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# Pythagorean Triangles

## 1 What about?

We give a visualization of the usual parameterization of the Pythagorean triangles.

## 2 Parameterization of the Pythagorean triangles

Consider integers $m > n > 0$, $m$ and $n$ relatively prime and with opposite parity.

Set $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$. The numbers $a$, $b$, and $c$ are a primitive Pythagorean triple and form a primitive Pythagorean triangle. Any primitive Pythagorean triple is given by such a pair $m$ and $n$ (Dickson 1920), (Dickson 1966)) und (Sierpiński 1962).

Table 1 gives the first examples.

| $m$ | $n$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 2 | 1 | 3 | 4 | 5 |
| 3 | 2 | 5 | 12 | 13 |
| 4 | 1 | 15 | 8 | 17 |
| 4 | 3 | 7 | 24 | 25 |
| 5 | 2 | 21 | 20 | 29 |
| 5 | 4 | 9 | 40 | 41 |

**Tab. 1: Pythagorean triples**

## 3 Visualization

The following figures refer to the cases $m = 2$, $n = 1$ (left), and $m = 5$, $n = 2$ (right), but the corresponding figures work for any Pythagorean triangle.

We consider not only the Pythagorean triangle itself (yellow), but also the right triangle with legs $m$ and $n$ (red).

Figure 1 gives the basic structure according to (Foster 2016).

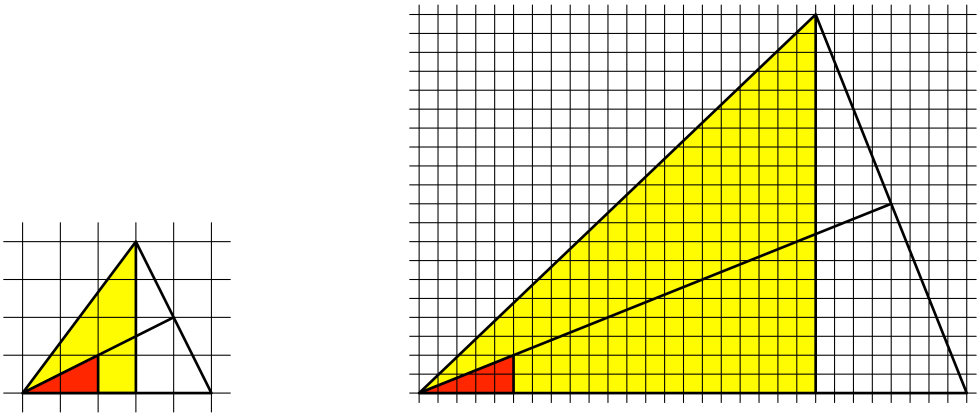

**Fig. 1: Pythagorean yellow triangle and red triangle**

And now a gallery of pictures. The proofs follow from the definitions of $a$, $b$, and $c$. For incircle and excircles compare (Baptist 1982).

