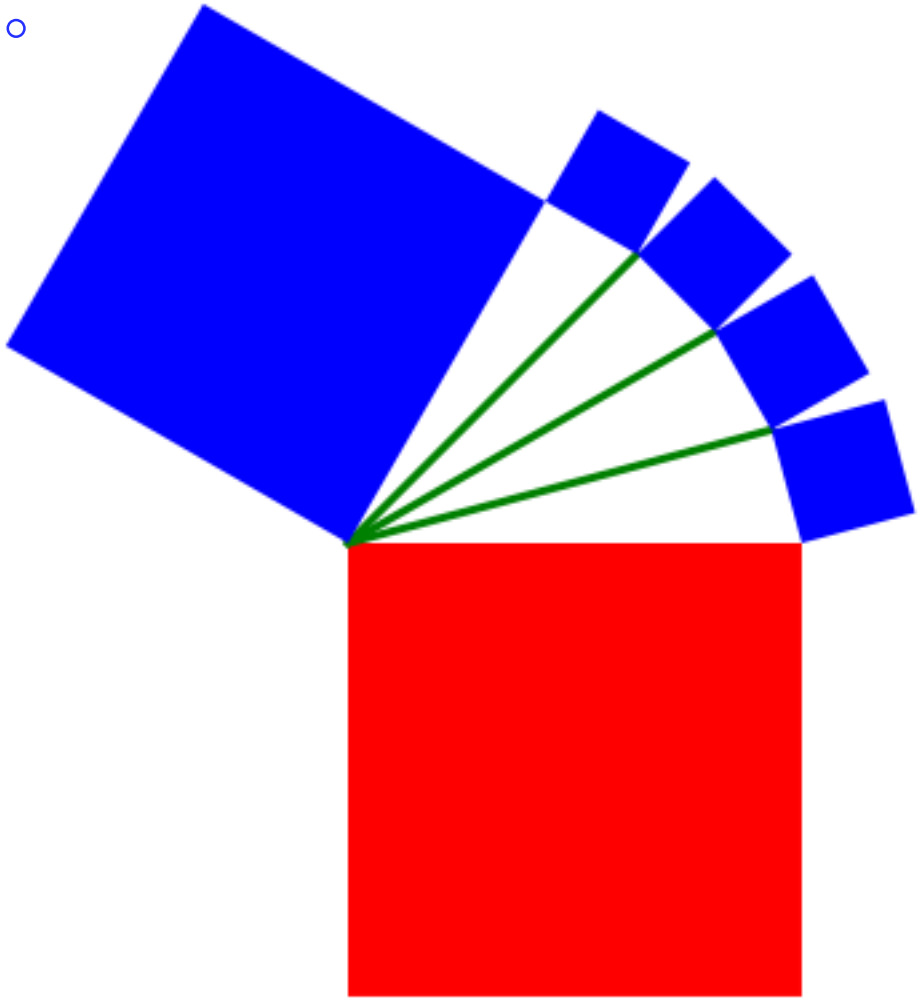


1. Sonderfall:

α konstant

blau + ⋯ + blau = rot

Beispiel: $\alpha = 15°$



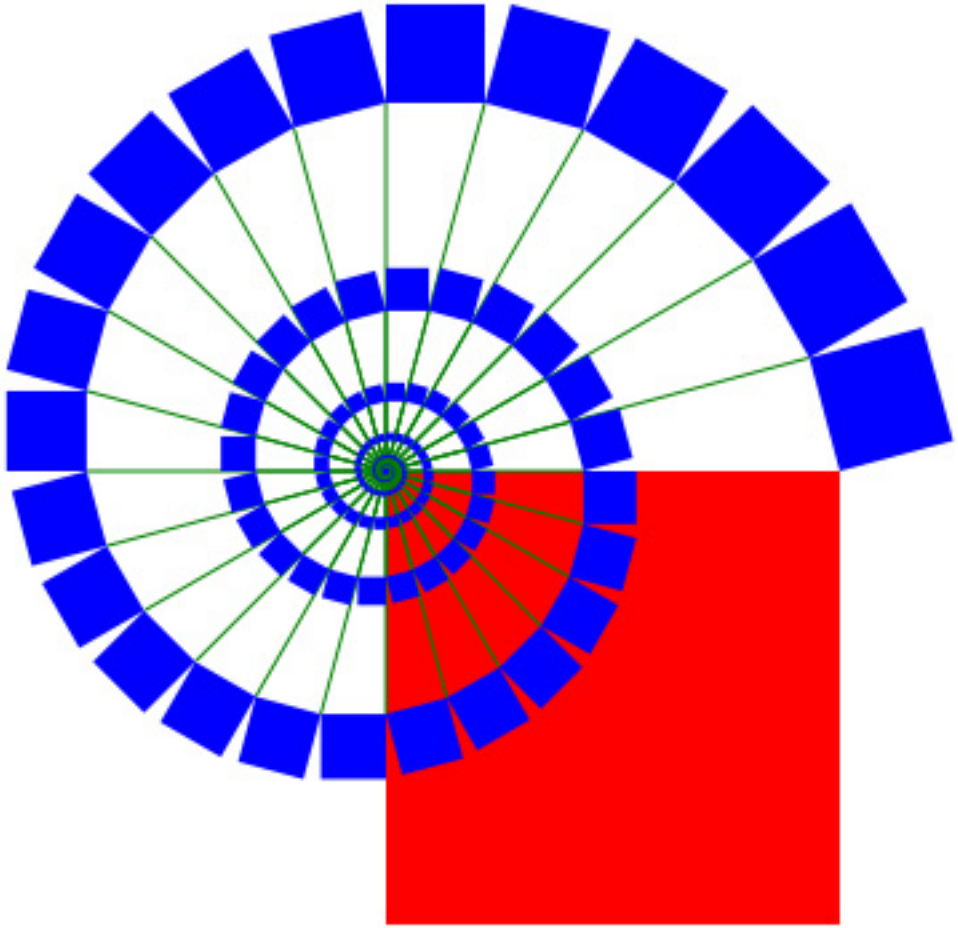
1. Sonderfall:

α konstant

blau + ⋯ + blau = rot

Logarithmische Spirale

Beispiel: $\alpha = 15°$



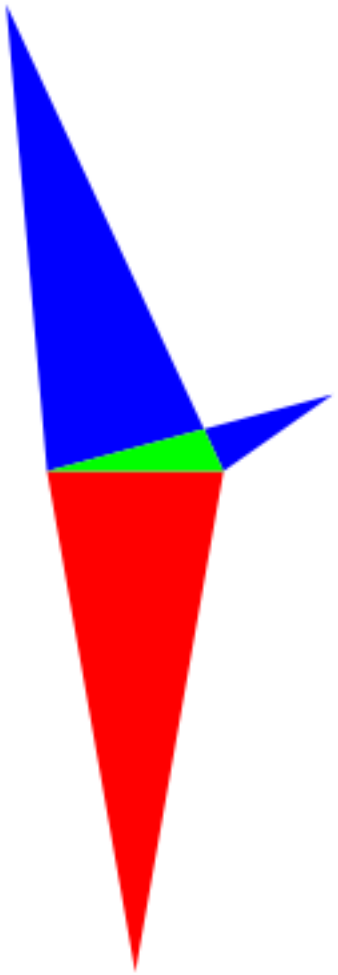
1. Sonderfall:

α konstant

blau + ⋯ + blau = rot

Stumpfer Winkel $\gamma = 100°$

Beispiel: $\alpha = 15°$



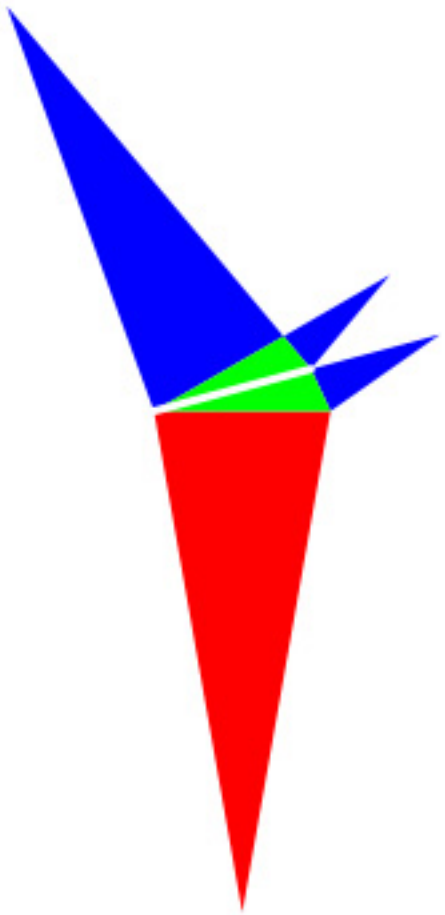
1. Sonderfall:

α konstant

blau + blau + grün = rot

Stumpfer Winkel $\gamma = 100°$

Beispiel: $\alpha = 15°$



1. Sonderfall:

α konstant

blau + ⋯ + blau + grün + grün = rot

Stumpfer Winkel $\gamma = 100°$

Beispiel: $\alpha = 15°$

1. Sonderfall:

α konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Stumpfer Winkel $\gamma = 100°$

Beispiel: $\alpha = 15°$

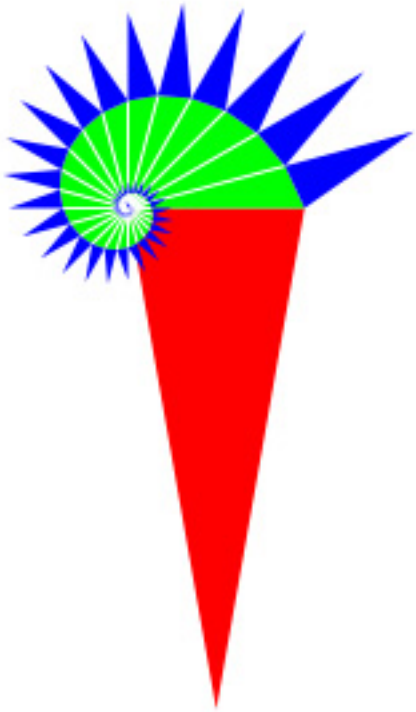
1. Sonderfall:

α konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Stumpfer Winkel $\gamma = 100°$ Logarithmische Spirale

Beispiel: $\alpha = 15°$



1. Sonderfall:

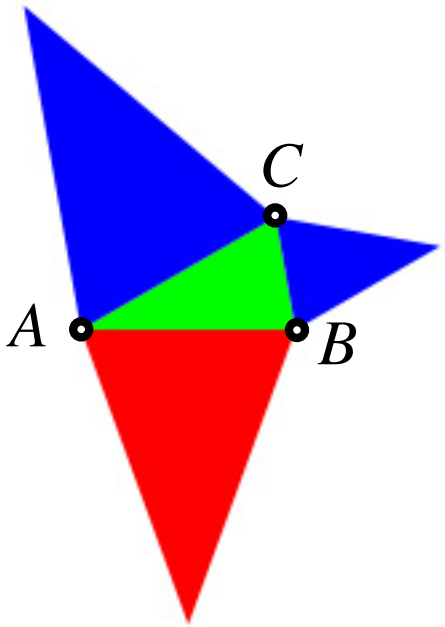
α konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



1. Sonderfall:

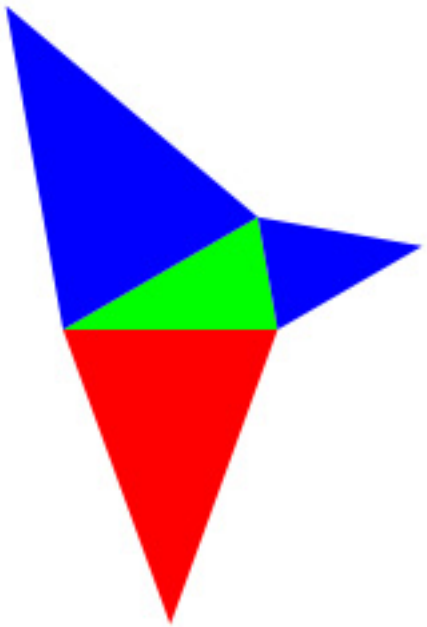
α konstant

blau + blau = grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



1. Sonderfall:

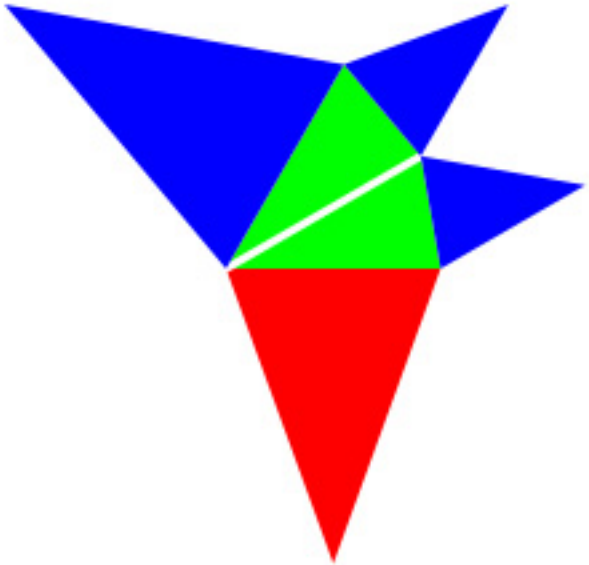
α konstant

blau + blau = grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



1. Sonderfall:

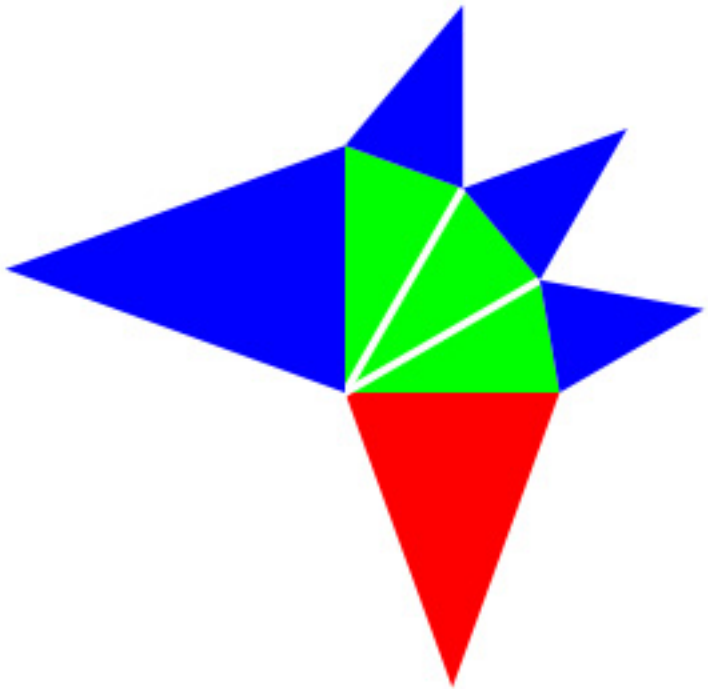
α konstant

blau + ⋯ + blau = grün + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



1. Sonderfall:

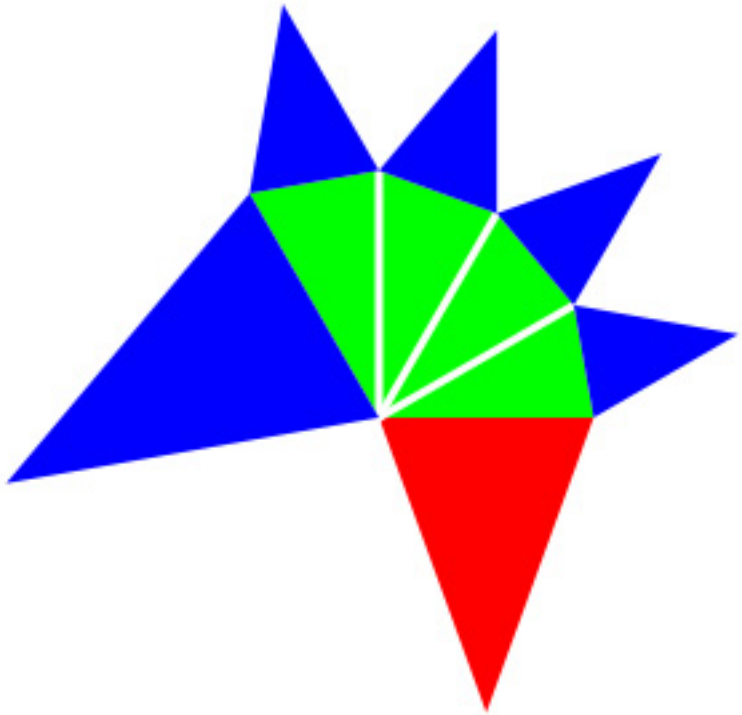
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$

1. Sonderfall:

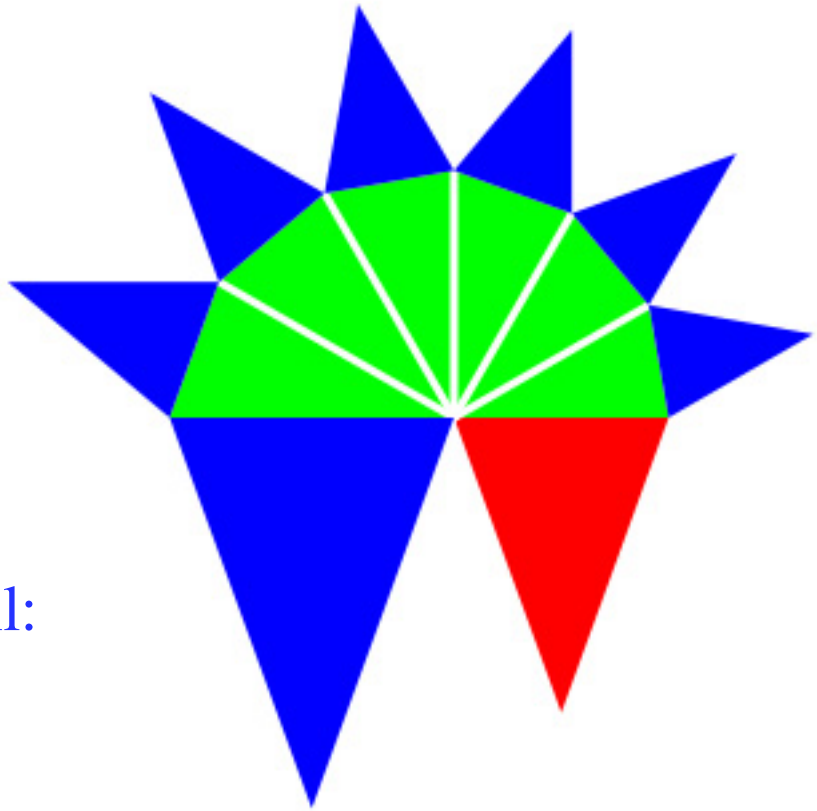
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$



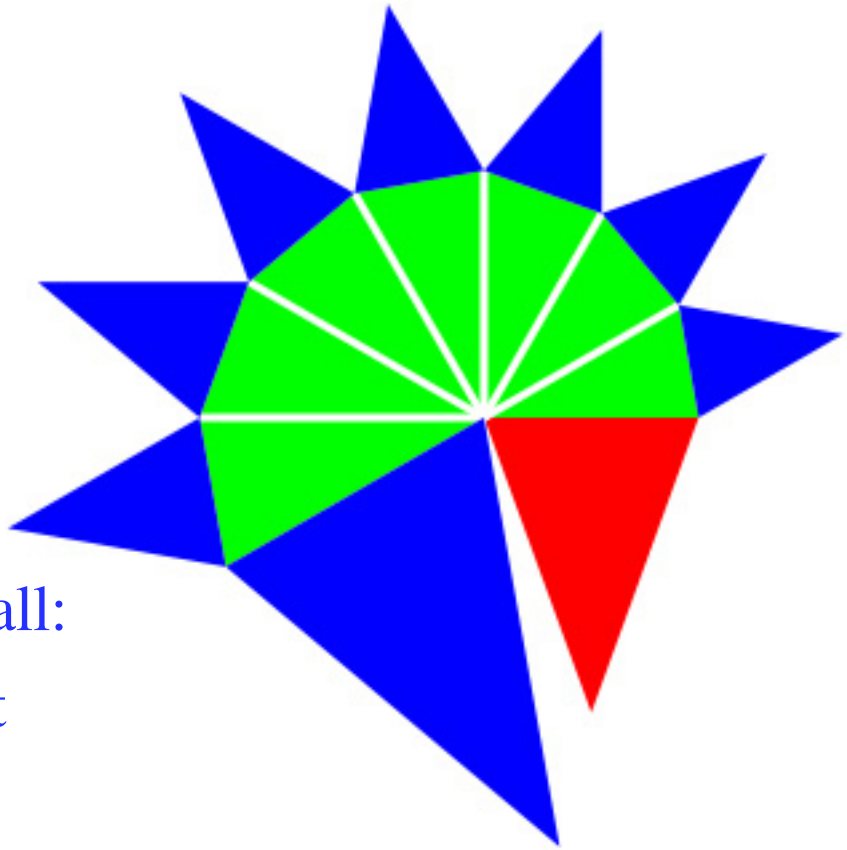
1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$ Spitzer Winkel γ