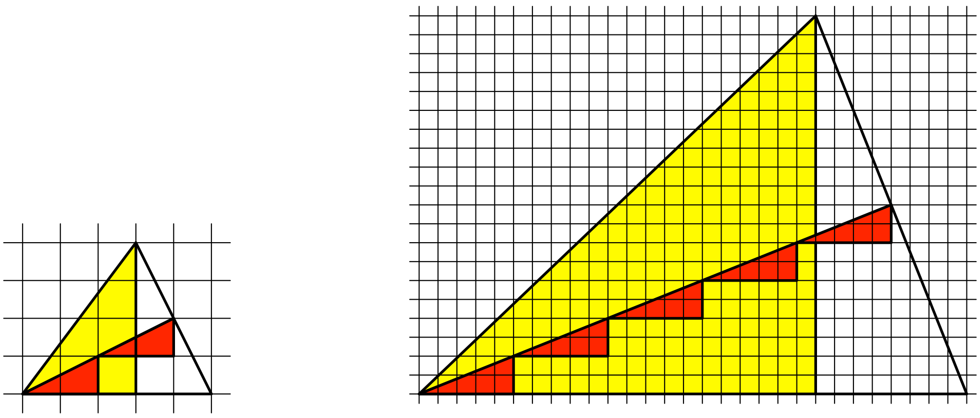

**Fig. 2: *m* red triangles**

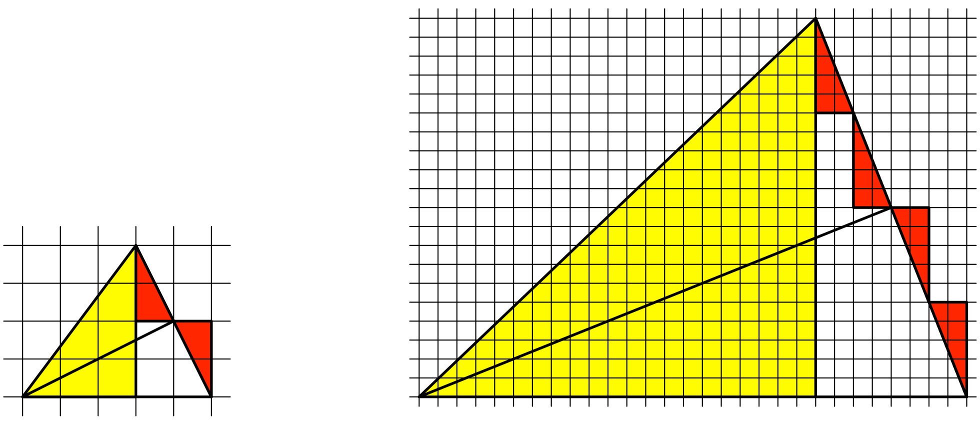

**Fig. 3: To times *n* red triangles**

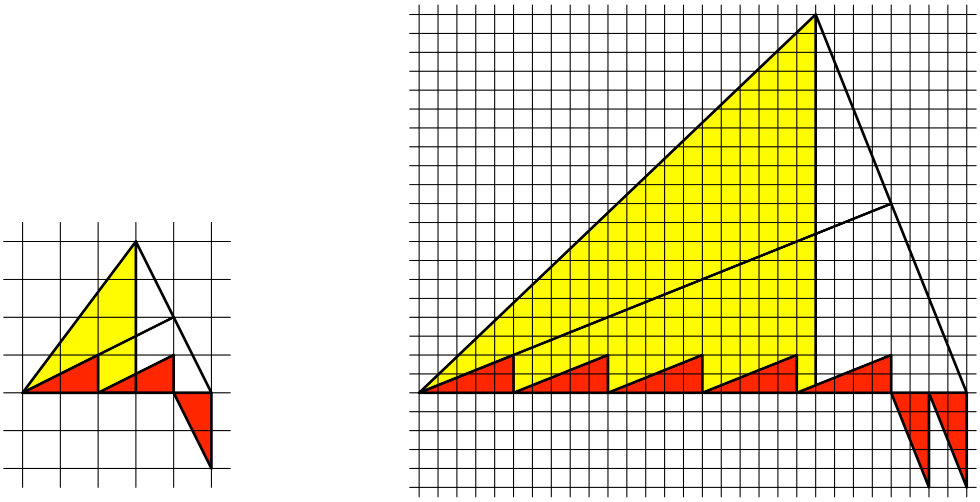

**Fig. 4: *m* red triangles plus *n* red triangles**

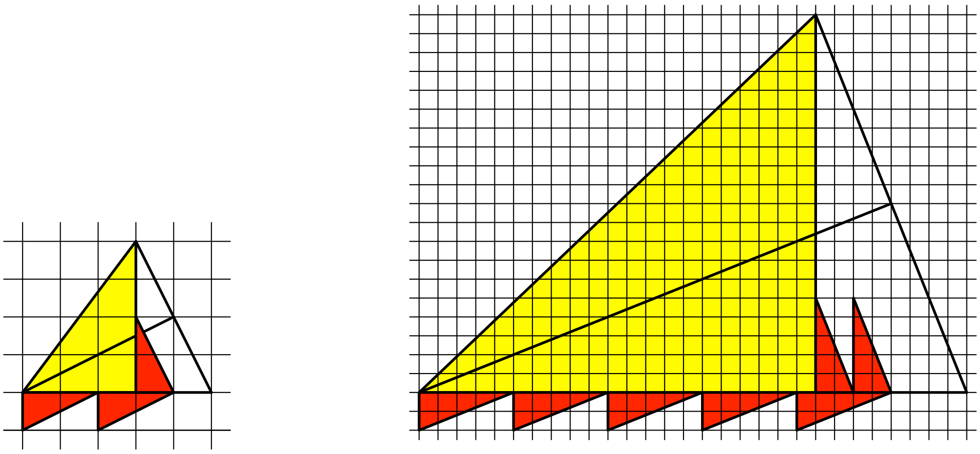

**Fig 5: *m* red triangles minus *n* red triangles**

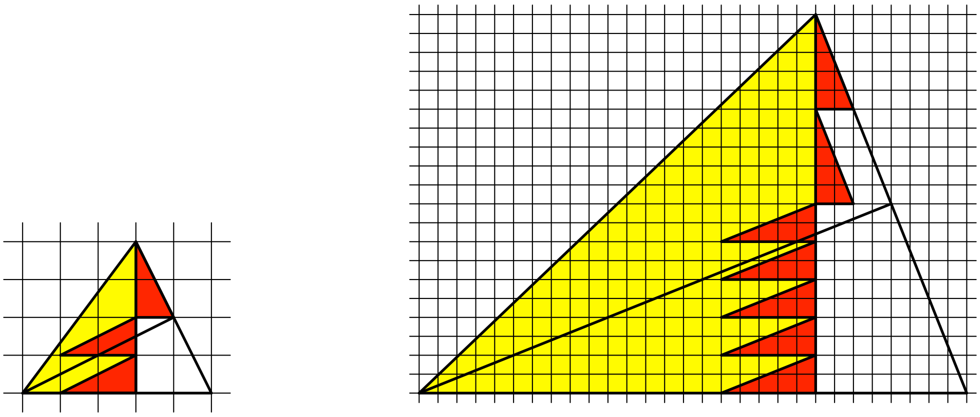

**Fig. 6: *m* red triangles plus *n* red triangles**