$\alpha = 30° < (360° - 2\gamma) = 40°$ Wachsende logarithmische Spirale



1. Sonderfall:

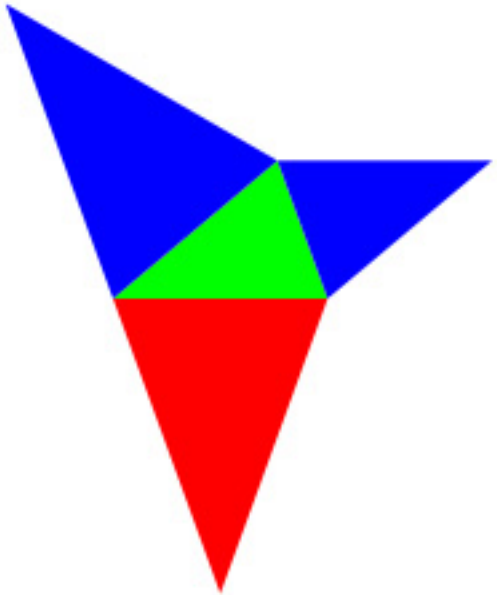
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$



1. Sonderfall:

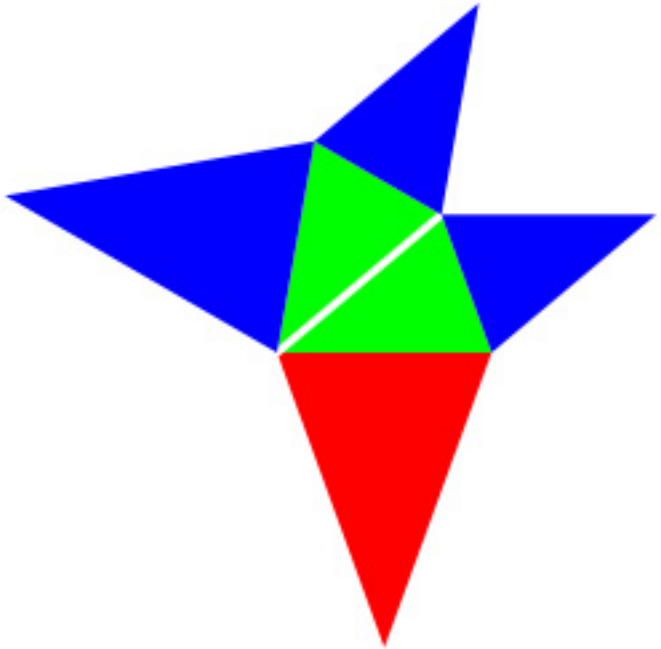
α konstant

blau + blau = grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$



1. Sonderfall:

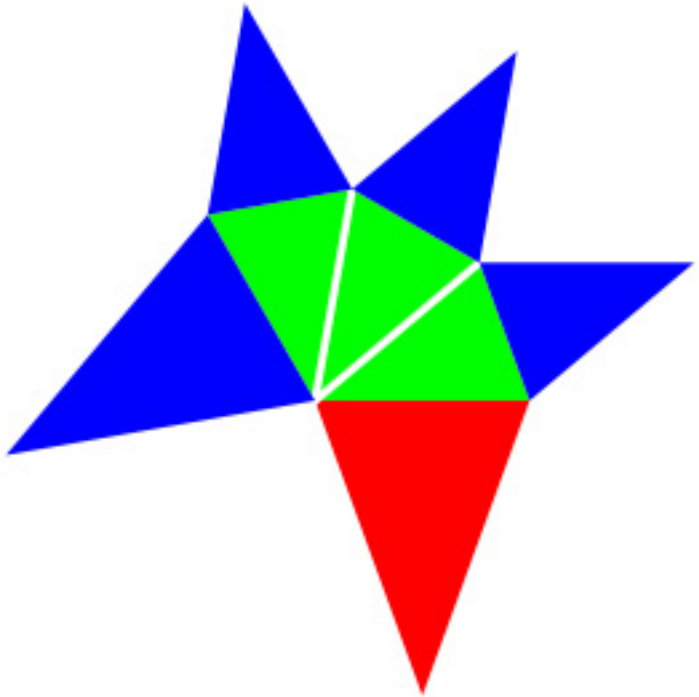
α konstant

blau + ⋯ + blau = grün + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$



1. Sonderfall:

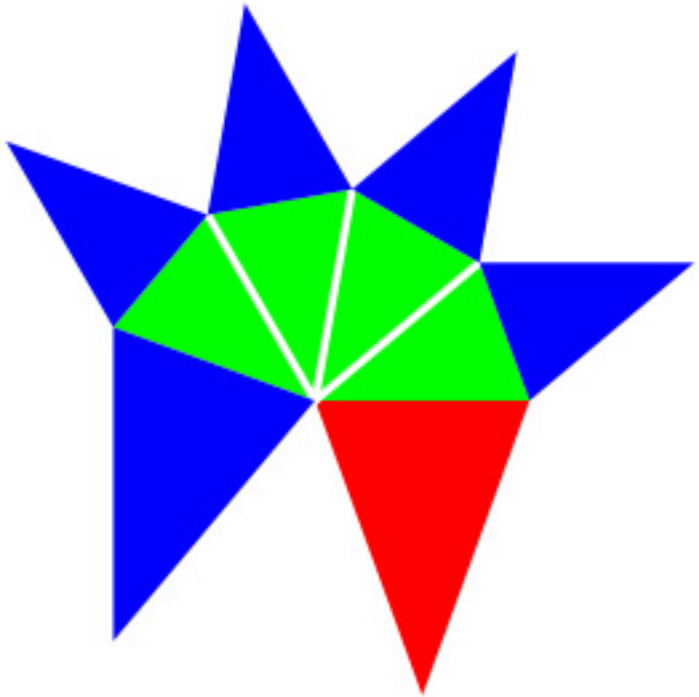
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$



1. Sonderfall:

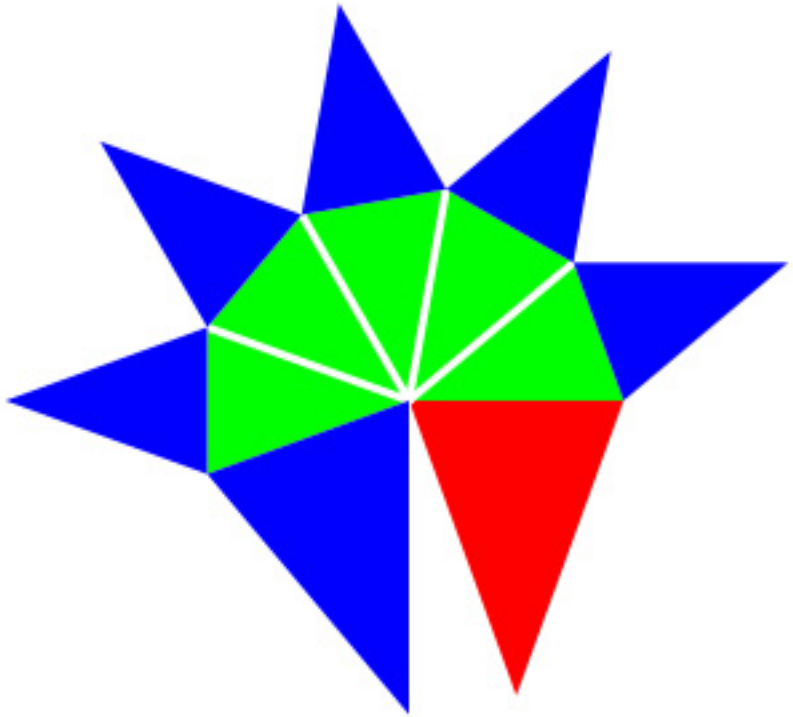
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$



1. Sonderfall:

α konstant

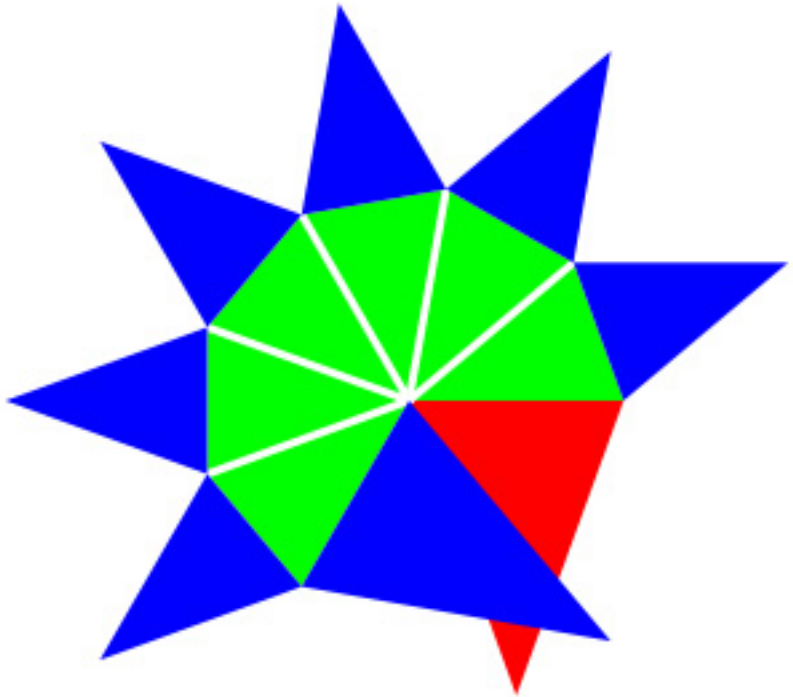
blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 40° = (360° - 2\gamma)$

„Kreis“, regelmäßiges Vieleck



1. Sonderfall:

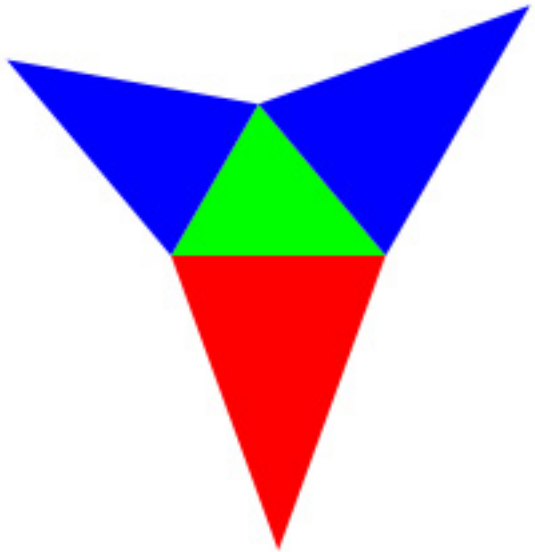
α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 60° > (360° - 2\gamma) = 40°$