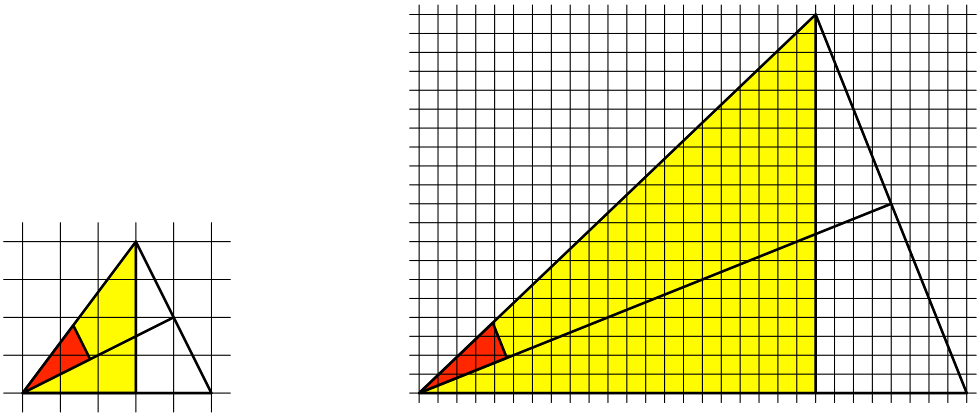

**Fig. 7: Rotated red triangle**

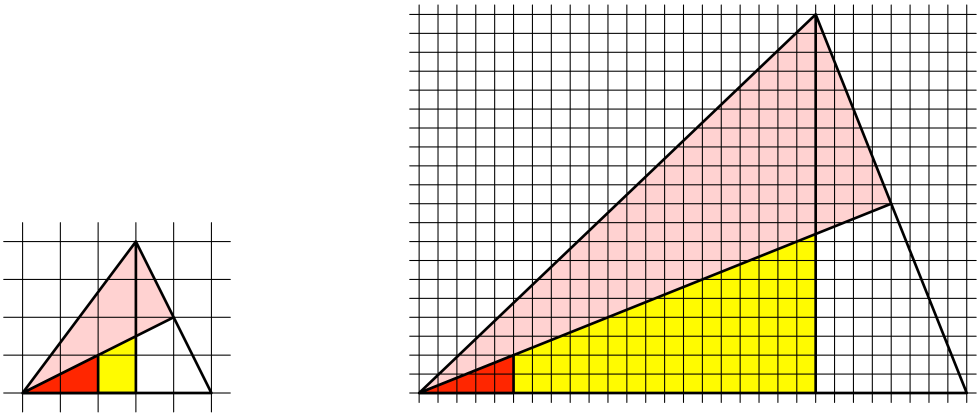

**Fig. 8: Rotated and stretched**

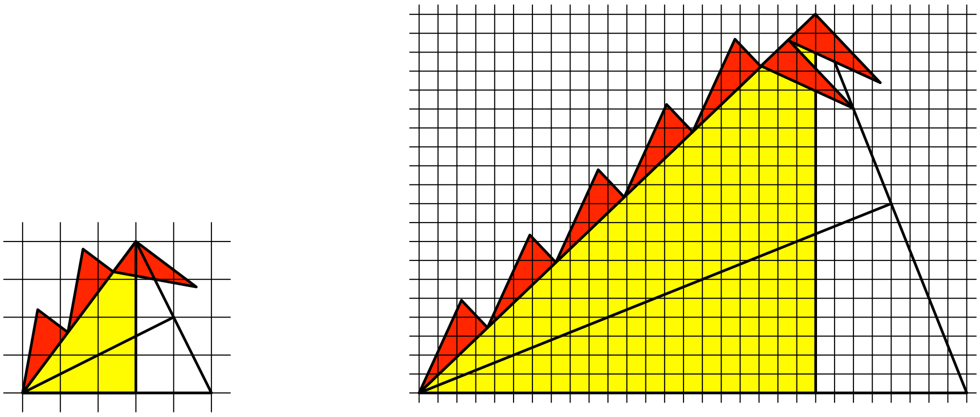

**Fig. 9: *m* rotated plus *n* rotated