1. Sonderfall:

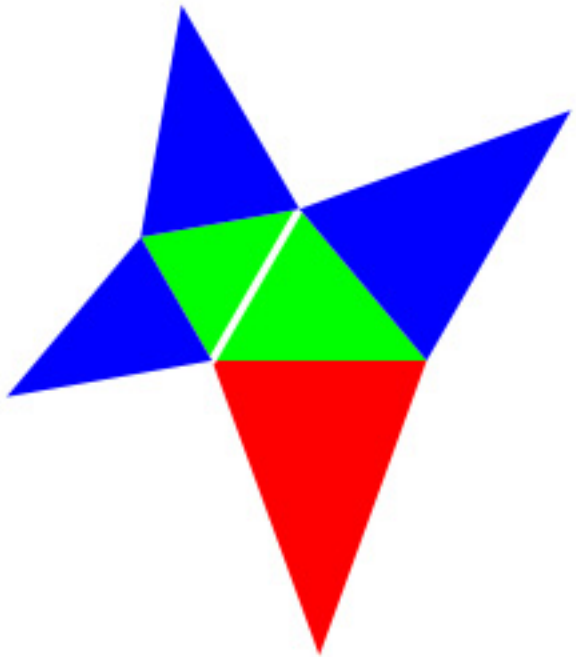
α konstant

blau + blau = grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 60° > (360° - 2\gamma) = 40°$



1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 60° > (360° - 2\gamma) = 40°$

1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$

Spitzer Winkel γ

$\alpha = 60° > (360° - 2\gamma) = 40°$

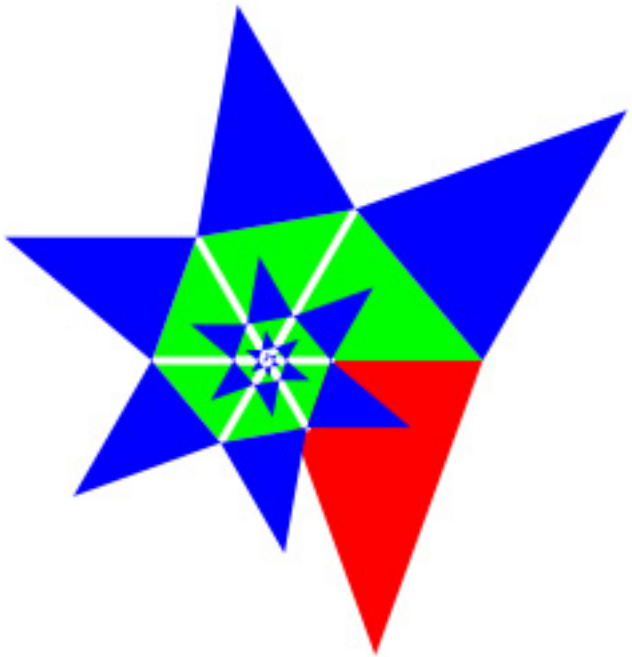
1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 70°$ Spitzer Winkel γ

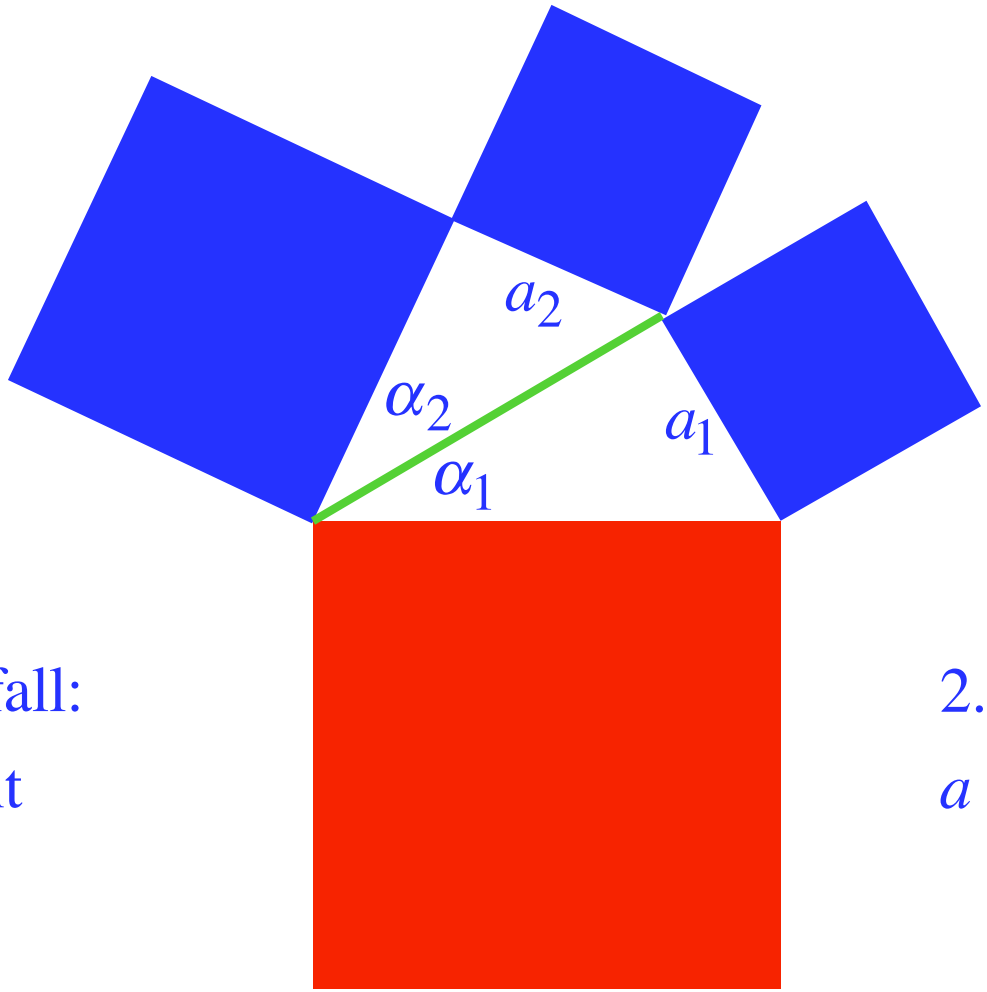
$\alpha = 60° > (360° - 2\gamma) = 40°$ Abnehmende logarithmische Spirale



1. Sonderfall:

α konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot



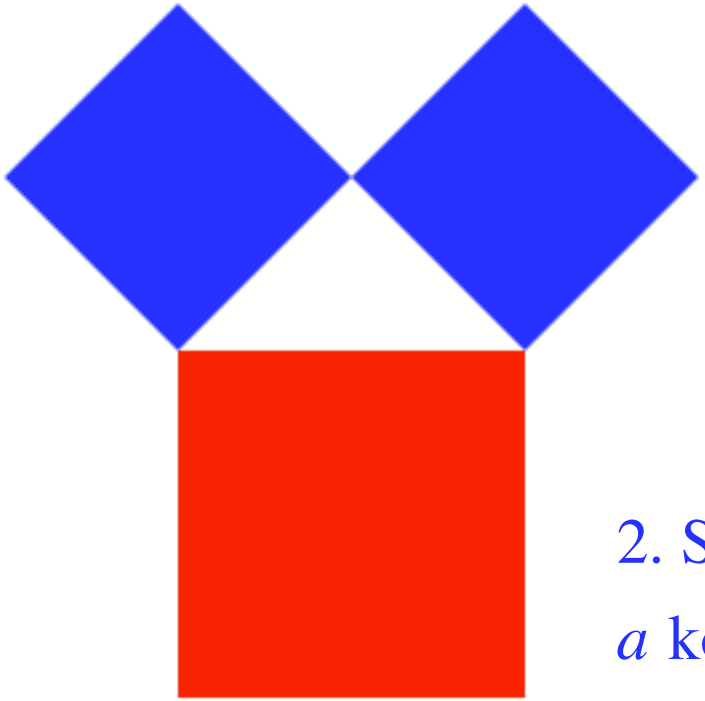
1. Sonderfall:

α konstant

2. Sonderfall:

a konstant

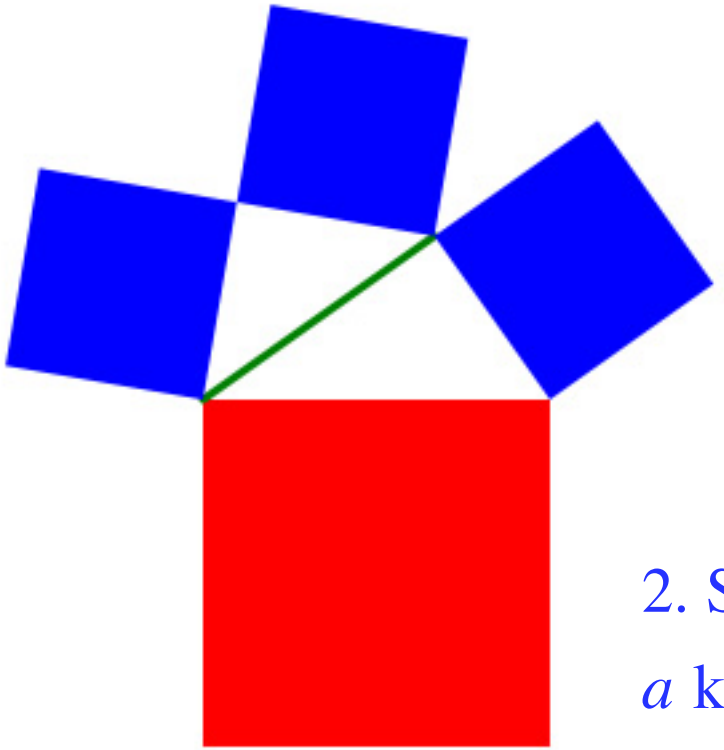
blau + blau + blau = rot



2. Sonderfall:

a konstant

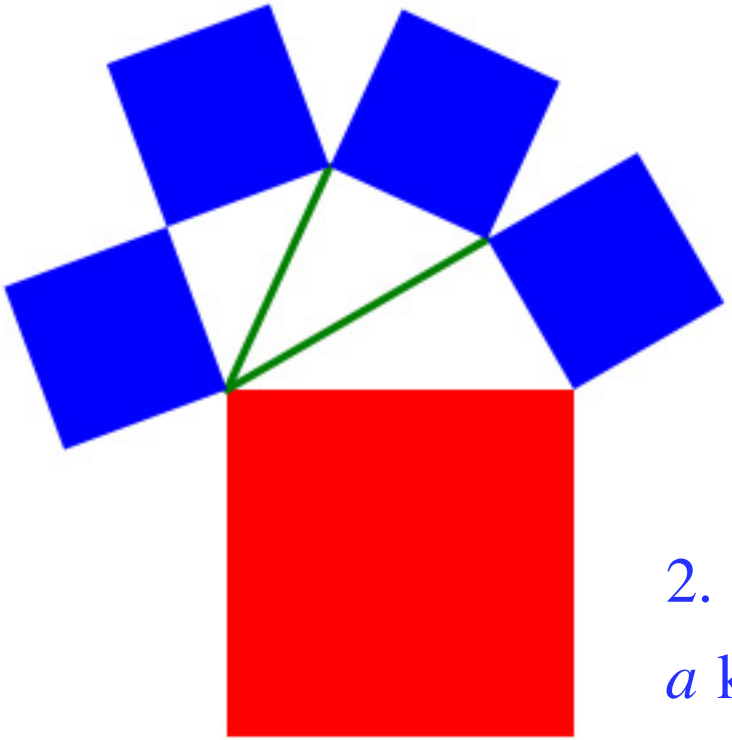
blau + blau = rot



2. Sonderfall:

a konstant

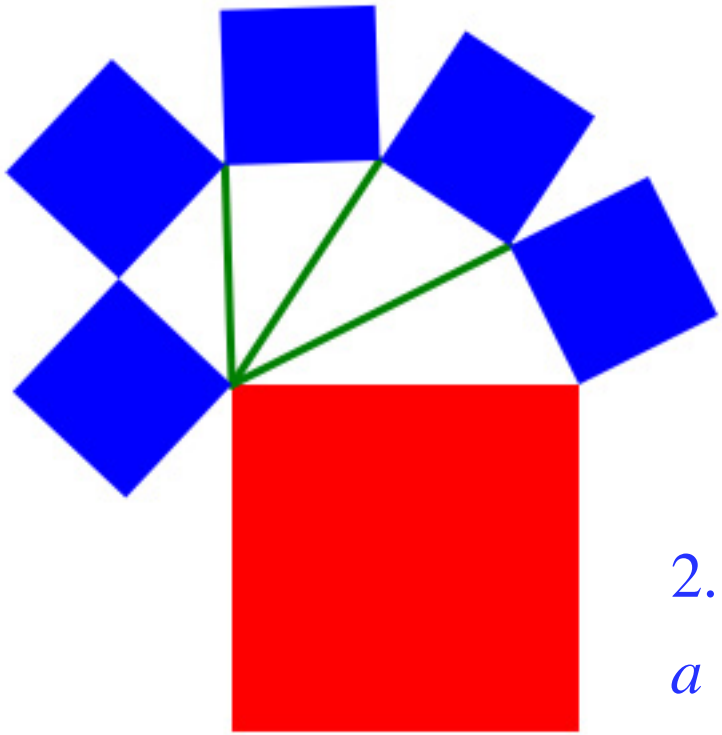
blau + ⋯ + blau = rot



2. Sonderfall:

a konstant

blau + ⋯ + blau = rot

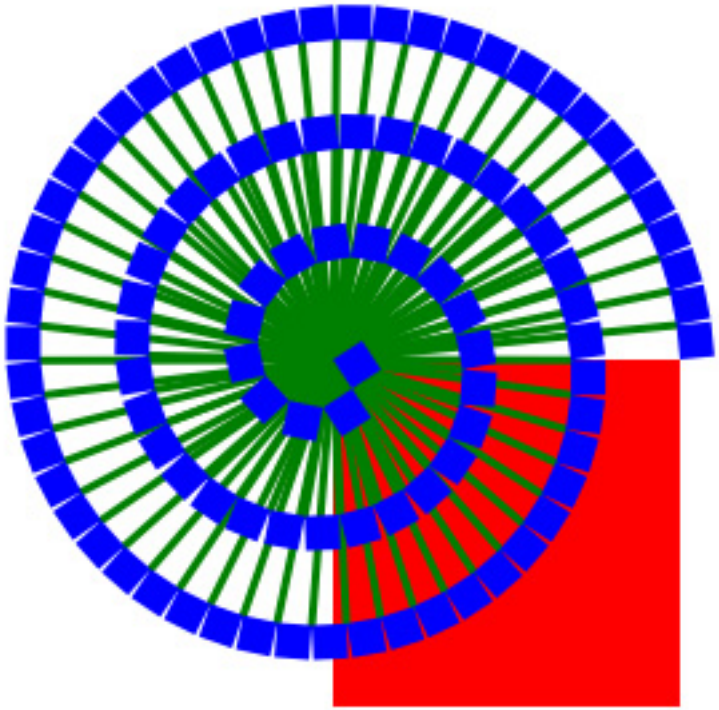


2. Sonderfall:

a konstant

blau + ⋯ + blau = rot

Archimedische Spirale

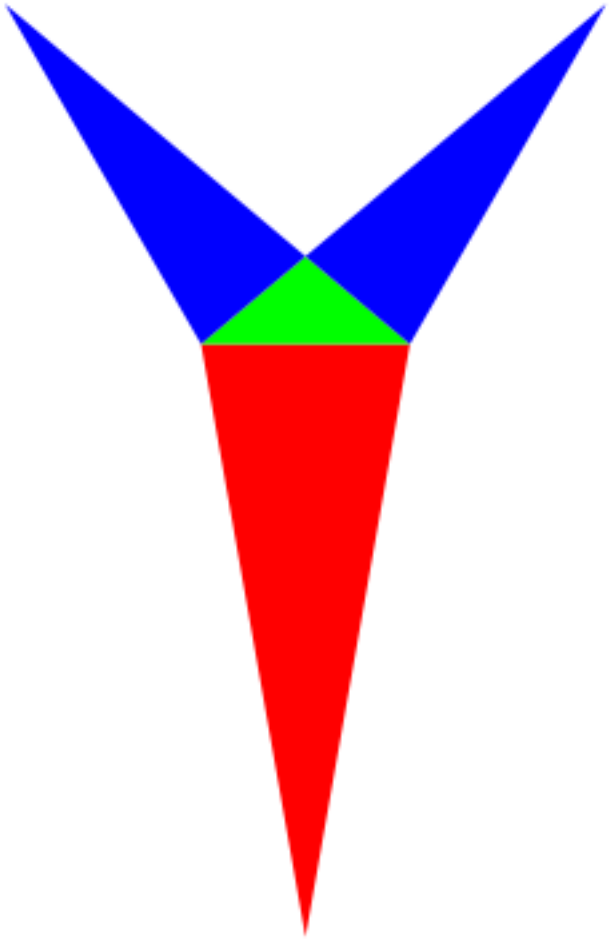


2. Sonderfall:

a konstant

blau + ⋯ + blau = rot

Stumpfer Winkel $\gamma = 100°$

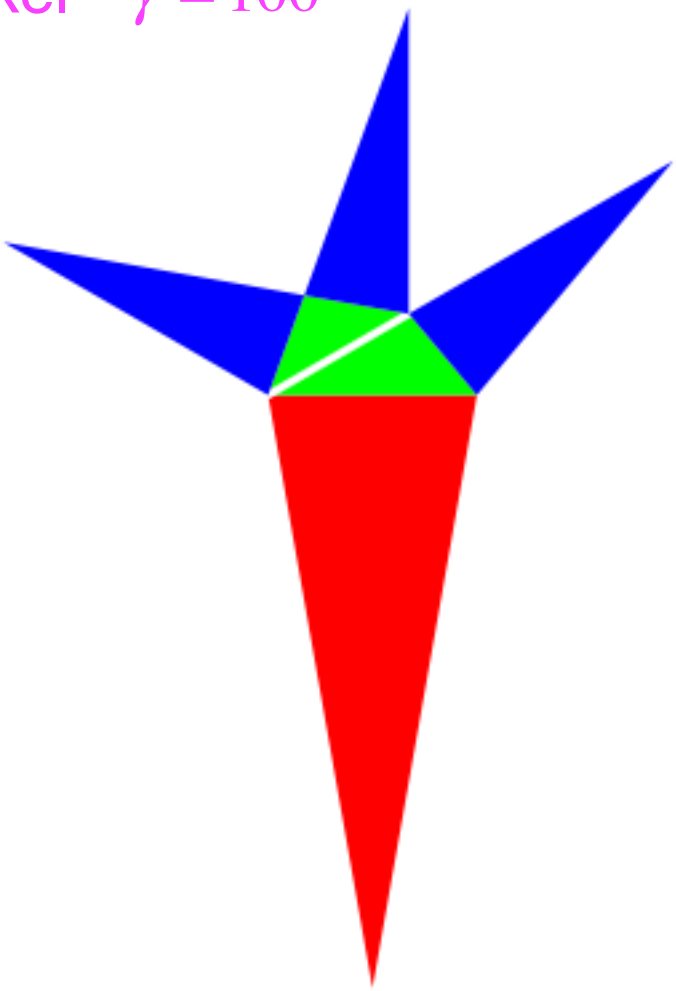


2. Sonderfall:

a konstant

blau + blau + grün = rot

Stumpfer Winkel $\gamma = 100°$

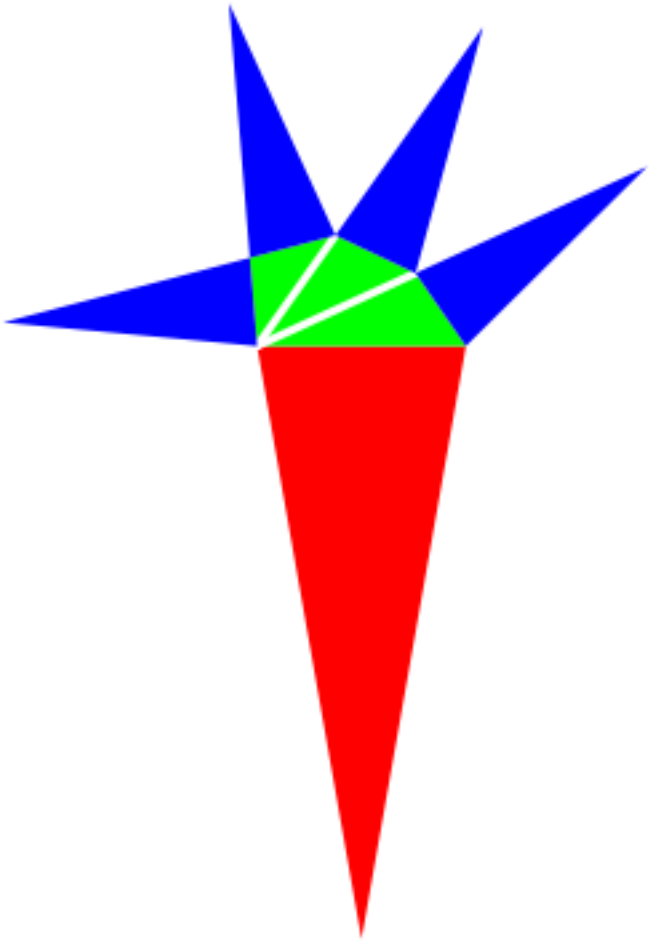


2. Sonderfall:

a konstant

blau + ⋯ + blau + grün + grün = rot

Stumpfer Winkel $\gamma = 100°$

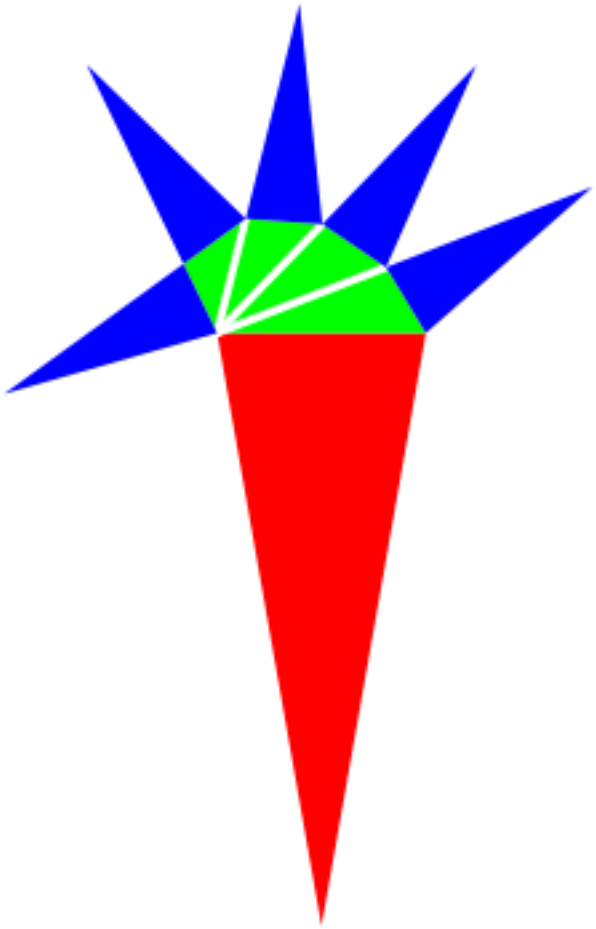


2. Sonderfall:

a konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Stumpfer Winkel $\gamma = 100°$