red triangles**

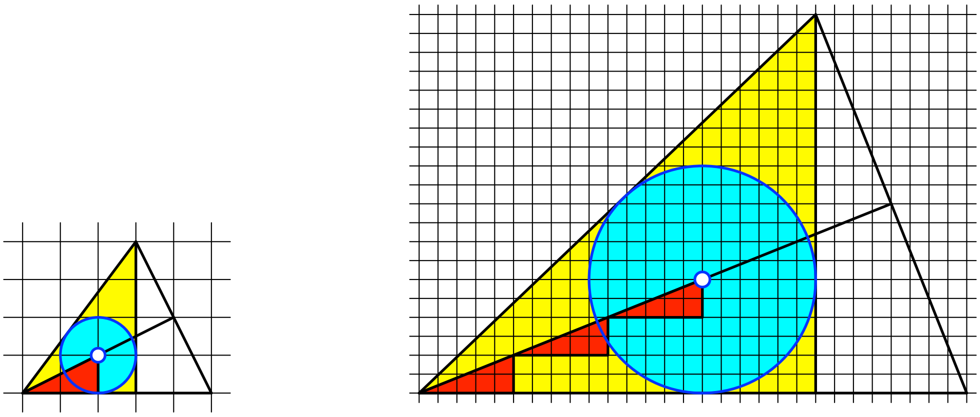

**Fig. 10: Incircle: ($m - n$) red triangles**

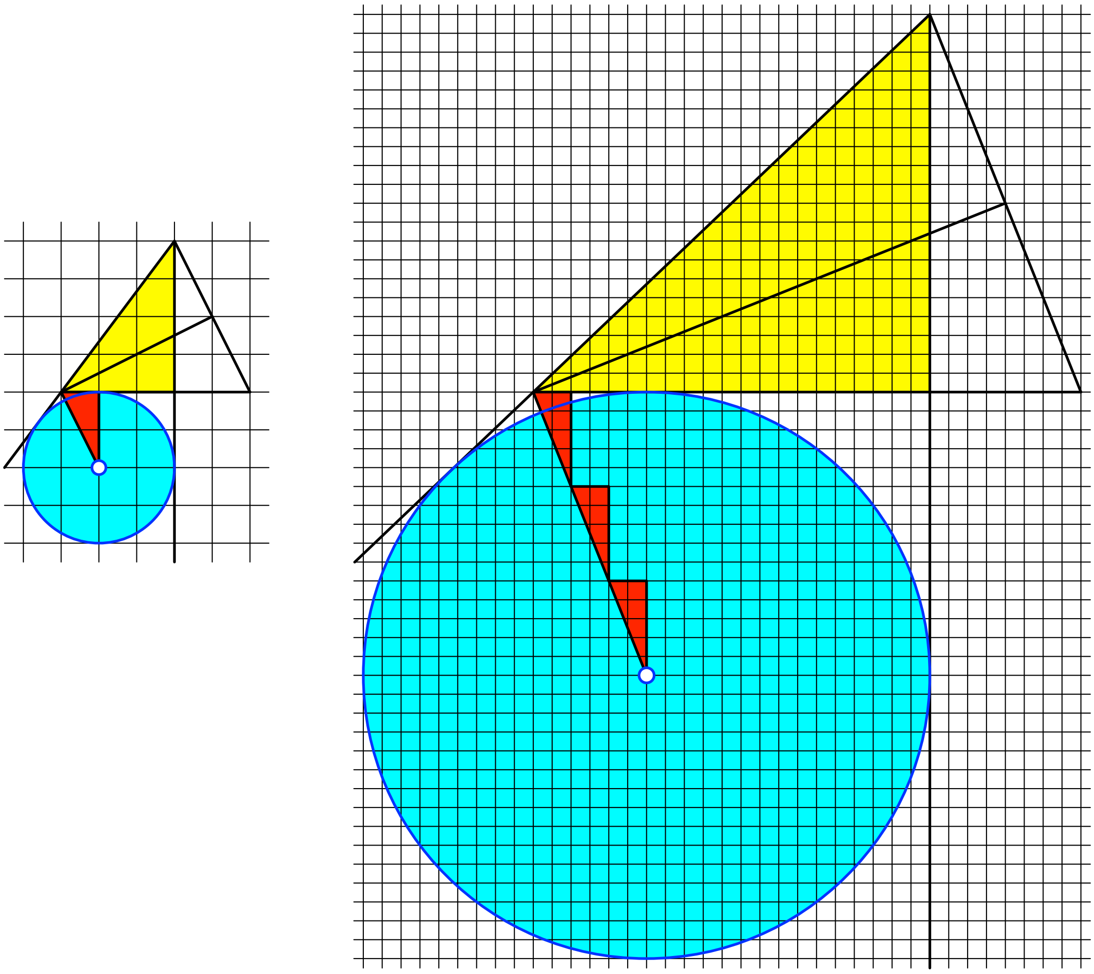

**Fig. 11: Excircle: ($m - n$) red triangles**

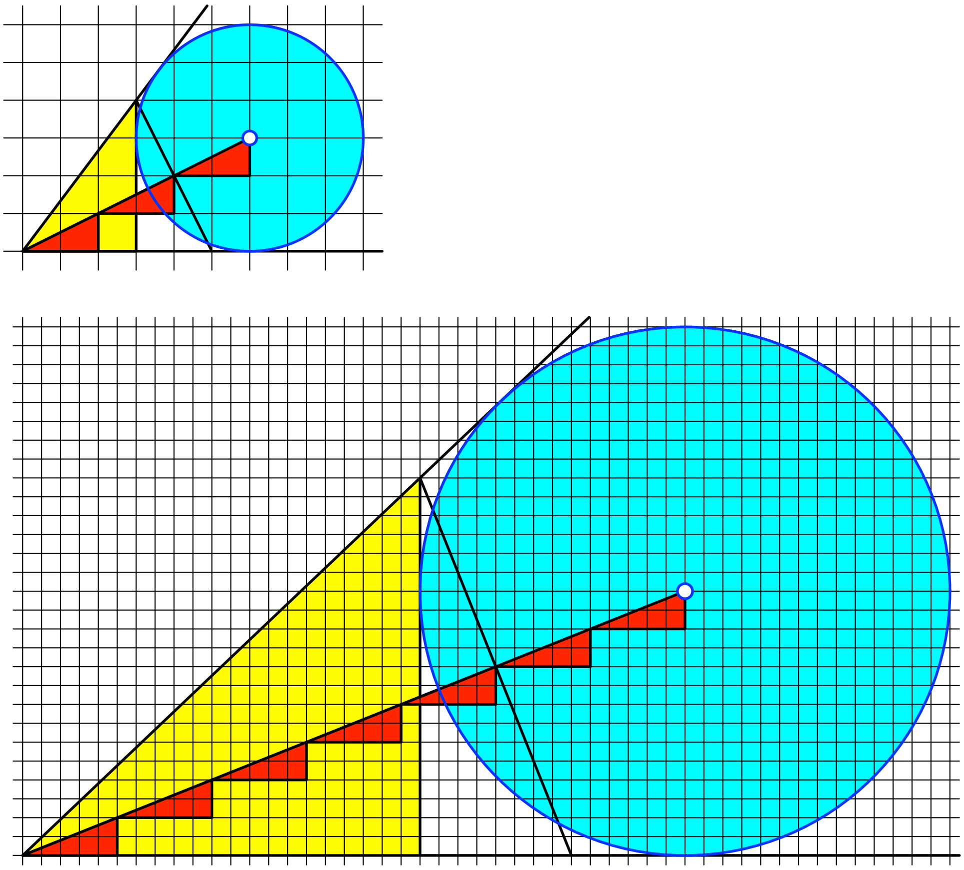