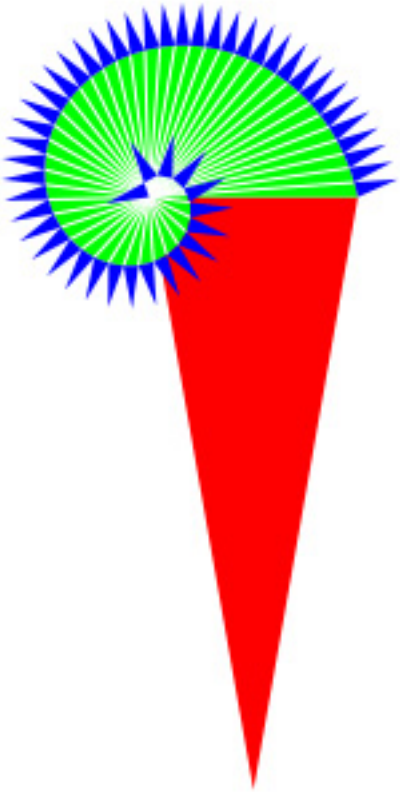
2. Sonderfall:

a konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Stumpfer Winkel $\gamma = 100°$

50 grüne Dreiecke



2. Sonderfall:

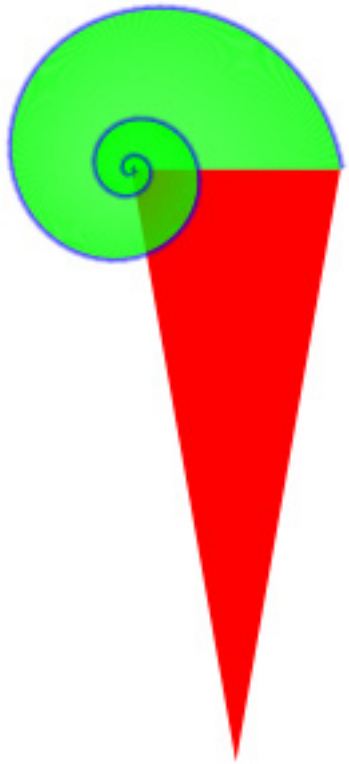
a konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Stumpfer Winkel $\gamma = 100°$

500 grüne Dreiecke

Logarithmische Spirale



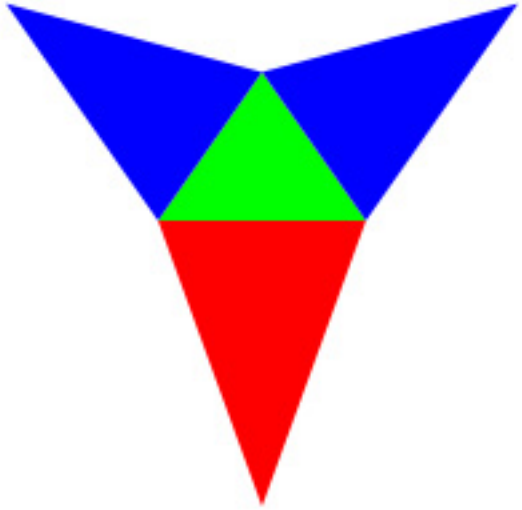
2. Sonderfall:

a konstant

blau + ⋯ + blau + grün + ⋯ + grün = rot

Spitzer Winkel $\gamma = 70°$

1 grünes Dreieck



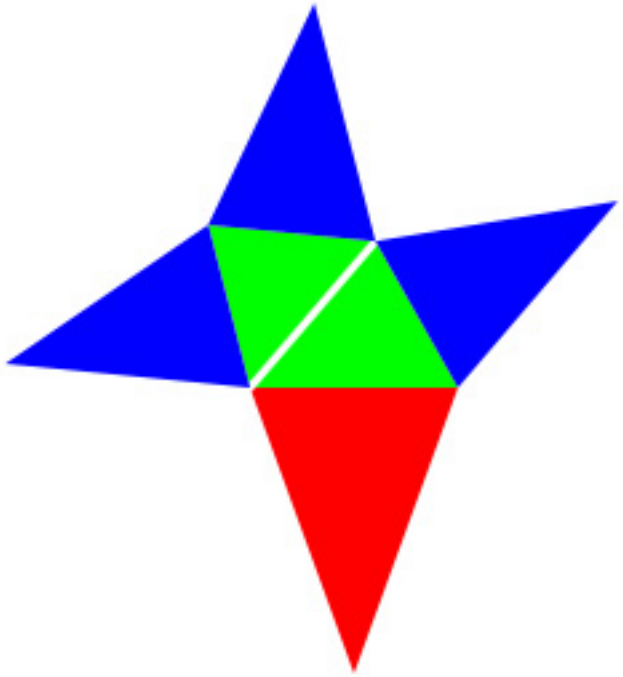
2. Sonderfall:

a konstant

blau + blau = grün + rot

Spitzer Winkel $\gamma = 70°$

2 grüne Dreiecke



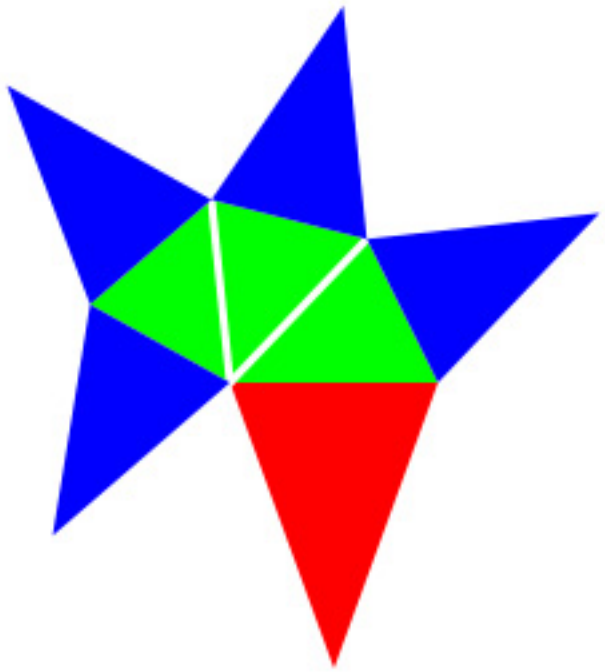
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + grün + rot

Spitzer Winkel $\gamma = 70°$

3 grüne Dreiecke



2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

4 grüne Dreiecke

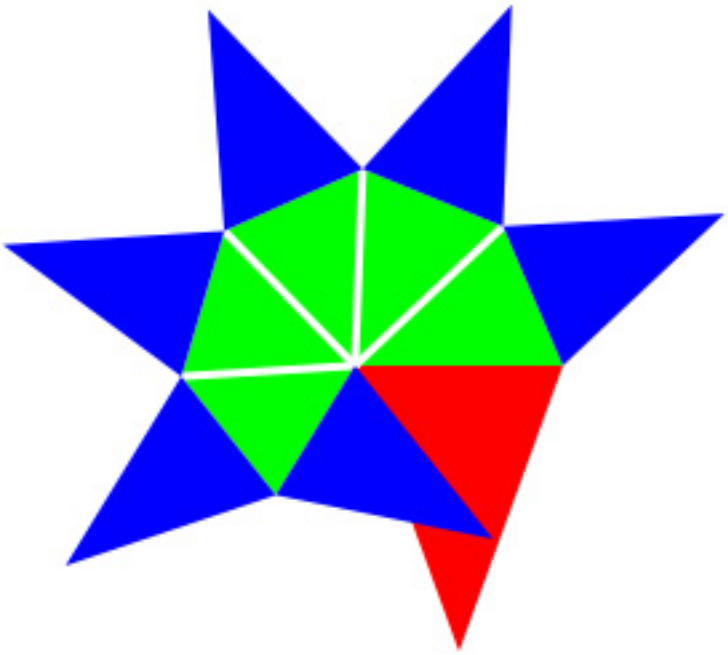
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

5 grüne Dreiecke



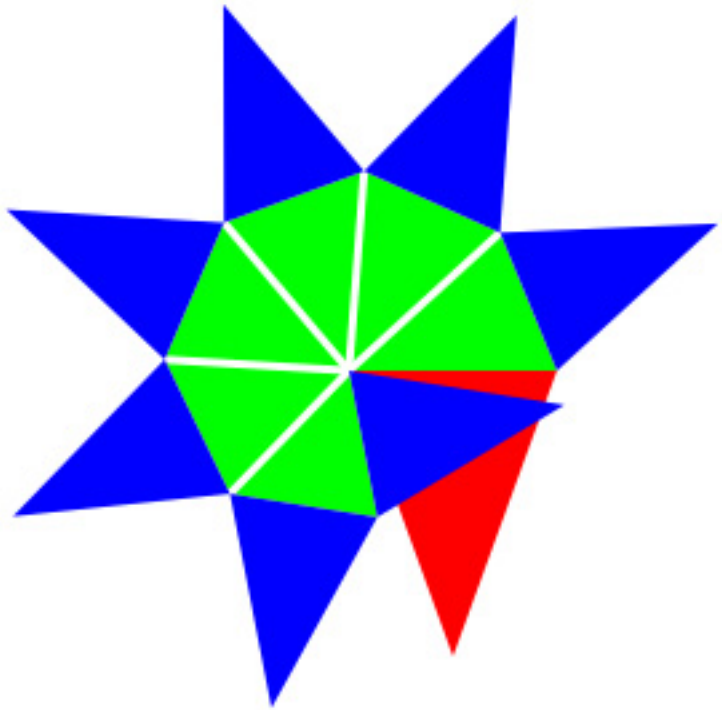
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