**Fig. 12: Excircle: (*m* + *n*) red triangles**

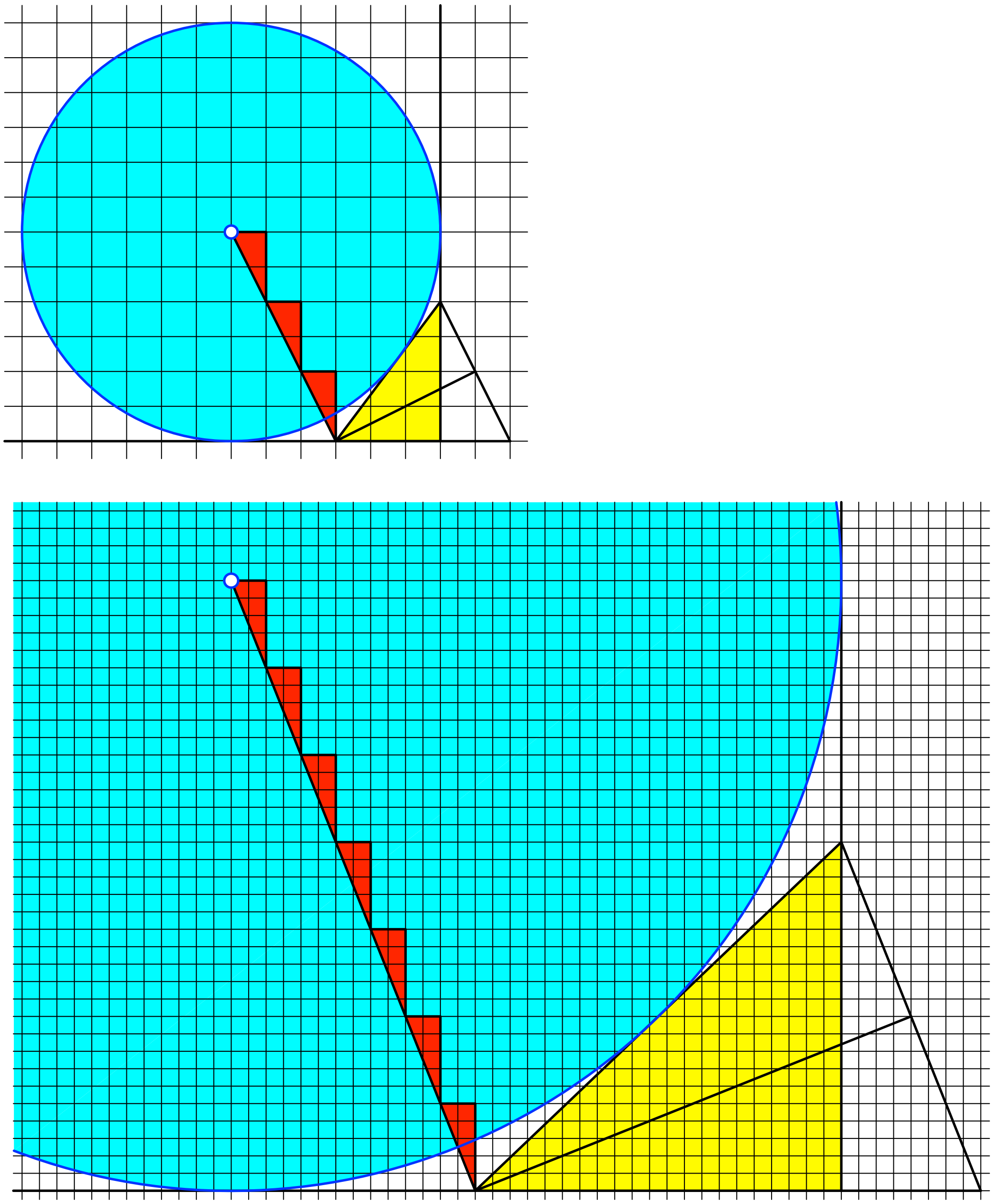

**Fig. 13: Excircle: ($m + n$) red triangles**

## References


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