6 grüne Dreiecke



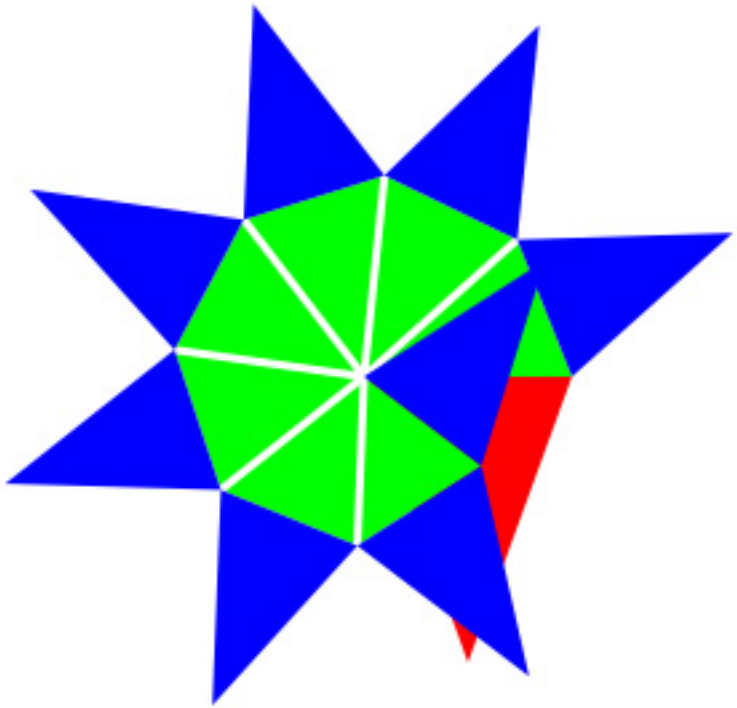
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

7 grüne Dreiecke



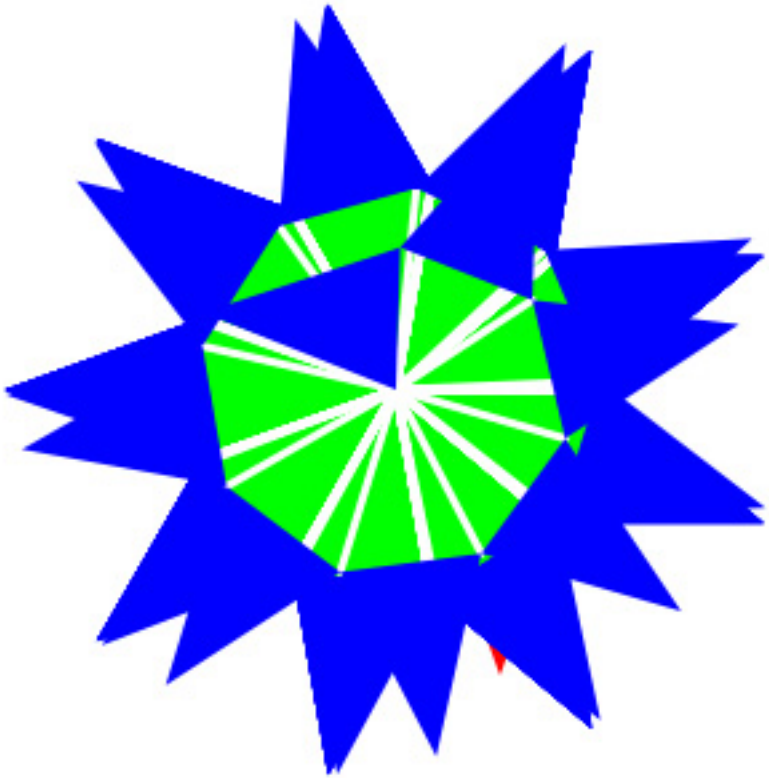
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

100 grüne Dreiecke



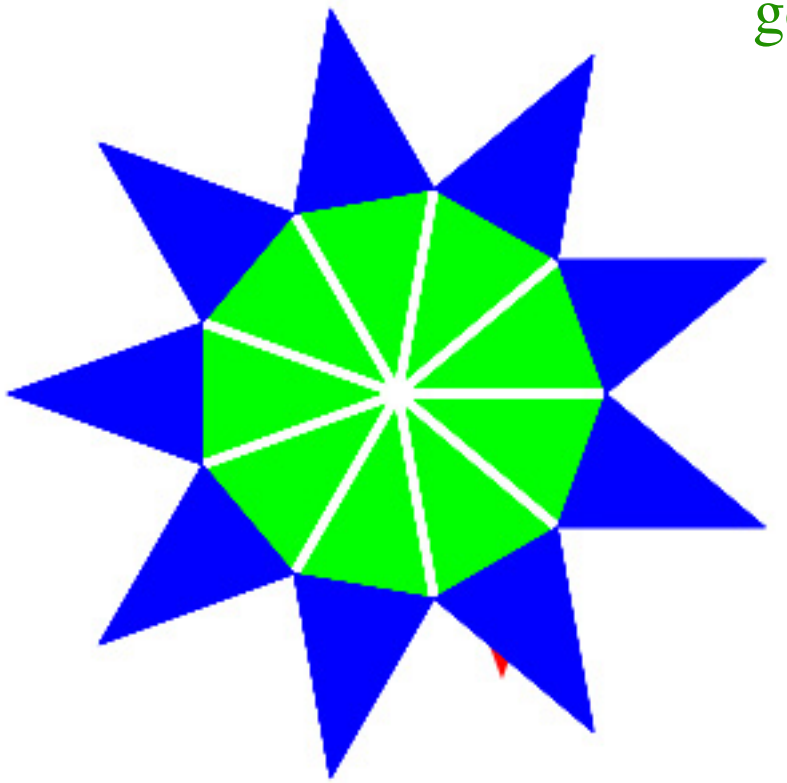
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

Spitzer Winkel $\gamma = 70°$

100 grüne Dreiecke
aber nur
Nr. 21 bis 100
gezeichnet



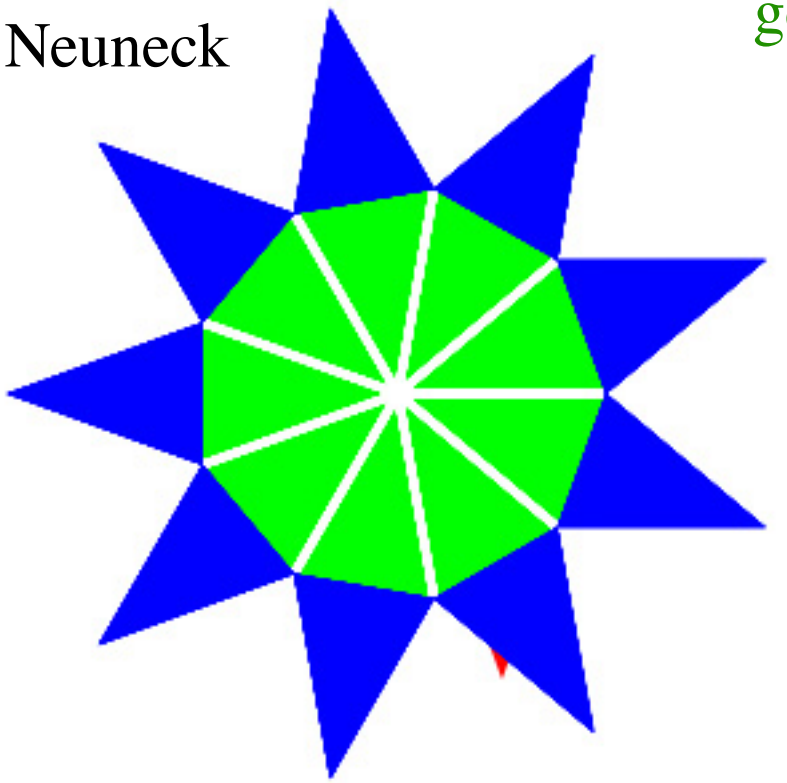
2. Sonderfall:

a konstant

Spitzer Winkel $\gamma = 70°$

$2\gamma = 140° =$ Innenwinkel

im regelmäßigen Neuneck



100 grüne Dreiecke
aber nur
Nr. 21 bis 100
gezeichnet

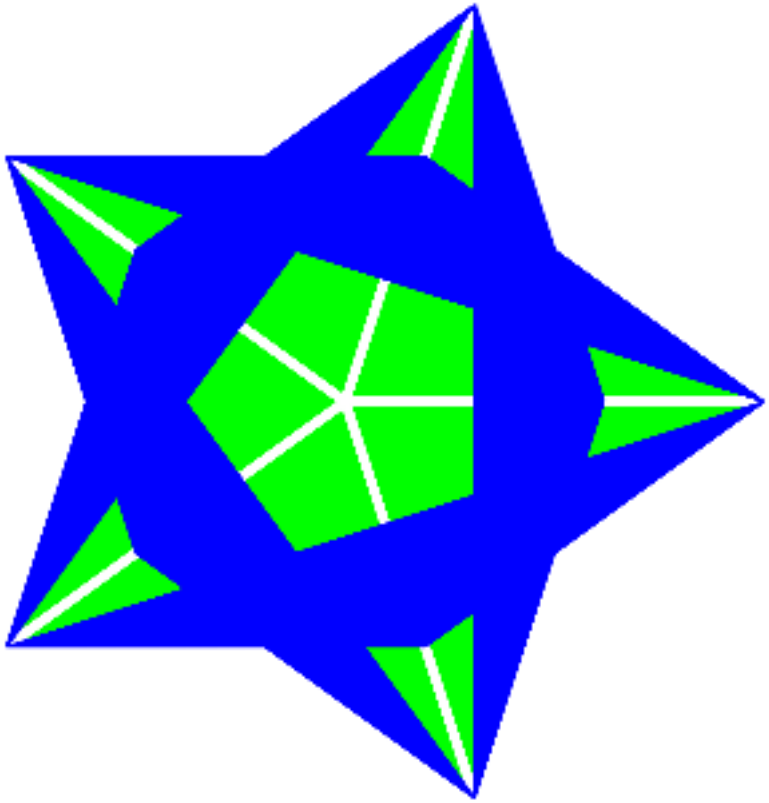
2. Sonderfall:

a konstant

Spitzer Winkel $\gamma = 18°$

$2\gamma = 36° =$ Innenwinkel

im Pentagramm



100 grüne Dreiecke
aber nur
Nr. 21 bis 100
zeichnet

2. Sonderfall:

a konstant

Spitzer Winkel $\gamma = 70°$

$\gamma = 70° \approx 1.2217304764$

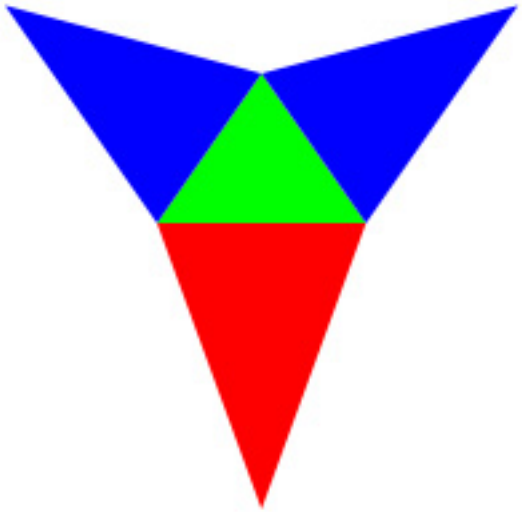
Neu: $\gamma = 1.22$, irrational zu 2π

2. Sonderfall:

a konstant

$\gamma = 1.22$, irrational zu 2π

1 grünes Dreieck



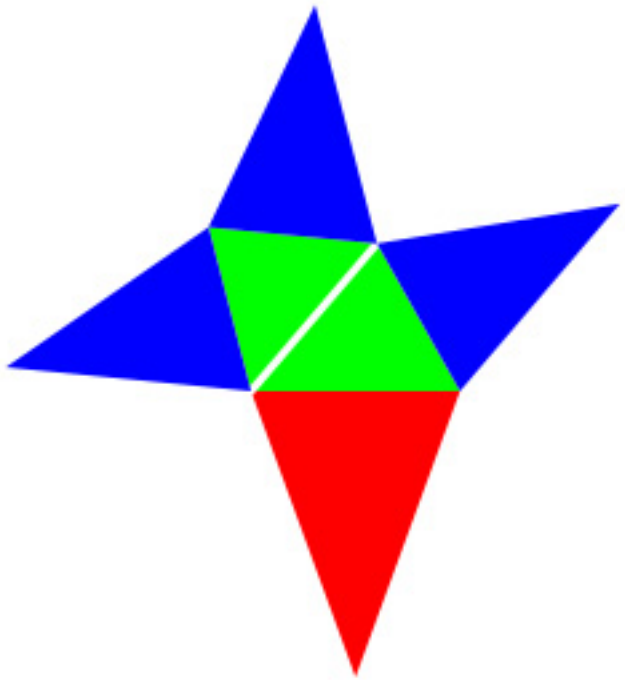
2. Sonderfall:

a konstant

blau + blau = grün + rot

$\gamma = 1.22$, irrational zu 2π

2 grüne Dreiecke



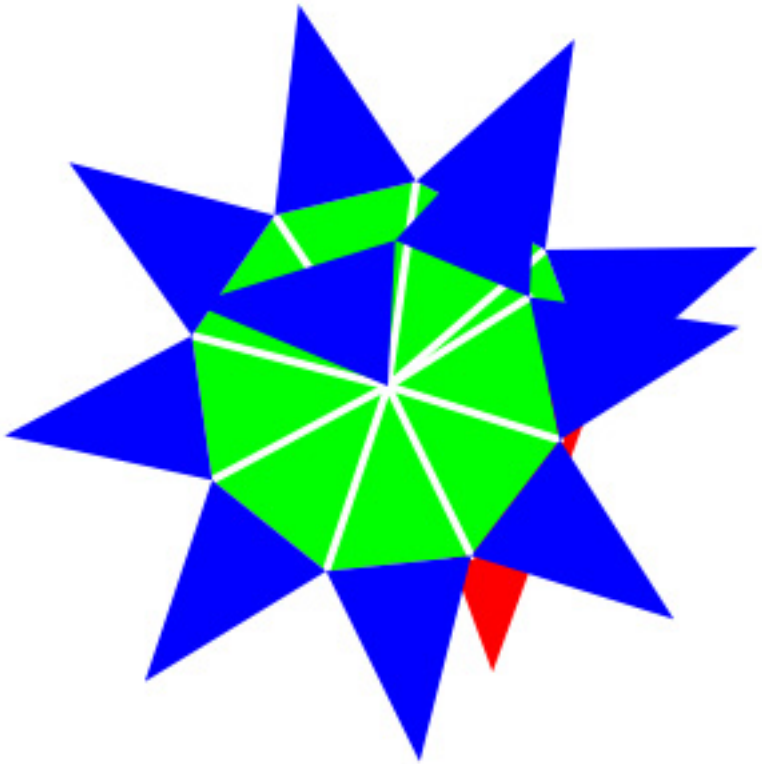
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + grün + rot

$\gamma = 1.22$, irrational zu 2π

10 grüne Dreiecke



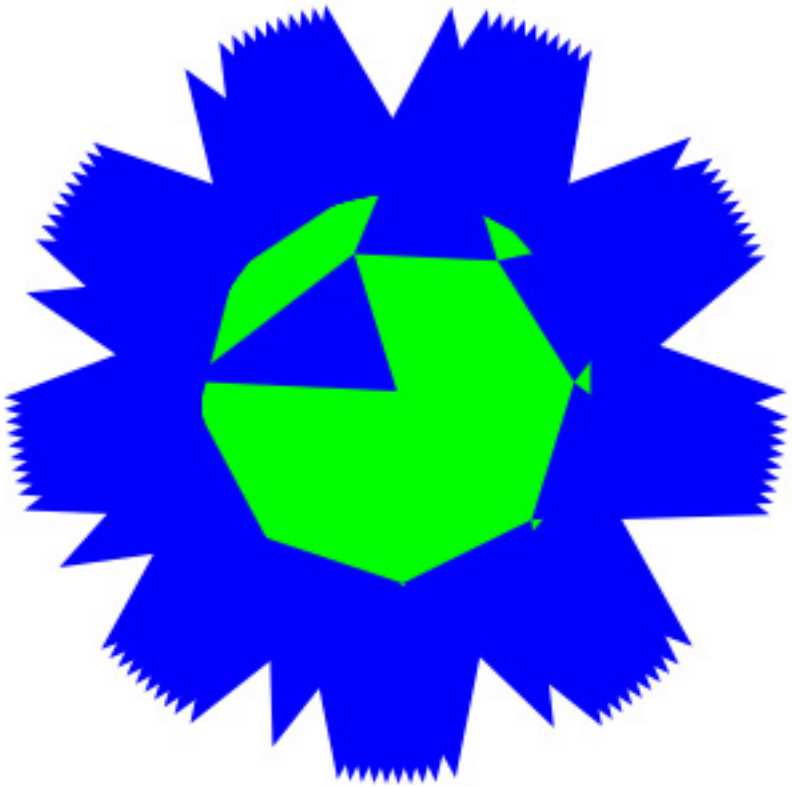
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 1.22$, irrational zu 2π

100 grüne Dreiecke



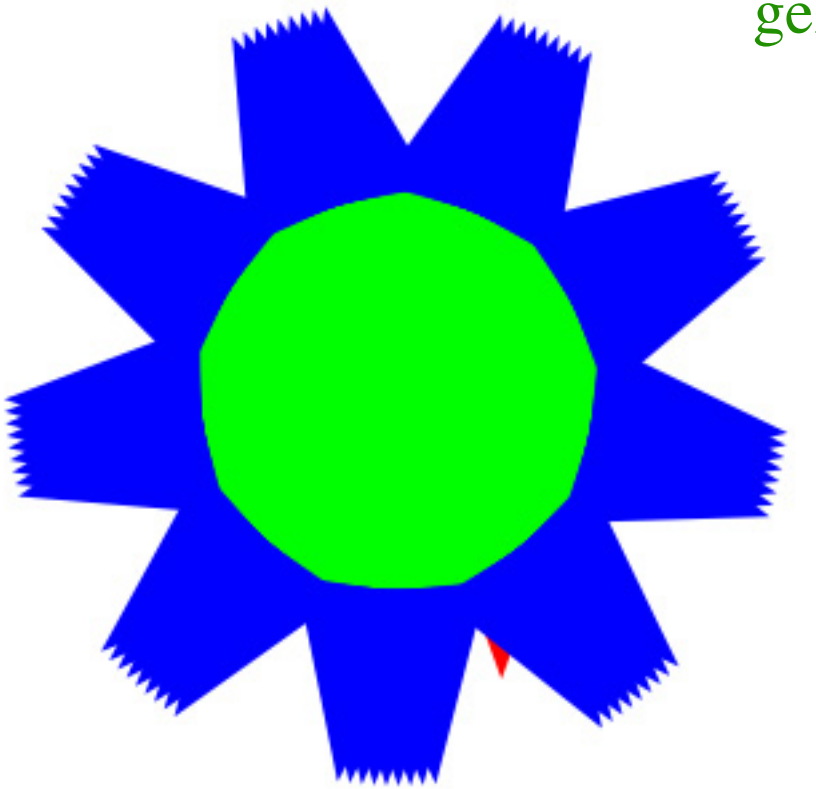
2. Sonderfall:

a konstant

blau + ⋯ + blau = grün + ⋯ + grün + rot

$\gamma = 1.22$, irrational zu 2π

100 grüne Dreiecke
aber nur
Nr. 21 bis 100
gezeichnet

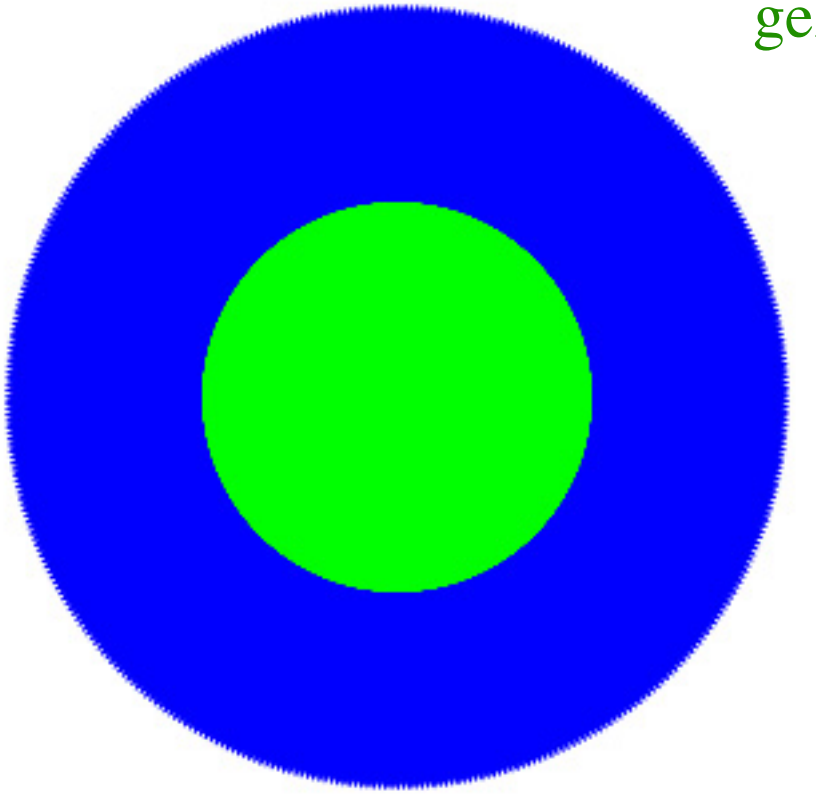


2. Sonderfall:

a konstant

$\gamma = 1.22$, irrational zu 2π

1000 grüne Dreiecke
aber nur
Nr. 21 bis 1000
gezeichnet



2. Sonderfall:

a konstant